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INTERNATIONAL A LEVEL MATHEMATICS • PURE 1 (WMA11)

Pure 1 Practice Paper 3

Worked Solutions

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA11/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	Hidden quadratics in indices	8	2
2	A right angle on a given line	9	3
3	Cosine rule with two positions	9	4
4	Normal meeting a curve again	10	6
5	Integrating twice	9	8
6	Exact radian geometry	10	9
7	A cubic in terms of a	10	11
8	Reciprocal inequality	10	12

Question paper & mark scheme | Exam questions by topic | More practice papers

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Solve the equation

$$3^{2x+1} + 9 = 28 \times 3^x$$

(4)

(b) Find all the real values of x for which

$$x^{\frac{4}{3}} - 10x^{\frac{2}{3}} + 9 = 0$$

(4)

(a) $3^{2x+1} = 3 \times (3^x)^2$. Let $u = 3^x$: $3u^2 + 9 = 28u$ M1

$3u^2 - 28u + 9 = 0 \Rightarrow (3u - 1)(u - 9) = 0$ M1

$u = \frac{1}{3}$ or $u = 9$, so $3^x = 3^{-1}$ or $3^x = 3^2$ A1

$x = -1$ or $x = 2$ A1

(b) $x^{4/3} = (x^{2/3})^2$. Let $u = x^{2/3}$: $u^2 - 10u + 9 = 0$ M1

$(u - 1)(u - 9) = 0 \Rightarrow u = 1$ or $u = 9$ A1

$x^{2/3} = 9$ means $(x^{1/3})^2 = 9$, so $x^{1/3} = 3$ or $-3 \Rightarrow x = 27$ or -27 M1

$x^{2/3} = 1$ gives $x^{1/3} = \pm 1 \Rightarrow x = \pm 1$. Negative cube roots are allowed.

$x = -27, -1, 1, 27$ A1

Check: $(-27)^{2/3} = (-3)^2 = 9$ ✓

(Total for Question 1 is 8 marks)

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The points A and B have coordinates (1, 0) and (9, 6) respectively.

The point C lies on the line with equation $y = 2x + 3$ and angle ACB is a right angle.

(a) Find the coordinates of the two possible positions of C.

(6)

(b) Find the area of the larger of the two possible triangles ABC.

(3)

(a) C is on $y = 2x + 3$, so write C as $(t, 2t + 3)$. M1

AC is perpendicular to BC, so the product of their gradients is -1:

$\frac{2t + 3 - 0}{t - 1} \times \frac{2t + 3 - 6}{t - 9} = -1$ M1

$(2t + 3)(2t - 3) = -(t - 1)(t - 9) \Rightarrow 4t^2 - 9 + t^2 - 10t + 9 = 0$ M1

$5t^2 - 10t = 0$ A1

$5t(t - 2) = 0 \Rightarrow t = 0$ or $t = 2$ dM1

$\boxed{C(0, 3)}$ or $\boxed{C(2, 7)}$ A1

(b) The right angle is at C, so area $= \frac{1}{2} \times AC \times BC$.

C(2, 7): $AC = \sqrt{1^2 + 7^2} = \sqrt{50}$, $BC = \sqrt{7^2 + 1^2} = \sqrt{50}$, area $= \frac{1}{2} \times 50 = 25$ M1

C(0, 3): $AC = \sqrt{1 + 9} = \sqrt{10}$, $BC = \sqrt{81 + 9} = \sqrt{90}$, area $= \frac{1}{2}\sqrt{900} = 15$ M1

Larger area = $\boxed{25}$ A1

(Total for Question 2 is 9 marks)

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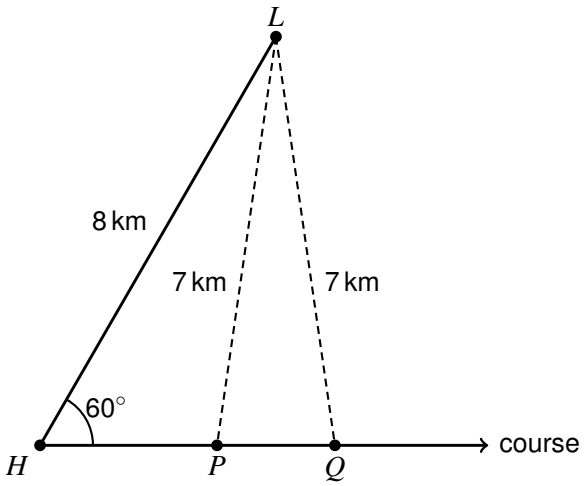

Diagram not drawn to scale

Figure 1

A boat leaves a harbour H and sails in a straight line. A lighthouse L is 8 km from H, and the course of the boat makes an angle of 60° with the line HL, as shown in Figure 1.

The boat is exactly 7 km from the lighthouse at two points on its course, P and Q, where P is closer to H.

In parts (a) and (b) you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

(a) Find the distances HP and HQ.

(4)

(b) Find the exact area of triangle LPQ.

(3)

(c) Find the size of angle HPL, giving your answer in degrees to one decimal place.

(2)

(a) Let HP = d. Cosine rule in triangle HLP, with the 60° angle at H:

$7^2 = 8^2 + d^2 - 2 \times 8 \times d \times \cos 60°$ M1

$49 = 64 + d^2 - 8d \Rightarrow d^2 - 8d + 15 = 0$ A1

$(d - 3)(d - 5) = 0 \Rightarrow d = 3$ or $d = 5$ M1

$\boxed{HP = 3 \text{ km}}$, $\boxed{HQ = 5 \text{ km}}$ A1

(b) PQ = 5 − 3 = 2 km, the base of triangle LPQ. B1

Height = distance from L to the course = $8 \sin 60° = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3}$ M1

Area = $\frac{1}{2} \times 2 \times 4\sqrt{3} = \boxed{4\sqrt{3} \text{ km}^2}$ A1

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Question 3 continued

(c) Cosine rule in triangle HPL: $\cos HPL = \frac{3^2 + 7^2 - 8^2}{2 \times 3 \times 7} = \frac{-6}{42} = -\frac{1}{7}$ M1

$HPL = \cos^{-1}(-\frac{1}{7}) = \boxed{98.2°}$ (obtuse, as the diagram shows) A1

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(Total for Question 3 is 9 marks)

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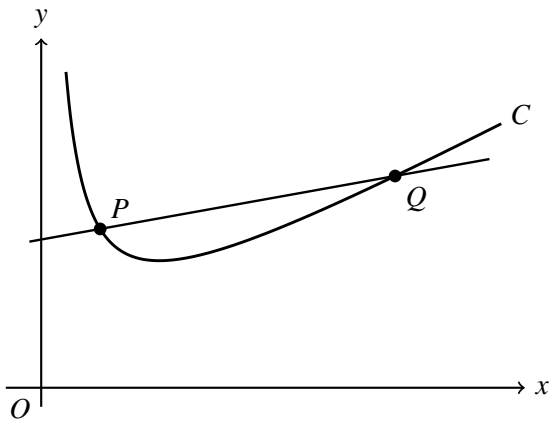

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = x + \frac{k}{x} \qquad x > 0$$

where k is a constant.

The point P on C has x coordinate 1. The gradient of C at P is -3

(a) Find the value of k.

(2)

The normal to C at P meets C again at the point Q, as shown in Figure 2.

(b) Find the exact coordinates of Q.

(6)

(c) Find the exact length of PQ, giving your answer as a simplified surd.

(2)

(a) $y = x + kx^{-1}$, so $\frac{dy}{dx} = 1 - kx^{-2} = 1 - \frac{k}{x^2}$ M1

At $x = 1$: $1 - k = -3 \Rightarrow \boxed{k = 4}$ A1

(b) At $x = 1$: $y = 1 + 4 = 5$, so $P(1, 5)$ B1

The tangent gradient is -3, so the normal gradient is $\frac{1}{3}$: $y - 5 = \frac{1}{3}(x - 1)$ M1

Normal meets C: $x + \frac{4}{x} = 5 + \frac{x-1}{3}$. Multiply by $3x$: $3x^2 + 12 = 15x + x^2 - x$ M1

$2x^2 - 14x + 12 = 0 \Rightarrow x^2 - 7x + 6 = 0 \Rightarrow (x - 1)(x - 6) = 0$ A1

$x = 1$ is P itself, so Q has $x = 6$ dM1

$y = 6 + \frac{4}{6} = \frac{20}{3}$, so $\boxed{Q(6, \frac{20}{3})}$ A1

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Question 4 continued

(c) $PQ^2 = (6-1)^2 + \left(\frac{20}{3} - 5\right)^2 = 25 + \frac{25}{9} = \frac{250}{9}$ M1

$PQ = \frac{\sqrt{250}}{3} = \boxed{\frac{5\sqrt{10}}{3}}$ A1

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(Total for Question 4 is 10 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$, where

$$f''(x) = \frac{3}{\sqrt{x}} + 2$$

The point $P(1, 2)$ lies on C.

The normal to C at P passes through the point $(5, 1)$.

(a) Find $f(x)$.

(7)

(b) Show that P is the only point on C at which the gradient of C is equal to the gradient at P.

(2)

(a) $f''(x) = 3x^{-1/2} + 2$. Integrate (add 1 to the power, divide by the new power): M1

$f'(x) = 3x^{1/2} \div \frac{1}{2} + 2x + A = 6x^{1/2} + 2x + A$ A1

Normal through $P(1, 2)$ and $(5, 1)$: gradient $= \frac{1-2}{5-1} = -\frac{1}{4}$, so the tangent gradient is 4 B1

$f'(1) = 4$: $6 + 2 + A = 4 \Rightarrow A = -4$ M1

Integrate again: $f(x) = 6x^{3/2} \div \frac{3}{2} + x^2 - 4x + B = 4x^{3/2} + x^2 - 4x + B$ M1 A1

$P(1, 2)$: $4 + 1 - 4 + B = 2 \Rightarrow B = 1$, so $\boxed{f(x) = 4x^{3/2} + x^2 - 4x + 1}$ A1

(b) Gradient 4: $6\sqrt{x} + 2x - 4 = 4 \Rightarrow x + 3\sqrt{x} - 4 = 0$, a quadratic in $\sqrt{x}$ M1

$(\sqrt{x} + 4)(\sqrt{x} - 1) = 0$. $\sqrt{x} = -4$ is impossible, so $\sqrt{x} = 1$ and $x = 1$: only at P ✓ A1

(Total for Question 5 is 9 marks)

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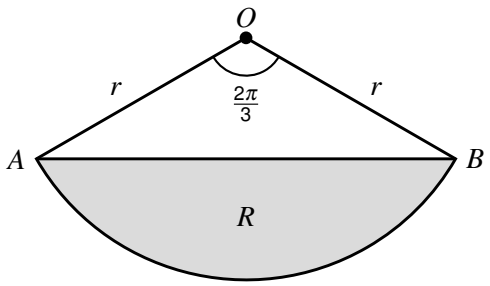

Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sector OAB of a circle with centre O and radius r cm.

The angle AOB is $\frac{2\pi}{3}$ radians.

The region R, shown shaded, is bounded by the arc AB and the chord AB.

The length of the chord AB is $(r+2)$ cm.

(a) Find the exact value of r, giving your answer in the form $a+\sqrt{b}$, where a and b are integers.

(4)

(b) Find the exact perimeter of R.

(3)

(c) Find the exact area of R, giving your answer in the form $(p+q\sqrt{3})\pi+s+t\sqrt{3}$, where p, q, s and t are rational numbers.

(3)

(a) Cosine rule in triangle AOB: $AB^2 = r^2 + r^2 - 2r^2\cos\frac{2\pi}{3}$ M1

$\cos\frac{2\pi}{3} = -\frac{1}{2}$, so $AB^2 = 3r^2$. But $AB = r + 2$: $(r+2)^2 = 3r^2$

$r^2 + 4r + 4 = 3r^2 \Rightarrow 2r^2 - 4r - 4 = 0 \Rightarrow r^2 - 2r - 2 = 0$ A1

$r = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$ M1

$r > 0$, so reject $1 - \sqrt{3}$: $\boxed{r = 1 + \sqrt{3}}$ A1

(b) Arc AB $= r\theta = \frac{2\pi}{3}(1+\sqrt{3})$ M1

Chord AB $= r + 2 = 3 + \sqrt{3}$ M1

Perimeter of R $= \boxed{\frac{2\pi}{3}(1+\sqrt{3}) + 3 + \sqrt{3}}$ A1

(c) Area of R $= \frac{1}{2}r^2(\theta - \sin\theta)$, where $r^2 = (1+\sqrt{3})^2 = 4 + 2\sqrt{3}$ M1

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Question 6 continued

$= (2 + \sqrt{3})(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}) = \frac{4\pi}{3} + \frac{2\sqrt{3}\pi}{3} - \sqrt{3} - \frac{3}{2}$ M1

$(\frac{4}{3} + \frac{2}{3}\sqrt{3})\pi - \frac{3}{2} - \sqrt{3}$ A1

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(Total for Question 6 is 10 marks)

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7.

$$f(x) = x(x-a)^2$$

where a is a positive constant.

(a) Sketch the curve with equation $y = f(x)$, showing the coordinates of the points where it meets the coordinate axes.

(2)

(b) On the same diagram, sketch the curve with equation $y = f(x+a)$, showing the coordinates of the points where it meets the coordinate axes.

(2)

In this question you must show all stages of your working.

(c) Find, in terms of a, the coordinates of the points where the two curves meet.

(3)

The curves meet at a point, other than the origin, which lies on the line $y = x$

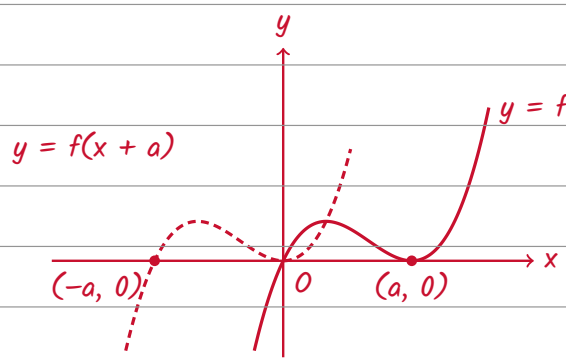
(d) Find the value of a.

(3)

(a) $f(x) = x(x-a)^2$: crosses the axis at 0 and touches it at $(a, 0)$. Positive cubic.

(b) $f(x+a) = (x+a)x^2$: crosses at $(-a, 0)$, touches at 0. Same shape, a to the left.

B1 B1 B1 B1



(c) The curves meet where $x(x-a)^2 = x^2(x+a)$ M1

$x[(x-a)^2 - x(x+a)] = 0 \Rightarrow x[x^2 - 2ax + a^2 - x^2 - ax] = 0 \Rightarrow x(a^2 - 3ax) = 0$ M1

$ax(a - 3x) = 0$: $x = 0$ or $x = \frac{a}{3}$. $f(\frac{a}{3}) = \frac{a}{3} \times (-\frac{2a}{3})^2 = \frac{4a^3}{27}$

$(0, 0)$ and $(\frac{a}{3}, \frac{4a^3}{27})$ A1

(d) The point is on $y = x$, so its coordinates are equal: $\frac{4a^3}{27} = \frac{a}{3}$ M1

$a \neq 0$, so divide by a: $12a^2 = 27 \Rightarrow a^2 = \frac{9}{4}$ M1

$a > 0$, so $a = \frac{3}{2}$ A1

(Total for Question 7 is 10 marks)

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8. The curve C has equation $y = \frac{2}{x}$, $x \neq 0$, and the line l has equation $y = 2x - 1$

(a) On the same axes, sketch C and l. Show the coordinates of the points where l meets the coordinate axes and state the equations of the asymptotes of C.

(3)

In this question you must show all stages of your working.

(b) Find the exact coordinates of the points where C and l meet.

(4)

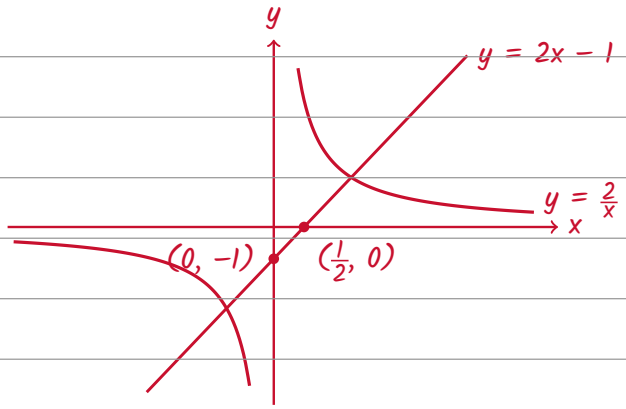
(c) Hence find the set of values of x for which

$$\frac{2}{x} > 2x - 1$$

(3)

(a) C: $y = \frac{2}{x}$ has asymptotes $x = 0$ and $y = 0$, with branches in the 1st and 3rd quadrants.

l: $y = 2x - 1$ meets the axes at $(\frac{1}{2}, 0)$ and $(0, -1)$.



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(b) $\frac{2}{x} = 2x - 1 \Rightarrow 2 = 2x^2 - x \Rightarrow 2x^2 - x - 2 = 0$ M1

$x = \frac{1 \pm \sqrt{1 + 16}}{4} = \frac{1 \pm \sqrt{17}}{4}$ M1 A1

$y = 2x - 1 = \frac{1 \pm \sqrt{17}}{2} - 1 = \frac{-1 \pm \sqrt{17}}{2}$

$\left(\frac{1 + \sqrt{17}}{4}, \frac{-1 + \sqrt{17}}{2}\right)$ and $\left(\frac{1 - \sqrt{17}}{4}, \frac{-1 - \sqrt{17}}{2}\right)$ A1

(c) Do NOT multiply by x: x can be negative, which would flip the inequality.

From the sketch, C is above l to the left of the lower crossing point, M1

and between $x = 0$ and the upper crossing point.

$x < \frac{1 - \sqrt{17}}{4}$ or $0 < x < \frac{1 + \sqrt{17}}{4}$ A1 A1

(Check: multiplying by x^2, which is always positive, gives $x(2x^2 - x - 2) < 0$:

negative left of the smallest root and between the other two, the same result.)

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Question 8 continued

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(Total for Question 8 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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