Algebraic Expressions

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 1

Question 1 — Worked Solution

(i) Find the value of *a*

Given:

$$27^{\frac{1}{3}} \times 3^{a} = 81$$

Rewrite 27 and 81 as powers of 3:

$$27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^1 = 3 \qquad \text{and} \qquad 81 = 3^4$$

So:

$$3 \times 3^a = 3^4 \;\Rightarrow\; 3^{1+a} = 3^4$$

Equating exponents: $1 + a = 4$

$$\therefore a = 3$$

(ii) Show that

$$\frac{8}{\sqrt{8} - 2} = 4\sqrt{2} + 4$$

Rationalise by multiplying numerator and denominator by ($\sqrt{8}$ + 2):

$$\frac{8(\sqrt{8} + 2)}{(\sqrt{8})^2 - 2^2} = \frac{8(\sqrt{8} + 2)}{8 - 4} = \frac{8(\sqrt{8} + 2)}{4} = 2(\sqrt{8} + 2)$$

Simplify $\sqrt{8} = 2\sqrt{2}$:

$$= 2(2\sqrt{2} + 2) = 4\sqrt{2} + 4 \quad \checkmark$$

$$\therefore \frac{8}{\sqrt{8} - 2} = 4\sqrt{2} + 4 \quad \checkmark$$

Question 2 — Worked Solution

Given:

$$y = \frac{1}{64}x^6$$

(a) $y^{1/3}$

$$y^{\frac{1}{3}} = \left(\frac{1}{64}x^6\right)^{\frac{1}{3}} = \left(\frac{1}{64}\right)^{\frac{1}{3}}x^2 = \frac{1}{4}x^2$$

$$\therefore y^{\frac{1}{3}} = \frac{1}{4}x^2$$

(b) $4y^{-1}$

$$4y^{-1} = 4 \times \left(\frac{1}{64}x^6\right)^{-1} = 4 \times 64\,x^{-6}$$

$$\therefore 4y^{-1} = 256x^{-6}$$

(c) √(64y)

$$\sqrt{64y} = \left(64 \cdot \frac{1}{64}x^6\right)^{\frac{1}{2}} = (x^6)^{\frac{1}{2}}$$

$$\therefore \sqrt{64y} = x^3$$

Question 3 — Worked Solution

(i) Show that

$$\frac{12}{3\sqrt{2}-\sqrt{6}} = \sqrt{2}\,(3+\sqrt{3})$$

Rationalise by multiplying by (3√2 + √6) / (3√2 + √6):

$$\frac{12(3\sqrt{2}+\sqrt{6})}{(3\sqrt{2})^2-(\sqrt{6})^2} = \frac{12(3\sqrt{2}+\sqrt{6})}{18-6} = \frac{12(3\sqrt{2}+\sqrt{6})}{12}$$

$$= 3\sqrt{2}+\sqrt{6} = 3\sqrt{2}+\sqrt{2}\cdot\sqrt{3} = \sqrt{2}(3+\sqrt{3}) \quad ✓$$

$$\therefore \frac{12}{3\sqrt{2}-\sqrt{6}} = \sqrt{2}\,(3+\sqrt{3}) \quad ✓$$

(ii) Show that

$$\sqrt{108}+\sqrt{84}\times\sqrt{7}-\frac{12}{\sqrt{3}} = 16\sqrt{3}$$

Simplify each term:

$$\sqrt{108} = 6\sqrt{3}, \quad \sqrt{84}\times\sqrt{7} = \sqrt{588} = 14\sqrt{3}, \quad \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

$$6\sqrt{3}+14\sqrt{3}-4\sqrt{3} = 16\sqrt{3} \quad ✓$$

Question 4 — Worked Solution

Solve:

$$x\sqrt{48} + 20 = \frac{10x}{\sqrt{3}}$$

Simplify: √48 = 4√3. Rationalise RHS:

$$\frac{10x}{\sqrt{3}} = \frac{10x\sqrt{3}}{3}$$

Equation becomes:

$$4x\sqrt{3} + 20 = \frac{10x\sqrt{3}}{3}$$

Multiply through by 3:

$$12x\sqrt{3} + 60 = 10x\sqrt{3}$$

$$60 = -2x\sqrt{3} \Rightarrow x = \frac{-30}{\sqrt{3}} = \frac{-30\sqrt{3}}{3}$$

$$\therefore x = -10\sqrt{3}$$

Question 5 — Worked Solution

(i) Solve $4^{x+2} = 8^{2x}$

Rewrite both sides in base 2:

$$(2^2)^{x+2} = (2^3)^{2x} \Rightarrow 2^{2x+4} = 2^{6x}$$

Equate exponents: $2x + 4 = 6x \rightarrow 4 = 4x$

$$\therefore x = 1$$

(ii)(a) Express 5√18 – √50 in the form k√2

$$5\sqrt{18} - \sqrt{50} = 5 \cdot 3\sqrt{2} - 5\sqrt{2} = 15\sqrt{2} - 5\sqrt{2} = 10\sqrt{2}$$

$$\therefore k = 10$$

(ii)(b) Solve 5√18 – √50 = √n

$$10\sqrt{2} = \sqrt{n} \Rightarrow n = (10\sqrt{2})^2 = 200$$

$$\therefore n = 200$$

Question 6 — Worked Solution

(a) Find *a* and *b*

Rewrite with base 2:

$$\frac{16^x}{4^{x-1}} = \frac{2^{4x}}{2^{2x-2}} = 2^{4x-(2x-2)} = 2^{2x+2}$$

$$\therefore a = 2, \quad b = 2$$

(b) Solve

$$\frac{16^x}{4^{x-1}} = 64$$

Using (a):

$$2^{2x+2} = 64 = 2^6 \Rightarrow 2x + 2 = 6 \Rightarrow x = 2$$

$$\therefore x = 2$$

Question 7 — Worked Solution

(a) Write 8/(√6 – √2) in the form a√6 + b√2

Rationalise by multiplying by (√6 + √2)/(√6 + √2):

$$\frac{8(\sqrt{6}+\sqrt{2})}{(\sqrt{6})^2-(\sqrt{2})^2}=\frac{8(\sqrt{6}+\sqrt{2})}{4}=2\sqrt{6}+2\sqrt{2}$$

$$\therefore a=2,\quad b=2$$

(b) Solve √6 x = √2 x + 8

Collect x terms:

$$x(\sqrt{6}-\sqrt{2})=8 \Rightarrow x=\frac{8}{\sqrt{6}-\sqrt{2}}$$

Using (a):

$$x=2\sqrt{6}+2\sqrt{2}=2(\sqrt{6}+\sqrt{2})$$

$$\therefore x=2(\sqrt{6}+\sqrt{2})$$

Question 8 — Worked Solution

Given:

$$y = \frac{9x^4}{16}$$

(a) $y^{-1/2}$

$$y^{-\frac{1}{2}} = \left(\frac{9x^4}{16}\right)^{-\frac{1}{2}} = \left(\frac{16}{9}\right)^{\frac{1}{2}} x^{-2} = \frac{4}{3}x^{-2}$$

$$\therefore y^{-\frac{1}{2}} = \frac{4}{3}x^{-2}$$

(b) $(16y)^{3/2}$

$$(16y)^{\frac{3}{2}} = \left(16 \cdot \frac{9x^4}{16}\right)^{\frac{3}{2}} = (9x^4)^{\frac{3}{2}} = 9^{\frac{3}{2}}x^6 = 27x^6$$

$$\therefore (16y)^{\frac{3}{2}} = 27x^6$$

Question 9 — Worked Solution

(a)

$$(x^8)^{\frac{1}{4}} = x^{8 \times \frac{1}{4}} = x^2$$

$$\therefore x^2$$

(b)

$$\sqrt{3}\,(x^4) \div \sqrt{\frac{27}{x^2}} = 3^{\frac{1}{2}}\,x^4 \div \frac{3^{\frac{3}{2}}}{x} = 3^{\frac{1}{2}-\frac{3}{2}}\,x^5 = 3^{-1}x^5$$

$$\therefore \frac{1}{3}x^5$$

Question 10 — Worked Solution

(a)

$$4^{2x+3} = (2^2)^{2x+3} = 2^{4x+6}$$

$$\therefore a = 4x + 6$$

(b) Solve

$$2^x \times 4^{2x+3} = 8^{2x}$$

Rewrite in base 2:

$$2^x \times 2^{4x+6} = 2^{6x} \Rightarrow 2^{5x+6} = 2^{6x}$$

Equate exponents: 5x + 6 = 6x

$$\therefore x = 6$$