eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 2

Mechanics 2

Chapter 3: Centres of Mass

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WME02/01) • Calculator allowed • Take $g = 9.8\,\text{m}\,\text{s}^{-2}$

Multi-step exam questions at the difficulty of the real WME02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Projectiles** Horizontal projection, projection at any angle, the equation of the path	10	84	separate file	
2	**Variable Acceleration** Calculus for motion in a line and in $\mathbf{i}, \mathbf{j}$ form	10	85	separate file	
3	**Centres of Mass** Particles, laminas, frameworks, suspended and tilting bodies	10	81	2	
4	**Work and Energy** Work done, kinetic and potential energy, the work–energy principle, power	10	82	separate file	
5	**Impulses and Collisions** Impulse as a vector, Newton's law of restitution, loss of kinetic energy	10	82	separate file	
6	**Statics of Rigid Bodies** Rods and ladders in equilibrium, friction, hinge forces	10	81	separate file	

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CHAPTER 3 • KEY SKILLS

3 Centres of Mass

Specification M2: Centres of mass • 10 questions • 81 marks • about 97 minutes

1 Particles

$\bar{x} = \frac{\sum mx}{\sum m}$ and $\bar{y} = \frac{\sum my}{\sum m}$, or $\bar{\mathbf{r}} = \frac{\sum m\mathbf{r}}{\sum m}$. Coordinates must be measured from the same origin.

2 Standard laminas

Rectangle: its centre. Triangle: $\frac{2}{3}$ of the way along a median from the vertex. Semicircle: $\frac{4r}{3\pi}$ from the diameter.

3 Composite laminas

Mass $\propto$ area for a uniform lamina. Make a table of masses and distances; a removed part has a negative mass.

4 Frameworks

For uniform rods or wire, mass $\propto$ length, acting at each midpoint. Do not assume rods have equal masses.

5 Different densities

Mass = (mass per unit area) $\times$ area. Attached particles add their own mass at their own point.

6 Freely suspended

The centre of mass hangs vertically below the point of suspension: find the angle from $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

7 Extra forces

Take moments about the point of suspension. Distances are measured perpendicular to each force's line of action.

8 Toppling

On the point of toppling about a corner, the centre of mass is vertically above that corner: $\tan\theta = \frac{\bar{x}}{\bar{y}}$ from that corner.

9 Symmetry

Use an axis of symmetry for one coordinate and moments for the other, and state which you used.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	11
1 Particles	•		•	•				•		
2 Standard laminas		•		•		•	•		•	•
3 Composite laminas		•			•	•	•		•	•
4 Frameworks			•					•		
5 Different densities				•			•	•		
6 Freely suspended				•		•	•	•	•	
7 Extra forces						•	•			
8 Toppling					•					•
9 Symmetry		•			•	•	•		•	
Interleaved: moments and friction (M1)						•	•	•		•
Interleaved: trigonometry and exact values (P1, P2)				•	•	•		•	•	•
Interleaved: straight lines and algebra (P1)	•	•	•					•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WME02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Mark your axes on the diagram and keep every term in terms of a. Use a table of mass ratios and distances. For a hanging body, draw the line from the point of suspension to the centre of mass and say which angle you have found.

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1. Three particles of masses 2 kg, 3 kg and 5 kg are placed at the points with coordinates $(1, 2)$, $(4, -1)$ and (p, q) respectively. The centre of mass of the three particles is at the point $(3, 1)$.

(a) Find the value of p and the value of q.

(4)

A fourth particle, of mass M kg, is placed at the origin.

(b) Show that, for every value of M, the centre of mass of the four particles lies on the line $x = 3y$

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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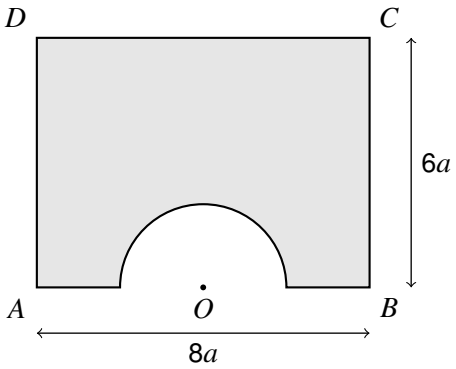


Figure 1

A uniform lamina is formed by removing a semicircle of radius $2a$ from a rectangle $ABCD$, where $AB = 8a$ and $BC = 6a$. The diameter of the semicircle lies along AB and its centre is the midpoint O of AB, as shown in Figure 1.

(a) Show that the centre of mass of the lamina is a distance $\dfrac{208a}{3(24-\pi)}$ from AB.

(5)

(b) State the distance of the centre of mass of the lamina from AD.

(1)

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Question 2 continued

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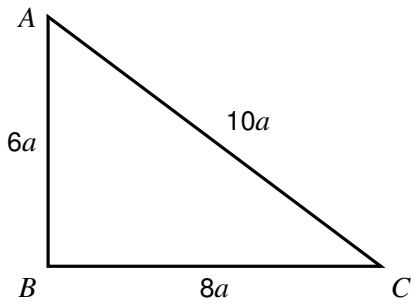


Figure 2

A uniform wire of total mass 12 kg is bent into the shape of a triangle ABC, with $AB = 6a$, $BC = 8a$ and $CA = 10a$, so that angle $ABC = 90^\circ$, as shown in Figure 2.

(a) Find the distance of the centre of mass of the framework from AB and from BC.

(4)

A particle of mass m kg is attached to the framework at A. The centre of mass of the loaded framework is equidistant from AB and BC.

(b) Find the value of m.

(2)

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Question 3 continued

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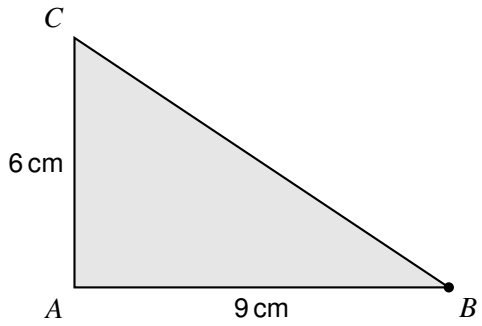


Figure 3

A uniform triangular lamina ABC, of mass $2M$, has $AB = 9$ cm, $AC = 6$ cm and angle $BAC = 90°$. A particle of mass M is attached to the lamina at B, as shown in Figure 3.

(a) Find the distance of the centre of mass of the loaded lamina from AC and from AB.

(4)

The loaded lamina is freely suspended from C and hangs in equilibrium.

(b) Find the angle between CA and the vertical, giving your answer to the nearest 0.1°

(2)

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Question 4 continued

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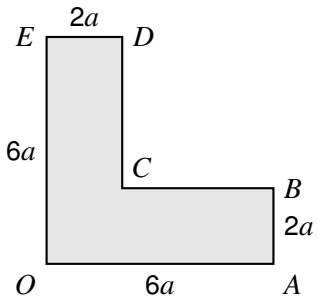


Figure 4

A uniform L-shaped lamina $OABCDE$ has $OA = OE = 6a$ and $AB = DE = 2a$, with all its angles right angles, as shown in Figure 4.

(a) Find the distance of the centre of mass of the lamina from OA and from OE.

(3)

The lamina is placed in a vertical plane with OA along a line of greatest slope of a rough plane inclined at an angle θ to the horizontal, with O below A. The plane is rough enough to prevent sliding.

(b) Find the greatest value of θ for which the lamina does not topple.

(3)

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Question 5 continued

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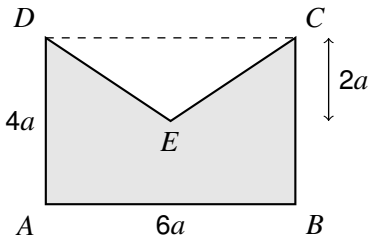


Figure 5

A uniform lamina $ABCED$ is formed by removing the triangle DCE from a rectangle $ABCD$, where $AB = 6a$ and $BC = 4a$. The point E is $2a$ from DC and lies on the perpendicular bisector of DC, as shown in Figure 5. The mass of the lamina is M.

(a) Show that the centre of mass of the lamina is a distance $\frac{14}{9}a$ from AB.

(4)

The lamina is freely suspended from A and hangs in equilibrium.

(b) Find the angle between AB and the vertical, giving your answer to the nearest degree.

(3)

The lamina is now suspended from A by a light string and is held in equilibrium, with AB horizontal and AD vertical, by a horizontal force of magnitude F applied at D, in the plane of the lamina.

(c) Find F in terms of M and g.

(3)

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Question 6 continued

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Question 6 continued

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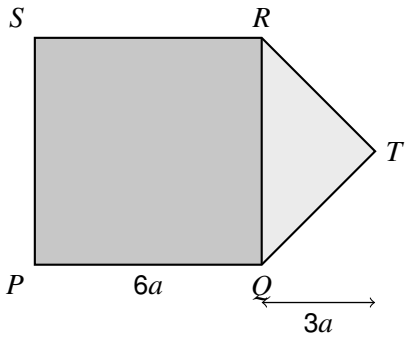


Figure 6

A lamina is formed by joining a square $PQRS$ of side $6a$ to an isosceles triangle QTR along QR, as shown in Figure 6. The perpendicular distance of T from QR is $3a$. The square and the triangle are each uniform, but the mass per unit area of the square is twice the mass per unit area of the triangle. The total mass of the lamina is M.

(a) Show that the centre of mass of the lamina is a distance $\frac{31}{9}a$ from PS.

(4)

The lamina is freely suspended from P and hangs in equilibrium.

(b) Find the angle between PQ and the vertical, giving your answer to the nearest degree.

(3)

The lamina is instead held in a vertical plane, with PQ horizontal and below SR, by two light vertical strings attached at P and at Q.

(c) Find, in terms of M and g, the tension in each string.

(3)

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Question 7 continued

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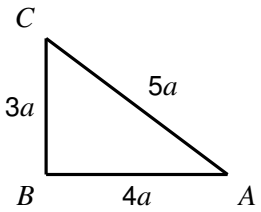


Figure 7

A framework ABC is made from three uniform rods AB, BC and CA, rigidly joined at their ends to form a triangle with $AB = 4a$, $BC = 3a$ and $CA = 5a$, as shown in Figure 7. The masses of AB, BC and CA are $2m$, m and $3m$ respectively.

(a) Show that the centre of mass of the framework is a distance $\frac{5}{3}a$ from BC and a distance a from AB.

(4)

The framework is suspended from the point D on AB by a single vertical string and hangs in equilibrium with AB horizontal.

(b) Find the distance AD.

(2)

A particle of mass km is now attached to the framework at C. The loaded framework is freely suspended from A and hangs in equilibrium with AB at an angle θ to the vertical, where $\tan\theta = \frac{3}{5}$

(c) Find the value of k.

(4)

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Question 8 continued

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Question 8 continued

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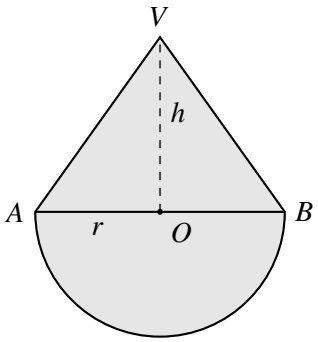


Figure 8

A uniform lamina is formed by joining a semicircle of radius r, with diameter AB and centre O, to an isosceles triangle ABV with $VA = VB$, along their common edge AB. The height of the triangle is h, as shown in Figure 8.

(a) Show that the distance of the centre of mass of the lamina from O, measured towards V, is

$$\frac{2\left(h^2 - 2r^2\right)}{3(2h + \pi r)}$$

(4)

(b) Find h in terms of r when the centre of mass of the lamina is at O.

(2)

(c) For this value of h, explain why the lamina can rest in equilibrium with any point of its curved edge in contact with a horizontal table.

(1)

Given instead that $h = 2r$, the lamina is freely suspended from A and hangs in equilibrium.

(d) Find the angle between AB and the vertical, giving your answer to the nearest 0.1°

(3)

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Question 9 continued

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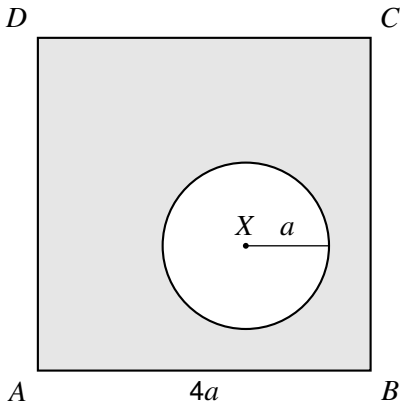


Figure 9

A uniform lamina is formed by cutting a circular hole of radius a from a square $ABCD$ of side $4a$. The centre X of the hole is a distance $2.5a$ from AD and $1.5a$ from AB, as shown in Figure 9.

(a) Show that the centre of mass of the lamina is a distance $\dfrac{(64-5\pi)a}{2(16-\pi)}$ from AD, and find its distance from AB.

(4)

The lamina is placed in a vertical plane with AB along a line of greatest slope of a rough plane inclined at an angle θ to the horizontal, with A below B. The angle θ is slowly increased.

(b) Assuming that the lamina does not slide, find the value of θ at which it is about to topple.

(3)

The coefficient of friction between the lamina and the plane is 0.9

(c) Determine whether the lamina slides or topples first as θ is increased, giving a reason.

(3)

(d) State how you have used the modelling assumption that the lamina is rigid.

(1)

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Question 10 continued

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TOTAL FOR CHAPTER 3 IS 81 MARKS

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