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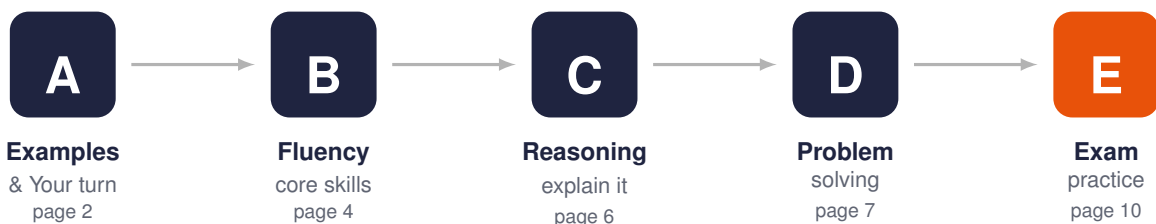
A-LEVEL MATHEMATICS • PURE 4

Vectors

Worksheet

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Free Vectors worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: find vectors, distances and unit vectors in three dimensions • write the vector equation of a line and test points on it • decide whether lines meet, and find the point • use the scalar product for angles and perpendiculars • *Stretch:* skew lines, the foot of a perpendicular, areas, and constants from geometric conditions.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Skew lines, feet of perpendiculars, constants fixed by an angle or a distance, and the shortest distance between two lines.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1: vectors and distance

$A(2, -1, 3)$ and $B(5, 3, -9)$. Find $\vec{AB}$, $|\vec{AB}|$ and a unit vector in the direction of $\vec{AB}$

End minus start: $\vec{AB} = \mathbf{b} - \mathbf{a}$

$$\vec{AB} = 3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}, \quad |\vec{AB}| = \sqrt{9 + 16 + 144} = 13$$

$$\frac{1}{13}(3\mathbf{i} + 4\mathbf{j} - 12\mathbf{k})$$

Your turn A1

$P(1, 4, -2)$ and $Q(-1, 10, 1)$. Find $\vec{PQ}$, $|\vec{PQ}|$ and a unit vector in the direction of $\vec{PQ}$

Worked example A2: the equation of a line

Find a vector equation of the line through $A(1, 2, -1)$ and $B(3, -1, 4)$, and decide whether $C(7, -7, 14)$ lies on it.

$$\mathbf{r} = (\mathbf{i} + 2\mathbf{j} - \mathbf{k}) + \lambda(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$$

$x: 1 + 2\lambda = 7$ gives $\lambda = 3$; check y and z : -7 and 14

yes, C lies on the line ($\lambda = 3$)

Your turn A2

Find a vector equation of the line through $P(0, 3, 2)$ and $Q(2, 1, 5)$, and decide whether $R(6, -3, 10)$ lies on it.

Worked example A3: the scalar product

Find the angle between $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $3\mathbf{i} + 4\mathbf{k}$, and the acute angle between lines in these directions.

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} = \frac{6 + 0 - 8}{3 \times 5} = -\frac{2}{15}$$

$\theta = 97.7^\circ$; lines meet at θ and $180^\circ - \theta$

97.7° ; acute angle 82.3°

Your turn A3

Find the angle between $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $4\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$, and the acute angle between lines in these directions.



STRETCH

Worked example A4: intersecting lines

$l_1: \mathbf{r} = (\mathbf{i} + 2\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$, $l_2: \mathbf{r} = (5\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$. Show that they meet, and find the point.

Solve two components, then check the third.

i: $1 + \lambda = 5 + 2\mu$; **j:** $\lambda = 1 - \mu$; so $\lambda = 2$, $\mu = -1$

k: $2 - 2 = 0$ and $2 - 2 = 0$ ✓

$(3, 2, 0)$

Your turn A4

$l_1: \mathbf{r} = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} - \mathbf{k})$, $l_2: \mathbf{r} = (\mathbf{i} + 5\mathbf{j} - \mathbf{k}) + \mu(-3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$. Show that they meet, and find the point.

Worked example A5: skew lines

Show that $l_1: \mathbf{r} = (\mathbf{i} + 2\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$ and $l_2: \mathbf{r} = (2\mathbf{i} + 3\mathbf{j}) + \mu(\mathbf{j} + \mathbf{k})$ are skew.

Not parallel: the directions are not multiples of each other.

i: $\lambda = 1$; **j:** $\lambda = 3 + \mu$, so $\mu = -2$; **k:** $1 \neq -2$

not parallel, do not meet: skew

Your turn A5

Show that $l_1: \mathbf{r} = (\mathbf{i} + \mathbf{j}) + \lambda(2\mathbf{i} + \mathbf{k})$ and $l_2: \mathbf{r} = (2\mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$ are skew.

Worked example A6: the foot of a perpendicular

Find the foot X of the perpendicular from $P(5, 2, 0)$ to $\mathbf{r} = (\mathbf{i} + \mathbf{j}) + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, and PX

$\overrightarrow{PX} \cdot \mathbf{d} = 0$, with $X = (1 + 2\lambda, 1 + \lambda, 2\lambda)$

$(2\lambda - 4)2 + (\lambda - 1) + (2\lambda)2 = 0$, so $\lambda = 1$

$X(3, 2, 2)$, $PX = 2\sqrt{2}$

Your turn A6

Find the foot X of the perpendicular from $P(5, 3, 5)$ to $\mathbf{r} = (2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, and PX

Worked example A7: dividing a line segment

$A(1, 2, 3)$ and $B(7, -1, 6)$. Find C on AB with $AC : CB = 2 : 1$

$\mathbf{c} = \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a})$

$(1, 2, 3) + \frac{2}{3}(6, -3, 3)$

$C(5, 0, 5)$

Your turn A7

$P(-2, 4, 1)$ and $Q(6, 0, 9)$. Find R on PQ with $PR : RQ = 1 : 3$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$. Find

(a) $\mathbf{a} + 2\mathbf{b}$

(b) $|\mathbf{a}|$

(c) $|\mathbf{a} - \mathbf{b}|$

(d) a unit vector in the direction of $\mathbf{b}$

[4 marks]

B2 $A(3, -2, 5)$ and $B(-1, 2, 7)$. Find

(a) $\overrightarrow{AB}$

(b) the distance AB

(c) the midpoint of AB

(d) the point D with $\overrightarrow{AD} = 3\overrightarrow{AB}$

[4 marks]

B3 (a) Write a vector equation of the line through $(2, 1, 0)$ parallel to $3\mathbf{i} - \mathbf{j} + 4\mathbf{k}$

(b) Find a vector equation of the line l through $A(1, 2, 3)$ and $B(4, 0, 5)$

(c) Show that $(10, -4, 9)$ lies on l

(d) The point $(p, 6, q)$ lies on l . Find p and q

[4 marks]

B4 (a) Find $(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) \cdot (\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$

(b) Show that $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + 5\mathbf{j} + \mathbf{k}$ are perpendicular.

(c) Find t so that $t\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + t\mathbf{j} + 4\mathbf{k}$ are perpendicular.

(d) Find the angle between $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$

[4 marks]

B5 $l_1: \mathbf{r} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{k})$ and $l_2: \mathbf{r} = (2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) + \mu\mathbf{j}$

(a) Show that l_1 and l_2 meet.

(b) Find the point where they meet.

(c) Find the angle between l_1 and l_2

[3 marks]

B6 (a) Find the acute angle between lines with directions $3\mathbf{i} + 4\mathbf{k}$ and $3\mathbf{j} + 4\mathbf{k}$

(b) Find angle ABC for $A(1, 0, 1)$, $B(2, 1, 0)$ and $C(4, 0, 2)$

(c) Find the angle between $2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and the x -axis.

(d) Find the exact cosine of the angle between $\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$

[4 marks]



STRETCH

B7 $l_1: \mathbf{r} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \lambda(2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$ and $l_2: \mathbf{r} = (\mathbf{j} + 5\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$

- (a) Show that l_1 and l_2 are parallel.
- (b) Show that l_1 and l_2 are not the same line.
- (c) The point $(a, 0, b)$ lies on l_1 . Find a and b
- (d) Find the point on l_1 with x -coordinate 7

[4 marks]

B8 Decide, with reasons, how each pair of lines is related.

- (a) $\mathbf{r} = (2\mathbf{i} + \mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = (3\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}) + \mu(\mathbf{i} - \mathbf{j} - 2\mathbf{k})$: find where they meet.
- (b) $\mathbf{r} = (\mathbf{i} + \mathbf{j} + p\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$ and $\mathbf{r} = (3\mathbf{i} + \mathbf{k}) + \mu(\mathbf{j} + \mathbf{k})$ meet. Find p
- (c) $\mathbf{r} = (\mathbf{i} + 2\mathbf{j}) + \lambda(\mathbf{i} + \mathbf{k})$ and $\mathbf{r} = 3\mathbf{k} + \mu(\mathbf{i} + \mathbf{j})$: show they are skew.
- (d) $\mathbf{r} = (\mathbf{j} + \mathbf{k}) + \lambda(2\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$ and $\mathbf{r} = \mathbf{i} + \mu(-\mathbf{i} - 2\mathbf{j} + \mathbf{k})$: parallel, and distinct?

[4 marks]

B9 $l: \mathbf{r} = (\mathbf{i} + \mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ and $P(4, 1, 3)$

- (a) Find the foot X of the perpendicular from P to l
- (b) Find the shortest distance from P to l
- (c) Find the reflection of P in l
- (d) Find the point of l nearest to the origin, and its distance from O

[4 marks]

B10 $ABCD$ is a parallelogram with $A(1, 0, 2)$, $B(4, 1, 0)$ and $C(5, 3, 1)$

- (a) Find the coordinates of D
- (b) Find angle DAB to 0.1°
- (c) Find the exact area of $ABCD$
- (d) Show that $(1, 2, 3)$, $(3, 3, 1)$ and $(7, 5, -3)$ are collinear.

[4 marks]



C

Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** For $A(1, 4, 2)$ and $B(3, 1, 5)$, Arun writes $\overrightarrow{AB} = \mathbf{a} - \mathbf{b} = -2\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$
Explain his error and give $\overrightarrow{AB}$ [2 marks]
- C2** Bea writes the line through $(1, 0, 2)$ in the direction $\mathbf{i} + 3\mathbf{k}$ as " $l = \mathbf{i} + 2\mathbf{k} + \mathbf{i} + 3\mathbf{k}$ "
Give two things that are wrong, and write the equation correctly. [2 marks]
- C3** Two lines have directions $\mathbf{i} + \mathbf{j}$ and $-\mathbf{i} + \mathbf{k}$. Cleo finds $\cos \theta = -\frac{1}{2}$ and says the angle between the lines is 120°
Explain why this is not the answer usually required, and give the acute angle between the lines. [3 marks]
- C4** Deji checks whether $(5, 3, 1)$ lies on $\mathbf{r} = (\mathbf{i} + \mathbf{j} + \mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - \mathbf{k})$. He finds $\lambda = 2$ from the x -components and says it does.
Explain his error, and decide correctly. [3 marks]

STRETCH

- C5** To show two lines are skew, Ella shows that the equations for λ and μ are inconsistent and writes "so the lines are skew"
Explain what is missing from her argument, and why it matters. [3 marks]
- C6** To find the foot X of the perpendicular from $P(3, 4, 0)$ to $\mathbf{r} = \lambda(\mathbf{i} + \mathbf{j})$, Femi solves $\overrightarrow{OX} \cdot (\mathbf{i} + \mathbf{j}) = 0$ and gets $X = O$
Explain his error and find X correctly. [3 marks]
- C7** To find the angle between $l_1: \mathbf{r} = (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) + \lambda(\mathbf{i} - \mathbf{k})$ and $l_2: \mathbf{r} = (4\mathbf{i} + \mathbf{k}) + \mu(\mathbf{j} + 2\mathbf{k})$, Gus uses $(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) \cdot (4\mathbf{i} + \mathbf{k})$
Explain his error and find the acute angle between the lines. [3 marks]
- C8** (a) Show that, for any vectors $\mathbf{a}$ and $\mathbf{b}$, $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = |\mathbf{a}|^2 - |\mathbf{b}|^2$ (2)
- (b) A parallelogram has sides given by the vectors $\mathbf{a}$ and $\mathbf{b}$, with $|\mathbf{a}| = |\mathbf{b}|$. Hence prove that its diagonals are perpendicular. (3)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** Relative to a fixed origin O , the point B has position vector $\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ and the point C has position vector $a\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$, use the scalar product to find the two possible values of a . You must show your working clearly. Hence, for the positive value of a , find the exact length of BC **[3 marks]**

- D2** Relative to a fixed origin O , the points A and B have coordinates $(3, -1, 2)$ and $(5, 3, -2)$ respectively. The point C , which does not lie on the line through A and B , has coordinates $(4, -3, 5)$

(a) Find the vector $\overrightarrow{AB}$, and hence write down a vector equation for the line l that passes through the points A and B , where λ is a scalar parameter.

(2)

(b) The point P lies on l and CP is perpendicular to l . Find the coordinates of P , showing clearly how you use the scalar product.

(2)

[4 marks]

- D3** The lines l_1 and l_2 meet at a point X and have equations

$$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 0 \\ -3 \end{pmatrix}$$

where λ and μ are scalar parameters. Show that the point with position vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ lies on both lines, and find the exact value of the cosine of the acute angle between l_1 and l_2 , giving your answer as a fraction in its simplest form. Explain why your answer must be positive. **[3 marks]**

- D4** With respect to a fixed origin O , the line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

where λ is a scalar parameter. The point A lies on l and the distance OA is $\sqrt{11}$ units. Find the two possible position vectors of A , showing clearly how you form and solve your equation in λ . Hence show that the distance between the two possible positions of A is 6 units. **[3 marks]**



STRETCH

D5 With respect to a fixed origin O , the lines l_1 and l_2 have equations

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ p \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$$

where p is a constant and λ and μ are scalar parameters. The lines l_1 and l_2 intersect at the point X

(a) Find the value of the constant p , and the coordinates of the point of intersection X . You must show your working clearly.

(3)

(b) Find the acute angle between the lines l_1 and l_2 , giving your answer in degrees to one decimal place.

(1)

[4 marks]

D6 With respect to a fixed origin O , the line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

The points A and B lie on l , where $\lambda = 0$ at A and $\lambda = 1$ at B . The point C has coordinates $(2, 2, 2)$

(a) Show that the line segment AC is perpendicular to l , and hence find the exact area of the triangle ABC , giving your answer as a simplified surd.

(2)

(b) The point D lies on l and the area of triangle ACD is twice the area of triangle ABC . Find the coordinates of each of the two possible positions of D

(2)

[4 marks]



STRETCH *continued*

D7 With respect to a fixed origin O , the line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, where λ is a scalar parameter,

and the point P has coordinates $(5, 4, 3)$

(a) Find the coordinates of the point X on l such that PX is perpendicular to l

(2)

(b) Find the exact shortest distance from P to l , giving your answer as a surd, and find the coordinates of the point P' , the reflection of P in the line l

(2)

[4 marks]

D8 Relative to a fixed origin O , the points A and B have coordinates $(1, 2, 0)$ and $(3, 3, 2)$. The point C has coordinates $(1, \alpha, 4)$, where α is a constant.

(a) Find a vector equation for the line through the points A and B , where λ is a scalar parameter.

(1)

(b) Given that angle $BAC = 45^\circ$, use the scalar product to show that $7\alpha^2 - 60\alpha + 108 = 0$, and hence find the two exact possible values of α

(4)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA14. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 4 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Show that l_1 and l_2 meet, and find the position vector of their point of intersection X . You must show clearly that your values of λ and μ satisfy all three components.

(3)

(b) The acute angle between l_1 and l_2 is θ . Find the exact value of $\cos \theta$, giving your answer as a fraction in its simplest form, and explain how you know that your angle is acute.

(2)

(Total for Question E1 is 5 marks)



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E2 Relative to a fixed origin O , the points A , B and C have coordinates $(2, 1, 0)$, $(4, -1, 1)$ and $(8, -2, 0)$ respectively.

(a) Find the vector $\overrightarrow{AB}$, and hence find a vector equation for the line l that passes through the points A and B , where λ is a scalar parameter.

(1)

(b) The point P lies on l such that CP is perpendicular to l . Find the coordinates of P , showing clearly how you use the scalar product.

(3)

(c) Explain why triangle ACP is right-angled, and find the exact area of triangle ACP

(2)

(Total for Question E2 is 6 marks)



STRETCH

E3 With respect to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) Prove that l_1 and l_2 are skew. You must show that the lines are not parallel and that they do not intersect.

(3)

(b) The point P on l_1 and the point Q on l_2 are such that $\overrightarrow{PQ}$ is perpendicular to both l_1 and l_2 . Using the scalar product, find the coordinates of P and of Q

(3)

(c) Hence find the exact shortest distance between the lines l_1 and l_2 , giving your answer as a simplified surd.

(2)

(Total for Question E3 is 8 marks)

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