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A-LEVEL MATHEMATICS • STATISTICS 2

Sampling and Sampling Distributions

Worksheet

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Free Sampling and Sampling Distributions worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

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Tap a section to jump straight to it

By the end of this worksheet you can: use the language of populations, censuses and samples • decide whether a quantity is a statistic • list a sampling distribution • find the mean and variance of a statistic • *Stretch:* medians, maxima, ranges and sampling without replacement.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Medians, maxima and ranges, sampling without replacement and unknown proportions.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: the vocabulary

A factory tests the breaking strength of a batch of 2000 cables, each with a serial number. Give the population, a sampling unit, a sampling frame, and a reason not to use a census.

Population: all 2000 cables. Unit: one cable. Frame: the list of serial numbers.

a census destroys every cable

Your turn A1

A library has 6000 members on its database and wants their views on opening hours. Give the population, a sampling unit, a sampling frame, and one advantage of a sample over a census.

Worked example A2: is it a statistic?

X_1, X_2, X_3 is a random sample from a population with unknown mean μ. Which of $X_1 + X_3$, $\bar{X} - \mu$ and $\max(X_1, X_2, X_3)$ are statistics?

A statistic contains no unknown parameters

$X_1 + X_3$ and the maximum

Your turn A2

X_1, X_2, X_3 is a random sample from a population with unknown standard deviation σ. Which of $\frac{1}{3}\sum X_i^2$, $\frac{\bar{X}}{\sigma}$ and $X_1 - X_2$ are statistics?

Worked example A3: listing a total

A large population contains values 1 and 3 in the ratio 3 : 2. Two values are chosen at random. Find the sampling distribution of their total T

$(1,3)$ *can happen in two orders*

$\left(\frac{3}{5}\right)^2$; $2\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)$; $\left(\frac{2}{5}\right)^2$

$T = 2, 4, 6$ with $\frac{9}{25}, \frac{12}{25}, \frac{4}{25}$

Your turn A3

A large population contains values 2 and 5 in the ratio 7 : 3. Two values are chosen at random. Find the sampling distribution of their total.

STRETCH

Worked example A4: a median of three

A large population contains values 1 and 2 in the ratio 1 : 4. Find the sampling distribution of the median M of three values chosen at random.

$M = 1$ needs at least two 1s

$3\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right) + \left(\frac{1}{5}\right)^3 = \frac{13}{125}$

$M = 1, 2$ with $\frac{13}{125}, \frac{112}{125}$

Your turn A4

A large population contains values 5 and 10 in the ratio 3 : 7. Find the sampling distribution of the median of three values chosen at random.

Worked example A5: the mean of a sample

A population has mean 4 and variance 2.4. A random sample of 3 is taken. Find $\mathrm{E}(\bar{X})$ and $\mathrm{Var}(\bar{X})$

$\mathrm{E}(\bar{X}) = \mu$, $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$

4, 0.8

Your turn A5

A population has mean 1.5 and variance 0.75. A random sample of 5 is taken. Find $\mathrm{E}(\bar{X})$ and $\mathrm{Var}(\bar{X})$

Worked example A6: a range

The values 1, 2 and 4 are equally likely. Two values are chosen at random, with replacement. Find the sampling distribution of the range R

$R = 0$: 3 pairs; $R = 1$: $(1,2),(2,1)$; $R = 2$: $(2,4),(4,2)$; $R = 3$: $(1,4),(4,1)$

$\frac{3}{9}, \frac{2}{9}, \frac{2}{9}, \frac{2}{9}$

Your turn A6

The values 0, 1 and 2 occur with probabilities 0.5, 0.3 and 0.2. Two values are chosen at random. Find the sampling distribution of the range.

Worked example A7: a proportion

30% of items are defective. In a sample of 20, Y are defective. Find the mean and variance of $\frac{Y}{20}$, and $\mathrm{P}\left(\frac{Y}{20} > 0.4\right)$

$\mathrm{E} = p = 0.3$, $\mathrm{Var} = \frac{0.3(0.7)}{20}$; $\mathrm{P}(Y > 8) = 1 - 0.8867$

0.3, 0.0105, 0.1133

Your turn A7

15% of items are defective. In a sample of 40, Y are defective. Find the mean and variance of $\frac{Y}{40}$, and $\mathrm{P}\left(\frac{Y}{40} \leqslant 0.1\right)$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A town council wants to survey the 12 000 households in its area about recycling. It has a list of the addresses of all the households.

(a) State the population.

(b) State a sampling unit.

(c) State a suitable sampling frame.

(d) Give one reason why a census might not be used.

[4 marks]

B2 X_1, X_2, X_3 is a random sample from a population with unknown mean μ and unknown standard deviation σ. State, with a reason, whether each is a statistic.

(a) $X_1 + X_2 + X_3$

(b) $\bar{X} - \mu$

(c) $\dfrac{(X_1 - X_2)^2}{\sigma}$

(d) $\max(X_i) - \min(X_i)$

[4 marks]

B3 A large bag contains coins of value 1, 2 and 5 in the ratio 5 : 3 : 2. Two coins are taken at random.

(a) List all the possible combinations of the two coins.

(b) Find P(total = 4)

(c) Find P(total = 6)

(d) Find the sampling distribution of the total.

[4 marks]

B4 For the coins in Question B3, find

(a) the population mean μ

(b) the population variance σ^2

(c) $\mathrm{E}(\bar{X})$ for samples of 2

(d) $\mathrm{Var}(\bar{X})$ for samples of 2

[4 marks]

B5 State whether a census or a sample should be used, giving a reason, to find

(a) the mean lifetime of the light bulbs made by a factory in a year.

(b) the mean height of the 25 students in one class.

(c) the proportion of all the voters in a country who support a new law. **[3 marks]**

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B6 The values 1, 2 and 3 occur with probabilities 0.5, 0.3 and 0.2. Two values are chosen at random, and M is the larger.

(a) Find P($M = 1$)

(b) Find P($M = 2$)

(c) Find P($M = 3$)

(d) Find E(M)

[4 marks]

STRETCH

B7 For the values in Question B6, three values are chosen at random, and N is their median.

(a) Find P($N = 1$)

(b) Find P($N = 3$)

(c) Find P($N = 2$)

(d) Find E(N)

[4 marks]

B8 In a large population a proportion $p = 0.4$ have a certain gene. A sample of 25 is taken, and Y have the gene.

(a) Find E(Y)

(b) Find Var $\left(\frac{Y}{25}\right)$

(c) Find P $\left(\frac{Y}{25} \geqslant 0.5\right)$

(d) If p were unknown, would $\frac{Y}{25} - p$ be a statistic? Explain.

[4 marks]

B9 A box holds five counters numbered 1, 1, 2, 3 and 3. Two are taken at random without replacement, and S is their total.

(a) How many different pairs of counters are there?

(b) Find P($S = 4$)

(c) Find the sampling distribution of S

(d) Find E(S)

[4 marks]

B10 The values 0, 1 and 2 occur with probabilities 0.4, 0.4 and 0.2. Two values are chosen at random.

(a) Find the population mean μ

(b) Find the sampling distribution of $\bar{X}$

(c) Show that E($\bar{X}$) = μ

(d) Find P($\bar{X} > \mu$)

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 A head teacher surveys a sample of the 900 students in a school. Amir says the sampling frame is "a student". Explain his error, and give a suitable sampling frame. **[2 marks]**

C2 $X_1, \ldots, X_n$ is a random sample from a population with unknown mean μ and unknown standard deviation σ. Bea says $\dfrac{\bar{X} - \mu}{\sigma}$ is a statistic. Explain her error. **[2 marks]**

C3 For the coins in Question B3, Cal gives the sampling distribution of the total as

t	2	3	4
P($T = t$)	0.25	0.3	0.09

Explain why this is not a sampling distribution, and complete it. **[3 marks]**

C4 A large population contains values 1 and 4 in the ratio 3 : 2. Two values are chosen at random. Dev writes P(total = 5) = $0.6 \times 0.4 = 0.24$

Explain his error, and find P(total = 5) **[3 marks]**

STRETCH

C5 A large population contains values 1 and 2 in the ratio 3 : 2. Eve finds the probability that the median of three values is 2 as $3(0.4)^2(0.6) = 0.288$

Explain her error, and find the probability. **[3 marks]**

C6 Bag A holds only 1p and 2p coins and bag B holds only 2p and 5p coins. One coin is taken from bag A and two from bag B. Fay gives a sampling distribution for the median that includes the value 1

Explain why the median cannot be 1, and state its possible values. **[3 marks]**

C7 A population has variance 6. Gus says that the mean of a random sample of 4 also has variance 6

Explain his error, and find Var($\bar{X}$) **[3 marks]**

C8 The values 0, 1 and 2 are equally likely. Two values are chosen at random, and R is their range.

(a) Show that $\mathrm{P}(R = 1) = \dfrac{4}{9}$

(2)

(b) Show that $\mathrm{E}(R) = \dfrac{8}{9}$ and $\mathrm{Var}(R) = \dfrac{44}{81}$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 A gym has 1800 members. The manager asks a random sample of 150 members about the classes it offers.

(a) Suggest a suitable sampling frame.

(1)

(b) Identify the sampling units.

(1)

(c) The manager has: A, the mean age of all 1800 members; B, the number of the 150 sampled members who attend a class; C, a list of the classes offered. State which of these is a statistic, giving a reason.

(1)

[3 marks]

D2 A bag contains a large number of red and blue counters in the ratio 3 : 1. A player takes a random sample of 3 counters and scores 2 points for each red counter and 6 points for each blue counter. The random variable X is the total score. Find the sampling distribution of X **[4 marks]**

D3 For the game in Question D2, the player takes n random samples of 3 counters, replacing each sample before taking the next. Find the least value of n for which the probability that at least one sample scores 18 points is greater than 0.9 **[3 marks]**

D4 A large population takes the values 3, 5 and 8 with probabilities $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{1}{4}$

(a) Find the population mean.

(1)

(b) Find the population variance.

(1)

(c) Find the variance of the mean of a random sample of 5 from this population.

(1)

[3 marks]

STRETCH

D5 A large bag contains 1p, 2p and 5p coins in the ratio 5 : 3 : 2. Three coins are taken at random from the bag, and R pence is the range of their values, that is, the largest value minus the smallest value. Find the sampling distribution of R **[4 marks]**

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D6 A bag contains a large number of 20p and 50p coins. A random sample of 3 coins is taken, and $\bar{X}$ is the mean value of the 3 coins. Given that $P(\bar{X} = 50) = 0.064$,

(a) find the proportion of 50p coins in the bag

(2)

(b) find $P(\bar{X} = 40)$ and $P(\bar{X} = 30)$

(2)

[4 marks]

D7 A box holds six counters numbered 1, 1, 2, 2, 2 and 4. Two counters are taken at random without replacement, and M is the mean of their numbers.

(a) Show that $P(M = 1) = \frac{1}{15}$

(1)

(b) Find the sampling distribution of M

(3)

[4 marks]

D8 A shop sells shirts at three prices: 30% are sold at £8, 50% at £12 and 20% at £20. A random sample of 3 shirt sales is taken.

(a) Find the mean price of a shirt sold.

(1)

(b) Find the sampling distribution of the median price of the 3 shirts.

(4)

[5 marks]

E Exam practice

Full-size exam questions in the style of WST02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 A bag contains a large number of counters, each either red or green. Red and green counters occur in the ratio 2 : 3 respectively. In a game, a player takes a random sample of 4 counters from the bag and scores 3 points for each red counter taken and 1 point for each green counter taken. The random variable T represents the total score for the 4 counters.

(a) Show that P(T = 8) = 0.3456

(2)

(b) Find the sampling distribution of T

(4)

(c) Find E(T)

(2)

The player plays 5 games. Before each game the counters are returned to the bag, so each game uses a new random sample of 4 counters, independently of the other games.

(d) Find the probability that the player scores at least 10 points in exactly 2 of the 5 games.

(3)

(Total for Question E1 is 11 marks)

E2 A college has 1500 students, and the college office keeps a list of the names of all of them. The principal wants to find out the students' opinions of the canteen, and decides to ask a random sample of 60 of the students to complete a short questionnaire.

(a) State the population.

(1)

(b) Suggest a suitable sampling frame.

(1)

(c) Give one advantage and one disadvantage of taking a sample rather than a census.

(2)

The principal has the following information. A is the proportion of all 1500 students who use the canteen. B is the number of the 60 sampled students who use the canteen.

(d) State which of A and B is a statistic, giving a reason.

(1)

Records from the canteen show that 70% of the students at the college use the canteen. The random variable Y is the number of the 60 sampled students who use the canteen.

(e) Using a suitable approximation, find the probability that fewer than 36 of the 60 students use the canteen.

(4)

(f) Find the mean and the variance of the proportion of the sampled students who use the canteen.

(2)

(Total for Question E2 is 11 marks)

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STRETCH

E3 A box contains six counters, which are identical apart from the number written on each one. Two of the counters are numbered 1, three are numbered 2 and one is numbered 5. A random sample of 3 counters is taken from the box without replacement, and the numbers on the 3 counters are recorded.

(a) Find the number of different samples of 3 counters that can be taken.

(1)

(b) Find the sampling distribution of the median of the numbers on the 3 counters.

(5)

(c) Find the sampling distribution of the largest number on the 3 counters.

(4)

(d) Find the expected value of the median of the numbers on the 3 counters, and compare it with the median of the numbers on all six counters in the box.

(2)

(e) Find the probability that the largest number on the 3 counters is 5, given that the median of the numbers on the 3 counters is 2

(2)

(Total for Question E3 is 14 marks)

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