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INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2

Statistics 2

Chapter 6: Sampling and Sampling Distributions

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WST02/01) • Calculator and Statistical Tables allowed

Multi-step exam questions at the difficulty of the real WST02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Binomial Distributions** Conditions, probabilities and tables, mean and variance, unknown n and p	10	80	separate file	
2	**Poisson Distributions** Probabilities, modelling conditions, scaling the rate, adding Poisson variables	10	80	separate file	
3	**Approximations** Poisson and normal approximations, continuity corrections	10	80	separate file	
4	**Continuous Random Variables** Probability density and cumulative distribution functions, mean, variance, median, mode	10	81	separate file	
5	**Continuous Uniform Distribution** Rectangular distribution, mean and variance, modelling	10	80	separate file	
6	**Sampling and Sampling Distributions** Populations and samples, statistics, sampling distributions	10	80	2	
7	**Hypothesis Testing** Critical regions, significance levels, binomial and Poisson tests	10	80	separate file	

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CHAPTER 6 • KEY SKILLS

6 Sampling and Sampling Distributions

Specification S2: Sampling distributions • 10 questions • 80 marks • about 96 minutes

1 Population and census

The population is the whole set of items. A census surveys every member; a sample surveys some of them.

2 Unit and frame

A sampling unit is one member of the population; the sampling frame is the list of sampling units used to choose the sample.

3 Census or sample

Census: accurate, but slow, costly, and impossible if testing destroys items. Sample: quicker and cheaper, but may not represent the population.

4 A statistic

A function of the sample values only. If it contains an unknown parameter such as μ, σ or p, it is not a statistic.

5 Sampling distribution

All possible values of the statistic, with their probabilities. Check the probabilities add to 1.

6 Listing samples

List every sample in a table. A sample such as $(1, 2)$ can occur in 2 orders; $(1, 2, 5)$ from mixed bags needs care with which bag each came from.

7 Different statistics

Mean, median, maximum, range and total each have their own distribution; some take only a few values.

8 Binomial samples

The number of successes in a sample of n is $\mathrm{B}(n, p)$; the sample proportion $\frac{Y}{n}$ has mean p and variance $\frac{p(1-p)}{n}$

9 Expectation of a statistic

Find E from its sampling distribution and compare with the population parameter, e.g. $\mathrm{E}(\bar{X}) = \mu$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Population and census	•							•		
2 Unit and frame	•							•		
3 Census or sample	•							•		
4 A statistic		•		•	•			•	•	
5 Sampling distribution		•	•	•	•	•	•		•	•
6 Listing samples			•	•		•	•		•	•
7 Different statistics				•		•			•	•
8 Binomial samples					•			•		
9 Expectation of a statistic		•		•	•		•		•	•
Interleaved: expectation and variance (S1)		•		•	•		•		•	•
Interleaved: binomial and approximations (Chapters 1, 3)					•			•		
Interleaved: conditional probability and algebra (S1, P1)									•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Draw up a table of every possible sample with its probability before finding any statistic. Definitions must refer to the context of the question.

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1. A head teacher wants to find out the views of the 1200 students in her school about a new timetable. She has a list of all the students.

(a) Describe the population.

(1)

(b) Describe a suitable sampling frame.

(1)

(c) State the sampling units.

(1)

(d) Give one advantage of taking a census and one advantage of taking a sample.

(2)

(e) Give one reason why a sample might not give a good representation of the views of all the students.

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. A random sample X_1, X_2, X_3 is taken from a population with unknown mean μ and unknown standard deviation σ.

(a) State, with a reason, which of the following are statistics.

(i) $X_1 + X_2 + X_3$ (ii) $(X_1 - \mu)^2$ (iii) $\max(X_1, X_2, X_3)$ (iv) $\dfrac{X_1 + X_2}{\sigma}$

(3)

(b) Explain what is meant by the sampling distribution of a statistic.

(1)

(c) Show that $\mathrm{E}\left(\dfrac{X_1 + X_2 + X_3}{3}\right) = \mu$

(2)

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Question 2 continued

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3. A bag contains a large number of counters, each numbered 1, 2 or 3. A counter is chosen at random, and its number X has the distribution

x	1	2	3
$P(X = x)$	0.5	0.3	0.2

A random sample of 2 counters is taken.

(a) List all the possible samples.

(2)

(b) Find the sampling distribution of the mean of the sample.

(4)

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Question 3 continued

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4. A warehouse stores a large number of sacks. Each sack weighs either 2 kg or 5 kg, and 70% of the sacks weigh 2 kg. A random sample of 3 sacks is chosen.

(a) Explain why the maximum weight in the sample is a statistic.

(1)

(b) Find the sampling distribution of the maximum weight M.

(2)

(c) Find the sampling distribution of the range R of the weights in the sample.

(2)

(d) Find E(R).

(1)

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Question 4 continued

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5. In a large town, 30% of residents use the library. A random sample of 10 residents is taken, and Y is the number who use the library.

(a) Write down the sampling distribution of Y.

(1)

(b) Find P($Y \geqslant 5$)

(2)

(c) Find the mean and the variance of the sample proportion $\frac{Y}{10}$

(2)

(d) In a different town the proportion p who use the library is unknown. Explain why $\frac{Y}{10}$ is a statistic but $Y - 10p$ is not.

(1)

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Question 5 continued

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6. Bag A contains a large number of 1p and 2p coins in the ratio 3 : 1. Bag B contains a large number of 2p and 5p coins in the ratio 1 : 1. Two coins are taken at random from bag A and one coin is taken at random from bag B.

(a) List all the possible samples of three coins, with their probabilities.

(3)

(b) Find the sampling distribution of the total value, T pence, of the three coins.

(3)

(c) Find the sampling distribution of the median value, M pence, of the three coins.

(3)

(d) Explain why M can take only two values.

(1)

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Question 6 continued

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7. A random variable X has the probability distribution

x	0	1	2
$P(X = x)$	0.4	0.4	0.2

(a) Find the mean μ and the variance σ^2 of X.

(2)

A random sample of 2 observations is taken from this population, and $\bar{X}$ is the sample mean.

(b) Find the sampling distribution of $\bar{X}$.

(4)

(c) Find $E(\bar{X})$ and $Var(\bar{X})$, and show that they are equal to μ and $\frac{\sigma^2}{2}$ respectively.

(4)

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Question 7 continued

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Question 7 continued

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8. A polling company wants to estimate the proportion p of the voters in a city who support a new tram line. It plans to telephone a random sample of 400 voters, chosen from the city's telephone directory.

(a) Give one reason why the company uses a sample rather than a census.

(1)

(b) Identify the sampling frame, and explain why using it may give a biased estimate of p.

(2)

Let Y be the number of the 400 voters who support the tram line. The company will use $\hat{P} = \frac{Y}{400}$ to estimate p.

(c) Given that $p = 0.35$, use a suitable approximation to find the probability that $\hat{P}$ is greater than 0.4

(4)

(d) Explain why $\hat{P}$ is a statistic, but $\hat{P} - p$ is not, when p is unknown.

(2)

(e) State the mean of the sampling distribution of $\hat{P}$.

(1)

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Question 8 continued

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9. A fair spinner is numbered 1, 2 and 3. It is spun 3 times, and the three scores form a random sample.

(a) Explain why the range R of the three scores is a statistic.

(1)

(b) Find the sampling distribution of R.

(5)

(c) Find E(R).

(2)

(d) Given that the median of the three scores is 2, find the probability that the range is 2

(2)

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Question 9 continued

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10. A large bag contains discs, each labelled 1 or 3. The proportion of discs labelled 1 is p. A random sample of 3 discs is taken, and $\bar{X}$ is the mean of the labels.

(a) Find, in terms of p, the sampling distribution of $\bar{X}$.

(4)

(b) Given that $\mathrm{P}\left(\bar{X} = 3\right) = \frac{1}{8}$, find the value of p.

(2)

(c) For this value of p, find the sampling distribution of the median of the sample.

(2)

(d) Show that, for any p, $\mathrm{E}\left(\bar{X}\right) = 3 - 2p$, and explain why this equals the mean of the population.

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 6 IS 80 MARKS

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