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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 1 (WFM01)

Further Pure 1 Practice Paper 3

Mark Scheme

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM01/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1 and 3 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

Rough grade guide (out of 75): A **53** • B **46** • C **39**

Based on recent WFM01 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Unknown coefficients from a complex root; a quartic factorised; the area of the shape the roots make	10	Ch 1 Complex Numbers; Ch 2 Roots of Quadratic Equations	
2	$\alpha^2+\beta^2$ with a parameter; rejecting a value that gives non-real roots; roots α^3, β^3	10	Ch 2 Roots of Quadratic Equations	
3	A series formula; a sum from n to $2n$; an equation in n with a non-integer root	11	Ch 7 Series	
4	A sign change across an asymptote; Newton–Raphson from a good and a poor start	11	Ch 3 Numerical Solutions	
5	A determinant with a parameter; $\lvert\det\rvert$ and area; a combined transformation	11	Ch 6 Transformations; Ch 5 Matrices	
6	The normal to a parabola; a triangle area with $\lvert p\rvert$; a locus	11	Ch 4 Coordinate Systems	
7	Induction for a two-step recurrence and for divisibility	11	Ch 8 Proof	
	Total	**75**		

Where did the marks go? Every question on this paper has a trap, and the marks lost at A* are nearly always there: a second value of k or p missed because a modulus was ignored, a value that solves the equation but breaks a condition in the question, a sign change across an asymptote taken as a root, or an induction with one base case when the recurrence needs two. Look back at each question and ask: did I check my answers against every condition I was given?

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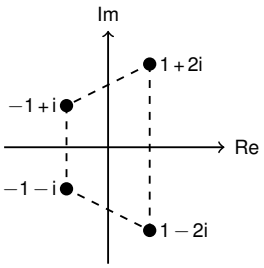
Question 1

10 marks

Part	Scheme	Marks
(a)	$(1+2\mathrm{i})^2 = -3+4\mathrm{i}$, $(1+2\mathrm{i})^4 = (-3+4\mathrm{i})^2 = -7-24\mathrm{i}$ $-7-24\mathrm{i}+a(-3+4\mathrm{i})+b(1+2\mathrm{i})+10=0$; real: $3-3a+b=0$, imaginary: $-24+4a+2b=0$ Solving: $10a=30$ $\boxed{a=3,\ b=6}$ *Alternative: z^2-2z+5 is a factor, so $\mathrm{f}(z)=(z^2-2z+5)(z^2+cz+2)$; the z^3 term gives $c=2$, then compare coefficients.*	**M1 A1** **M1** **M1** **A1** **(5)**
(b)	$1-2\mathrm{i}$, and $\mathrm{f}(z)=(z^2-2z+5)(z^2+2z+2)$ $\boxed{z=-1\pm\mathrm{i}}$	**B1 M1** **A1** **(3)**
(c)	Vertices $(1,\pm2)$ and $(-1,\pm1)$ form a trapezium with parallel sides 4 and 2, a distance 2 apart: $\frac{1}{2}(4+2)\times 2$  $\boxed{6}$ *The vertices must be taken in order round the shape. Taking A, B, C, D in the order $1+2\mathrm{i}$, $1-2\mathrm{i}$, $-1+\mathrm{i}$, $-1-\mathrm{i}$ in the shoelace formula gives a crossed quadrilateral and the wrong value 2: M0.*	**M1** **A1** **(2)**
		Total for Question 1: 10 marks

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Question 2

10 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=-k,\ \alpha\beta=k+6;\ \alpha^2+\beta^2=k^2-2(k+6)$	M1
	$\boxed{k^2-2k-12}$	A1
		(2)
(b)	$k^2-2k-12=12,\ k^2-2k-24=0$	M1
	$(k-6)(k+4)=0,\ k=6$ or $k=-4$	A1
	Real roots need $b^2-4ac\geqslant 0$: $k^2-4(k+6)\geqslant 0$	M1
	$k=6$: $36-48=-12<0$ (roots not real); $k=-4$: $16+16-24=8\geqslant 0$	A1
	$\boxed{k=-4}$, rejecting $k=6$ because the roots would not be real	A1
	With $k=6$ the non-real roots still satisfy $\alpha^2+\beta^2=12$, so the condition that the roots are real must be used.	
		(5)
(c)	$\alpha+\beta=4,\ \alpha\beta=2$: $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)=64-24=40$	M1 A1
	Product $(\alpha\beta)^3=8$: $\boxed{x^2-40x+8=0}$	A1
		(3)
		Total for Question 2: 10 marks

Question 3

11 marks

Part	Scheme	Marks
(a)	$\sum r^2+\sum r=\frac{1}{6}m(m+1)(2m+1)+\frac{1}{2}m(m+1)$	M1
	$=\frac{1}{6}m(m+1)[(2m+1)+3]$	M1
	$\boxed{\frac{1}{3}m(m+1)(m+2)}$	A1
		(3)
(b)	$\sum_{r=n}^{2n}=\sum_{1}^{2n}-\sum_{1}^{n-1}$	M1
	$\frac{1}{3}(2n)(2n+1)(2n+2)-\frac{1}{3}(n-1)n(n+1)$	A1
	$=\frac{1}{3}n(n+1)[4(2n+1)-(n-1)]$	M1
	$=\frac{1}{3}n(n+1)(7n+5)$	A1*
	Subtracting the sum to n instead of $n-1$ scores M0: the term $r=n$ is part of the sum.	
		(4)
(c)	$\sum_{r=1}^{n}(28r-15)=14n(n+1)-15n=14n^2-n$	B1
	$\frac{1}{3}n(n+1)(7n+5)=n(14n-1)$; divide by $n\neq 0$: $7n^2+12n+5=42n-3$	M1
	$7n^2-30n+8=0,\ (n-4)(7n-2)=0$	A1
	$\boxed{n=4}$, rejecting $n=\frac{2}{7}$ (and $n=0$) because n must be a positive integer	A1
		(4)
		Total for Question 3: 11 marks

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Question 4

11 marks

Part	Scheme	Marks
(a)	f(0.5) = −5.75 and f(1.5) = 0.25: opposite signs f is not continuous on [0.5, 1.5]: it has an asymptote (discontinuity) at $x = 1$ *The sign changes because* f(x) *goes from* $-\infty$ *to* $+\infty$ *across* $x = 1$. *(In fact* f$(x) = 0$ *has no root there.)*	**M1 A1** **B1** **(3)**
(b)	f(−2.1) = 0.0874… and f(−2) = $-\frac{1}{3}$ Change of sign and f is continuous on [−2.1, −2], so there is a root α in the interval	**M1** **A1** **(2)**
(c)	$f'(x) = 2x - (x-1)^{-2}$, $f'(-2) = -4 - \frac{1}{9} = -\frac{37}{9}$ $-2 - \dfrac{f(-2)}{f'(-2)} = -2 - \left(-\frac{1}{3}\right) \div \left(-\frac{37}{9}\right) = -2 - \frac{3}{37}$ $\boxed{-2.081}$	**B1** **M1** **A1** **(3)**
(d)	$1.6 - \dfrac{f(1.6)}{f'(1.6)} = 1.6 - \dfrac{0.2266\ldots}{0.4222\ldots}$ $\boxed{1.063}$ $f'(1.6)$ is close to 0, so the tangent is nearly flat and the next value jumps a long way, close to the asymptote at $x = 1$ (and there is no root of $f(x) = 0$ for $x > 1$)	**M1** **A1** **B1** **(3)**
		Total for Question 4: 11 marks

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Question 5

11 marks

Part	Scheme	Marks
(a)	$\det \mathbf{P} = 3k + (k+2) = 4k+2$	B1
	$\boxed{k = -\frac{1}{2}}$	B1
		(2)
(b)	Area of $T = 3$, so $\lvert\det \mathbf{P}\rvert = \frac{30}{3} = 10$	M1
	$4k+2 = 10$ or $4k+2 = -10$	M1
	$\boxed{k = 2}$, $\boxed{k = -3}$	A1 A1
	Using only $4k+2 = 10$ loses the last two marks.	
		(4)
(c)	$k = -3$: $\begin{pmatrix} -3 & -1 \\ -1 & 3 \end{pmatrix}$; $\boxed{A'(-9, -3)}$, $\boxed{B'(-2, 6)}$	B1 B1
		(2)
(d)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -3 & -1 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ -3 & -1 \end{pmatrix}$	M1 A1
	$\mathbf{R}^{-1}\begin{pmatrix} 5 \\ 5 \end{pmatrix} = \frac{1}{-10}\begin{pmatrix} -1 & 3 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 5 \end{pmatrix}$: $\boxed{(-1, -2)}$	A1
	The rotation happens second, so its matrix goes on the left. Multiplying the other way round gives $\begin{pmatrix} -1 & 3 \\ 3 & 1 \end{pmatrix}$*: M0.*	
		(3)
	Total for Question 5: 11 marks	

Question 6

11 marks

Part	Scheme	Marks
(a)	$2y\frac{dy}{dx} = 8,\ \frac{dy}{dx} = \frac{4}{y} = \frac{1}{p}$ at P	M1 A1
	Normal gradient $-p$: $y - 4p = -p\left(x - 2p^2\right)$	M1
	$y + px = 2p^3 + 4p$	A1*
		(4)
(b)	$y = 0$: $N\left(2p^2 + 4, 0\right)$; $S(2, 0)$	B1
	$SN = 2p^2 + 2$, height $4p$: $\frac{1}{2}\left(2p^2 + 2\right)(4p)$	M1
	$\boxed{4p\left(p^2 + 1\right)}$	A1
		(3)
(c)	Area $= 4\lvert p\rvert\left(p^2 + 1\right) = 40,\ \lvert p\rvert^3 + \lvert p\rvert - 10 = 0,\ \lvert p\rvert = 2$	M1
	$\boxed{(8, 8) \text{ and } (8, -8)}$	A1
	The area for $p < 0$ is $-4p(p^2 + 1)$. Giving only $(8, 8)$ scores M1 A0.	
		(2)
(d)	$M\left(\frac{2p^2 + 2p^2 + 4}{2}, \frac{4p}{2}\right) = \left(2p^2 + 2, 2p\right)$; $p = \frac{y}{2}$ so $x = \frac{y^2}{2} + 2$	M1
	$\boxed{y^2 = 2x - 4}$, a parabola	A1
		(2)
		Total for Question 6: 11 marks

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Question 7

11 marks

Part	Scheme	Marks
(a)	$n = 1$: $2 + 3 = 5 = u_1$; $n = 2$: $4 + 9 = 13 = u_2$	**B1**
	Assume $u_k = 2^k + 3^k$ **and** $u_{k+1} = 2^{k+1} + 3^{k+1}$	**M1**
	$u_{k+2} = 5\left(2^{k+1} + 3^{k+1}\right) - 6\left(2^k + 3^k\right) = 10 \times 2^k + 15 \times 3^k - 6 \times 2^k - 6 \times 3^k$	**M1**
	$= 4 \times 2^k + 9 \times 3^k$	**A1**
	$= 2^{k+2} + 3^{k+2}$	**A1**
	True for $n = 1$ and $n = 2$, and true for $n = k$ and $n = k + 1$ implies true for $n = k + 2$, so true for all positive integers n	**A1**
	Both base cases are needed for B1, and the conclusion must refer to two consecutive cases.	**(6)**
(b)	$\mathrm{f}(m) = u_{2m-1} = 2^{2m-1} + 3^{2m-1}$; $\mathrm{f}(1) = 5$, so true for $m = 1$	**B1**
	$\mathrm{f}(k+1) = 2^{2k+1} + 3^{2k+1} = 4 \times 2^{2k-1} + 9 \times 3^{2k-1}$	**M1**
	$= 4\left(2^{2k-1} + 3^{2k-1}\right) + 5 \times 3^{2k-1} = 4\mathrm{f}(k) + 5 \times 3^{2k-1}$	**A1**
	If $5 \mid \mathrm{f}(k)$, both terms are divisible by 5, so $5 \mid \mathrm{f}(k+1)$	**A1**
	True for $m = 1$, and true for $m = k$ implies true for $m = k + 1$, so true for all positive integers m	**A1**
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 3 \times 2^{2k-1} + 8 \times 3^{2k-1}$ *is not a multiple of 5 in an obvious way: take out* $4\mathrm{f}(k)$ *instead.*	**(5)**
		Total for Question 7: 11 marks

TOTAL FOR PAPER: 75 MARKS