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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	2	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

[Full mark scheme](#)

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1

CHAPTER 1 • KEY SKILLS

Proof

Specification P4: Proof by contradiction • 10 questions • 73 marks • about 88 minutes

1 The method

Assume the statement is false. Use correct algebra to reach something impossible. Finish: “this is a contradiction, so the statement is true”.

2 Writing the assumption

“For all $x \dots$ ” becomes “there is some x for which it is false”.

“There are no integers. . .” becomes “assume there are integers. . .”.

3 Parity and divisibility

Even = $2k$, odd = $2k + 1$.

$(2k + 1)^2 = 4k^2 + 4k + 1$ is odd, so if n^2 is even, n is even. Not a multiple of 3: $3k + 1$ or $3k + 2$.

4 Irrational numbers

Write $\sqrt{6} = \frac{a}{b}$ where a and b are integers with *no common factors*. Show a and b share a factor.

5 Integer equations

Factorise, e.g. $x^2 - y^2 = (x - y)(x + y)$, then use factor pairs or odd and even. 2^a is even but 3^b is odd.

6 Inequalities

Assume it fails for some x . Multiply only by something you know is positive, then complete the square: $(x - 2)^2 < 0$ is impossible.

7 No real solutions

Assume the curves meet and eliminate y . A quadratic in x^2 with a negative discriminant has no real solutions.

8 Calculus and ranges

A stationary point means $\frac{dy}{dx} = 0$. Use $e^{kx} > 0$, $-1 \leq \cos x \leq 1$ and $\sin^2 x \leq 1$

9 Infinitely many primes

Assume p is the largest prime. $p! + 1$ is not divisible by $2, 3, \dots, p$, so it has a prime factor bigger than p .

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	6	10	10	8	9	10
1 The method	•	•	•	•	•	•	•	•	•	•
2 Writing the assumption	•	•	•	•			•	•	•	•
3 Parity and divisibility	•		•		•	•		•		
4 Irrational numbers	•				•	•				•
5 Integer equations			•		•					
6 Inequalities				•			•			
7 No real solutions		•							•	
8 Calculus and ranges		•					•			
9 Infinitely many primes								•		
Interleaved: deduction, exhaustion and counter-examples (P2)			•	•		•		•		•
Interleaved: logarithms and change of base (P2)					•					
Interleaved: quadratics, discriminant, completing the square (P1)				•					•	
Interleaved: differentiation, e and trigonometric identities (P3)		•					•			

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Start with the assumption in words (“Assume there is . . . for some x ”, never “for all x ”), and include “no common factors” for an irrational number. Use algebra, not a calculator or a sketch, and end with the contradiction and a conclusion.



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1. A student writes the following attempt to prove that $\sqrt{6}$ is irrational.

Line 1 Assume that $\sqrt{6} = \frac{a}{b}$, where a and b are integers.

Line 2 Then $a^2 = 6b^2$, so a^2 is even and therefore a is even.

Line 3 Let $a = 2m$. Then $4m^2 = 6b^2$, so $2m^2 = 3b^2$

Line 4 Therefore b is even.

Line 5 So a and b are both even. This is a contradiction, so $\sqrt{6}$ is irrational.

- (a) State what is missing from Line 1 and explain why the proof needs it. (2)
- (b) Explain why “ a^2 is even and therefore a is even” in Line 2 is true. (1)
- (c) Line 4 does not follow from Line 3 as it is written. Write out the missing steps. (2)



Question 1 continued

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(Total for Question 1 is 5 marks)



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2. The curve C has equation

$$y = e^{3x} + 6x - 2\sin 3x \quad x \in \mathbb{R}$$

Use proof by contradiction to prove that C has no stationary points.

(5)

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Question 2 continued

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(Total for Question 2 is 5 marks)



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3. (a) Prove that, for all integers x and y , the numbers $x + y$ and $x - y$ are either both even or both odd.

(2)

- (b) Hence use proof by contradiction to prove that there are no positive integers x and y such that

$$x^2 - y^2 = 18$$

(3)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. (a) Prove by contradiction that, for all real numbers x with $x > 1$,

$$\frac{x^2}{x-1} \geq 4$$

(4)

- (b) Show that the result in part (a) is not true for all real numbers x with $x \neq 1$

(1)

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Question 4 continued

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(Total for Question 4 is 5 marks)



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5. (a) Use proof by contradiction to prove that $\log_2 3$ is irrational.

(4)

(b) Hence prove that $\log_8 9$ is irrational.

(2)

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Question 5 continued

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(Total for Question 5 is 6 marks)



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- (b)** Hence use proof by contradiction to prove that $\sqrt[3]{3}$ is irrational. **(5)**

A student tries to use the same method to prove that $\sqrt[3]{8}$ is irrational, starting with $\sqrt[3]{8} = \frac{a}{b}$, where a and b are integers with no common factors, and deducing that a is even.

- (c) Explain why the method does not lead to a contradiction for $\sqrt[3]{8}$ (2)

Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that

$$\sec^2 x + \operatorname{cosec}^2 x \equiv 4 \operatorname{cosec}^2 2x \quad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(3)

(b) Hence use proof by contradiction to prove that

$$\sec^2 x + \operatorname{cosec}^2 x \geq 4$$

for all values of x for which it is defined.

(3)

(c) Solve, for $0 < x < \pi$,

$$\sec^2 x + \operatorname{cosec}^2 x = \frac{16}{3}$$

giving your answers as exact multiples of π .

(4)



Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)



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8. In this question n is an integer with $n \geq 2$, and $n! = 1 \times 2 \times 3 \times \dots \times n$

(a) Prove that none of the integers $2, 3, \dots, n$ is a factor of $n! + 1$

(2)

(b) Hence use proof by contradiction to prove that there are infinitely many prime numbers.

(4)

A student says “ $n! + 1$ is always a prime number”.

(c) Show that the student is wrong.

(2)

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Question 8 continued

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(Total for Question 8 is 8 marks)



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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

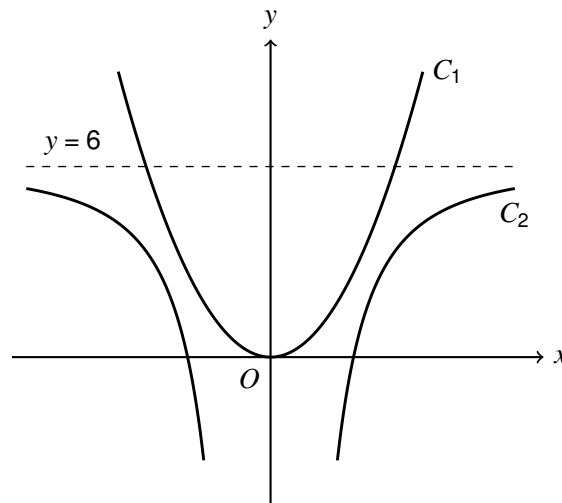


Figure 1

Figure 1 shows a sketch of the curves C_1 and C_2 with equations

$$C_1: y = 2x^2 \qquad C_2: y = 6 - \frac{8}{x^2} \quad x \neq 0$$

The curves appear not to meet.

- (a) Use algebra and proof by contradiction to prove that C_1 and C_2 do not meet.

(4)

The equation of C_2 is changed to $y = k - \frac{8}{x^2}$, $x \neq 0$, where k is a constant.

- (b) Find the set of values of k for which C_1 and the new curve meet.

(3)

- (c) In the case $k = 10$, find the coordinates of all the points where C_1 and the new curve meet.

(2)



Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)



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10. (a) Prove by contradiction that the sum of a rational number and an irrational number is irrational.

(3)

- (b) Use proof by contradiction to prove that $\sqrt{2} + \sqrt{3}$ is irrational.

You may assume that $\sqrt{6}$ is irrational.

(4)

- (c) Give an example to show that the sum of two irrational numbers can be rational.

(1)

- (d) Hence, or otherwise, prove that $\sqrt{3} - \sqrt{2}$ is irrational.

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 1 IS 73 MARKS

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