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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 1 (WFM01)

Further Pure 1 Practice Paper 2

Mark Scheme

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM01/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 4 and 7 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

Rough grade guide (out of 75): A **58** • B **51** • C **44**

Based on recent WFM01 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Singular values of k; an inverse with a parameter; finding a point from its image	7	Ch 5 Matrices · Ch 6 Transformations	
2	A sign change and Newton–Raphson, then the exact root and complex roots	8	Ch 3 Numerical Solutions · Ch 1 Complex Numbers	
3	$\alpha^3+\beta^3$ from the sum and product; roots α/β and β/α	8	Ch 2 Roots of Quadratic Equations	
4	A series formula; an equation in n with invalid solutions rejected	9	Ch 7 Series	
5	The tangent to a parabola; the focus and directrix; a triangle area	9	Ch 4 Coordinate Systems	
6	A reflection matrix; the order of a combined transformation; area and $\lvert\det\rvert$	9	Ch 6 Transformations · Ch 5 Matrices	
7	An equation with z^* and a division; modulus-argument form	8	Ch 1 Complex Numbers	
8	Proof by induction for a matrix power; the inverse of $\mathbf{M}^n$	8	Ch 8 Proof · Ch 5 Matrices	
9	The normal to $xy=c^2$ meeting the curve again, using roots of a quadratic	9	Ch 4 Coordinate Systems · Ch 2 Roots of Quadratic Equations	
	Total	**75**		

Where did the marks go? At grade A the lost marks are usually single steps, not whole questions: the matrices multiplied in the wrong order, a negative determinant used as an area, an argument given in the wrong quadrant, 40 written where the sum needs $40n$, or an answer such as $n=-11$ not rejected. Find the line where your working first differs from the scheme, then practise that chapter.

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Question 1

7 marks

Part	Scheme	Marks
(a)	$\det \mathbf{M} = k(k+1) - 6 = k^2 + k - 6$	M1
	$k^2 + k - 6 = 0,\ (k+3)(k-2) = 0$	M1
	$\boxed{k = 2 \text{ or } k = -3}$	A1
		(3)
(b)	$\frac{1}{k^2+k-6} \times \ldots$ with the leading diagonal swapped and the other signs changed	M1
	$\boxed{\mathbf{M}^{-1} = \frac{1}{k^2+k-6}\begin{pmatrix} k+1 & -2 \\ -3 & k \end{pmatrix}}$	A1
		(2)
(c)	$k = 4$: $\frac{1}{14}\begin{pmatrix} 5 & -2 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} 10 \\ 18 \end{pmatrix}$	M1
	$\boxed{P(1, 3)}$	A1
	Multiplying $\mathbf{M}$ by $(10, 18)$ instead of $\mathbf{M}^{-1}$ scores M0.	
		(2)
	Total for Question 1: 7 marks	

Question 2

8 marks

Part	Scheme	Marks
(a)	$f(1.5) = -3.625$ and $f(2.5) = 5.625$	M1
	Change of sign and f is continuous, so there is a root α in $[1.5, 2.5]$	A1
		(2)
(b)	$f'(x) = 3x^2 - 4x + 5$	B1
	$1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{-3.625}{5.75}$	M1
	$\boxed{2.130}$	A1
		(3)
(c)	$f(2) = 8 - 8 + 10 - 10 = 0$, so $\alpha = 2$	B1
	$f(x) = (x-2)(x^2+5)$	M1
	$\boxed{x = \pm\sqrt{5}\,i}$	A1
	The quadratic factor may be found by division or by comparing coefficients. $x^2 + 5 = 0$ has no real roots, so α is the only real root.	
		(3)
	Total for Question 2: 8 marks	

Question 3

8 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=\frac{2}{3},\ \alpha\beta=\frac{4}{3}$	B1
		(1)
(b)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1
	$=\frac{8}{27}-3\times\frac{4}{3}\times\frac{2}{3}=\frac{8}{27}-\frac{8}{3}$	M1
	$\boxed{-\frac{64}{27}}$	A1
		(3)
(c)	Sum: $\dfrac{\alpha^2+\beta^2}{\alpha\beta}$, with $\alpha^2+\beta^2=\frac{4}{9}-\frac{8}{3}=-\frac{20}{9}$	M1
	Sum $=-\frac{20}{9}\div\frac{4}{3}=-\frac{5}{3}$	A1
	Product: $\dfrac{\alpha}{\beta}\times\dfrac{\beta}{\alpha}=1$	B1
	$\boxed{3x^2+5x+3=0}$	A1
	The equation must have integer coefficients and "$=0$".	
		(4)
		Total for Question 3: 8 marks

Question 4

9 marks

Part	Scheme	Marks
(a)	$2\sum r^2-\sum r=\frac{1}{3}n(n+1)(2n+1)-\frac{1}{2}n(n+1)$	M1
	$=\frac{1}{6}n(n+1)[2(2n+1)-3]$	M1
	$=\frac{1}{6}n(n+1)(4n-1)$	A1
	$\boxed{a=4,\ b=-1}$	A1
		(4)
(b)	$\sum(r^2+40)=\frac{1}{6}n(n+1)(2n+1)+40n$	M1 B1
	$\frac{1}{6}n(n+1)(4n-1)-\frac{1}{6}n(n+1)(2n+1)=40n$, so $\frac{1}{6}n(n+1)(2n-2)=40n$	M1
	Divide by n: $(n+1)(n-1)=120,\ n^2=121$	A1
	$\boxed{n=11}$, rejecting $n=0$ and $n=-11$ (n must be a positive integer)	A1
	B1 is for $40n$: writing $+40$ is a common error. The final A1 needs the rejection of -11 (and 0) to be seen.	
		(5)
		Total for Question 4: 9 marks

Question 5

9 marks

Part	Scheme	Marks
(a)	$2y\frac{dy}{dx} = 12$, so $\frac{dy}{dx} = \frac{6}{y} = \frac{1}{p}$ at P (or $\frac{dy}{dp} \div \frac{dx}{dp} = \frac{6}{6p}$) $y - 6p = \frac{1}{p}(x - 3p^2)$, $\boxed{py = x + 3p^2}$	M1 A1 A1 (3)
(b)	$x = -3$: $py = -3 + 3p^2$ $\boxed{Q\left(-3, \frac{3p^2 - 3}{p}\right)}$	M1 A1 (2)
(c)	SP = distance from P to the directrix: $3p^2 + 3 = 15$ (or $(3p^2 - 3)^2 + 36p^2 = 225$) $\boxed{p = 2}$	M1 A1 (2)
(d)	$P(12, 12)$, $S(3, 0)$, $Q\left(-3, \frac{9}{2}\right)$; $SP = 15$, $SQ = \frac{15}{2}$ and $\angle PSQ = 90°$ (or the shoelace formula) $\boxed{\frac{225}{4} = 56.25}$ *$\overrightarrow{SP} = (9, 12)$ and $\overrightarrow{SQ} = \left(-6, \frac{9}{2}\right)$, so the gradients $\frac{4}{3}$ and $-\frac{3}{4}$ multiply to -1.*	M1 A1 (2)
		Total for Question 5: 9 marks

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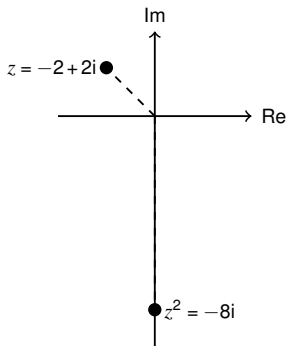
Question 6

9 marks

Part	Scheme	Marks
(a)	$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 (1)
(b)	$\mathbf{C} = \mathbf{RS} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} k & -1 \\ 1 & 2 \end{pmatrix}$ $\begin{pmatrix} -1 & -2 \\ -k & 1 \end{pmatrix}$ $\mathbf{SR}$ *gives* $\begin{pmatrix} 1 & -k \\ -2 & -1 \end{pmatrix}$*: M0 A0.*	M1 A1 (2)
(c)	$\det \mathbf{C} = -1 - 2k$ $\mathbf{C}^{-1} = \frac{1}{-1-2k}\begin{pmatrix} 1 & 2 \\ k & -1 \end{pmatrix}$ oe	M1 A1 (2)
(d)	$\lvert\det \mathbf{C}\rvert = 2k + 1 = \frac{33}{3} = 11$ $k = 5$ $-1 - 2k = 11$ *gives* $k = -6$*, which is not positive: M0.*	M1 A1 (2)
(e)	$\frac{1}{-11}\begin{pmatrix} 1 & 2 \\ 5 & -1 \end{pmatrix}\begin{pmatrix} -8 \\ -7 \end{pmatrix}$ (or solve $-x - 2y = -8$, $-5x + y = -7$) $(2, 3)$	M1 A1 (2)
		Total for Question 6: 9 marks

Question 7

8 marks

Part	Scheme	Marks
(a)	$\frac{(-12+8\mathrm{i})(1-\mathrm{i})}{(1+\mathrm{i})(1-\mathrm{i})} = \frac{-4+20\mathrm{i}}{2} = -2+10\mathrm{i}$	M1 A1
	$z = a+b\mathrm{i}$: $3(a+b\mathrm{i}) - 2(a-b\mathrm{i}) = a+5b\mathrm{i}$, so $a=-2$, $5b=10$	M1
	$\boxed{z=-2+2\mathrm{i}}$	A1
		(4)
(b)	$r = 2\sqrt{2}$ and $\theta = \pi - \frac{\pi}{4}$	M1
	$\boxed{z = 2\sqrt{2}\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)}$	A1
	$\theta = -\frac{\pi}{4}$ is the common error: z is in the second quadrant.	
		(2)
(c)	$z^2 = 4 - 8\mathrm{i} + 4\mathrm{i}^2 = \boxed{-8\mathrm{i}}$	B1
	z in the second quadrant and z^2 on the negative imaginary axis	
		B1
		(2)
	Total for Question 7: 8 marks	

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Question 8

8 marks

Part	Scheme	Marks
(a)	$n=1$: $\begin{pmatrix} 3^1 & 3^1-1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix} = \mathbf{M}$, so true for $n=1$	B1
	Assume true for $n=k$: $\mathbf{M}^{k+1} = \begin{pmatrix} 3^k & 3^k-1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3^{k+1} & 2\times 3^k + 3^k - 1 \\ 0 & 1 \end{pmatrix}$	A1
	$2\times 3^k + 3^k = 3\times 3^k = 3^{k+1}$	M1
	$\mathbf{M}^{k+1} = \begin{pmatrix} 3^{k+1} & 3^{k+1}-1 \\ 0 & 1 \end{pmatrix}$	A1
	True for $n=1$, and true for $n=k$ implies true for $n=k+1$, so true for all positive integers n	A1
		(6)
(b)	$\det \mathbf{M}^n = 3^n$	M1
	$(\mathbf{M}^n)^{-1} = \frac{1}{3^n}\begin{pmatrix} 1 & 1-3^n \\ 0 & 3^n \end{pmatrix}$ oe	A1
		(2)
		Total for Question 8: 8 marks

Question 9

9 marks

Part	Scheme	Marks
(a)	$y = 9x^{-1}$, $\frac{dy}{dx} = -9x^{-2} = -\frac{1}{p^2}$ at P	M1 A1
	Normal gradient p^2: $y - \frac{3}{p} = p^2(x - 3p)$	M1
	$py - 3 = p^3x - 3p^4$, so $p^3x - py = 3p^4 - 3$	A1*
		(4)
(b)	Substitute $\left(3q, \frac{3}{q}\right)$ (or $y = \frac{9}{x}$): $3p^3q - \frac{3p}{q} = 3p^4 - 3$	M1
	$p^3q^2 + \left(1 - p^4\right)q - p = 0$	A1
	$q = p$ is one root, so $q = -\frac{1}{p^3}$ (factorise as $(q-p)(p^3q+1) = 0$, or product of roots $= -\frac{p}{p^3}$)	M1
	$\boxed{Q\left(-\frac{3}{p^3}, -3p^3\right)}$	A1
		(4)
(c)	$p = 2$: $\boxed{Q\left(-\frac{3}{8}, -24\right)}$	B1
		(1)
		Total for Question 9: 9 marks

TOTAL FOR PAPER: 75 MARKS

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