eClassroom

A-LEVEL MATHEMATICS • FURTHER PURE 1

Series

Worksheet

Pearson Edexcel International A Level (WFM01) • Calculator allowed: show every step

Free Series worksheet for A-Level and International A-Level Maths, with full worked solutions

A Examples & Your turn page 2 → **B** Fluency core skills page 4 → **C** Reasoning explain it page 6 → **D** Problem solving page 7 → **E** Exam practice page 9

Tap a section to jump straight to it

By the end of this worksheet you can: use the standard results for $\sum r$, $\sum r^2$ and $\sum r^3$ • sum related series • find sums between limits • *Stretch:* simplify to a factorised form, sums from $n+1$ to $2n$, and solve equations involving sums.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Given factorised forms, sums between limits including from $n+1$ to $2n$, and equations in n

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	13
Stretch marks	16	14	17	9
My score				

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Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: a linear sum

Show that $\sum_{r=1}^{n}(6r+1) = n(3n+4)$

$\sum_{r=1}^{n} 1 = n$, *not* 1

$6 \times \frac{1}{2}n(n+1) + n = 3n^2 + 4n$

$n(3n+4)$

Your turn A1

Show that $\sum_{r=1}^{n}(4r+3) = n(2n+5)$

Worked example A2: a standard result

Find $\sum_{r=1}^{12} r^2$

$\frac{1}{6}(12)(13)(25)$

650

Your turn A2

Find $\sum_{r=1}^{9} r^3$

Worked example A3: between limits

Find $\sum_{r=11}^{20} r$

f(20) − f(10), *subtracting up to* $a-1$

$210 - 55$

155

Your turn A3

Find $\sum_{r=6}^{15} r^2$

STRETCH

Worked example A4: a factorised form

Show that $\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$

Take out $\frac{1}{6}n(n+1)$ straight away

$\frac{1}{6}n(n+1)(2n+1)+n(n+1) = \frac{1}{6}n(n+1)\big((2n+1)+6\big)$

$\frac{1}{6}n(n+1)(2n+7)$

Your turn A4

Show that $\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$

Worked example A5: with cubes

Show that $\sum_{r=1}^{n} (r^3+r) = \frac{1}{4}n(n+1)(n^2+n+2)$

$\frac{1}{4}n^2(n+1)^2+\frac{1}{2}n(n+1) = \frac{1}{4}n(n+1)\big(n(n+1)+2\big)$

$\frac{1}{4}n(n+1)(n^2+n+2)$

Your turn A5

Show that $\sum_{r=1}^{n} (r^3-2r) = \frac{1}{4}n(n+1)(n^2+n-4)$

Worked example A6: from $n+1$ to $2n$

Find $\sum_{r=n+1}^{2n} r$ in terms of n

$\mathrm{f}(2n)-\mathrm{f}(n)$

$\frac{1}{2}(2n)(2n+1)-\frac{1}{2}n(n+1)$

$\frac{1}{2}n(3n+1)$

Your turn A6

Find $\sum_{r=n+1}^{2n} r^2$ in terms of n

Worked example A7: solving for n

Find n given that $\sum_{r=1}^{n} r = 153$

$\frac{1}{2}n(n+1) = 153,\ n^2+n-306 = 0,\ (n+18)(n-17) = 0$

$n = 17$

Your turn A7

Find n given that $\sum_{r=1}^{n} (2r+1) = 168$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 Find

(a) $\sum_{r=1}^{20} r$

(b) $\sum_{r=1}^{10} r^2$

(c) $\sum_{r=1}^{6} r^3$

(d) $\sum_{r=1}^{50} 3$

[4 marks]

B2 Find, in simplest form in terms of n,

(a) $\sum_{r=1}^{n} (2r+5)$

(b) $\sum_{r=1}^{n} (3r^2+1)$

(c) $\sum_{r=1}^{n} (r+1)^2$

(d) $\sum_{r=1}^{n} r(r-3)$

[4 marks]

B3 Find

(a) $\sum_{r=21}^{40} r$

(b) $\sum_{r=5}^{12} r^2$

(c) $\sum_{r=4}^{8} r^3$

(d) $\sum_{r=10}^{30} 2r$

[4 marks]

B4 Show that

(a) $\sum_{r=1}^{n} (r^3+3r^2) = \frac{1}{4}n(n+1)(n^2+5n+2)$

(b) $\sum_{r=1}^{n} r^2(r+1) = \frac{1}{12}n(n+1)(n+2)(3n+1)$

(c) $\sum_{r=1}^{n} (2r+1)^2 = \frac{1}{3}n(4n^2+12n+11)$

(d) $\sum_{r=1}^{n} r(r-1) = \frac{1}{3}n(n-1)(n+1)$

[4 marks]

B5 Find, in terms of n,

(a) $\sum_{r=n+1}^{2n} 1$

(b) $\sum_{r=n+1}^{2n} 4r$

(c) $\sum_{r=n+1}^{2n} r^3$, fully factorised

[3 marks]

B6 Find n given that

(a) $\sum_{r=1}^{n} r = 325$

(b) $\sum_{r=1}^{n} (2r-1) = 400$

(c) $\sum_{r=1}^{n} r^3 = 3025$

(d) $\sum_{r=1}^{n} (6r-2) = 310$

[4 marks]

STRETCH

B7 $\sum_{r=1}^{n} (ar+b) = \frac{1}{2}n(3n+7)$ for all n

(a) Find a

(b) Find b

(c) Find $\sum_{r=11}^{20} (ar+b)$

(d) Find the least n for which $\sum_{r=1}^{n} (ar+b) > 1000$

[4 marks]

B8 Find, in terms of n,

(a) $\sum_{r=n}^{2n} r$

(b) $\sum_{r=n}^{2n} 1$

(c) $\sum_{r=n}^{3n} r$

(d) $\sum_{r=1}^{2n} r - 2\sum_{r=1}^{n} r$

[4 marks]

B9 Write each series in sigma notation and find its value.

(a) $1 \times 2 + 2 \times 3 + \cdots + 20 \times 21$

(b) $2^2 + 4^2 + \cdots + 40^2$

(c) $1 + 3 + 5 + \cdots + 59$

(d) $5 + 8 + 11 + \cdots + 62$

[4 marks]

B10 $f(n) = \sum_{r=1}^{n} r^2(r-1)$

(a) Show that $f(n) = \frac{1}{12}n(n-1)(n+1)(3n+2)$

(b) Find f(10)

(c) Find $\sum_{r=11}^{20} r^2(r-1)$

(d) Check part (a) directly when $n = 3$

[4 marks]

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 Amir writes $\sum_{r=1}^{n}(2r+5) = n(n+1)+5$. Explain his error, and give the correct sum. **[2 marks]**

C2 Bea finds $\sum_{r=10}^{20} r$ as $\sum_{r=1}^{20} r - \sum_{r=1}^{10} r = 155$. Explain her error, and find the sum. **[2 marks]**

C3 Cal says that $\sum_{r=1}^{n} r^2 = \left(\sum_{r=1}^{n} r\right)^2$

Use $n = 4$ to show that he is wrong, and state the correct formula. **[3 marks]**

C4 Dev writes $\sum_{r=1}^{n} r(r+1) = \left(\sum_{r=1}^{n} r\right)\left(\sum_{r=1}^{n}(r+1)\right)$

Use $n = 2$ to show that he is wrong, and give the correct method. **[3 marks]**

STRETCH

C5 Eve finds $\sum_{r=n+1}^{2n} r$ as $\sum_{r=1}^{2n} r + \sum_{r=1}^{n} r$

Explain her error, and find the sum in terms of n **[3 marks]**

C6 Fay is asked for n such that $\sum_{r=1}^{n} r = 100$. She writes $n \approx 13.65$

Explain why there is no such n **[3 marks]**

C7 Asked to show that $\sum_{r=1}^{n} r(r+3) = \frac{1}{3}n(n+1)(n+5)$, Gus writes $\frac{1}{3}n^3 + 2n^2 + \frac{5}{3}n$ and stops.

Explain why this would not score full marks, and complete the working. **[3 marks]**

C8 **(a)** Show that $\sum_{r=1}^{n} r(n-r) = \frac{1}{6}n(n-1)(n+1)$, where n is treated as a constant inside the sum.

(3)

(b) Hence find $1\times 9 + 2\times 8 + 3\times 7 + \cdots + 9\times 1$

(2)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$$

You must show each stage of your working, including the common factor you take out. **[3 marks]**

D2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Use the standard results for $\sum_{r=1}^{n} r^3$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+1)(r+3) = \frac{1}{12}n(n+1)(3n^2+19n+26)$$

(3)

(b) Hence find the value of $\sum_{r=1}^{10} r(r+1)(r+3)$, showing the substitution you make.

(1)

[4 marks]

D3 Given that, for all positive integers n, $\sum_{r=1}^{n} r(r+2) = \frac{1}{6}n(n+1)(2n+7)$, write the following series in sigma notation and hence determine its value, showing your working clearly:

$$3\times5+4\times6+5\times7+\cdots+30\times32$$

[3 marks]

D4 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (5r-2)^2 = \frac{n}{6}(an^2+bn+c)$$

where a, b and c are integers to be determined. **[3 marks]**

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STRETCH

D5 **(a)** Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (6r^2 + 2r) = 2n(n+1)^2$$

(2)

(b) Hence find the positive integer n for which $\sum_{r=1}^{n} (6r^2 + 2r) = 20 \sum_{r=1}^{n} r$

(2)

[4 marks]

D6 **(a)** Use the standard results to show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+5) = \frac{1}{3}n(n+1)(n+8)$$

(2)

(b) Hence find the value of the positive integer n for which $\sum_{r=1}^{n} r(r+5) = 12 \sum_{r=1}^{n} r$, explaining why any other solution is rejected.

(2)

[4 marks]

D7 **(a)** Use the standard results to show that, for all positive integers n,

$$\sum_{r=1}^{n} (6r^2 - 1) = n^2(2n+3)$$

(2)

(b) Hence, without expanding the sum again, show that $\sum_{r=n+1}^{2n} (6r^2 - 1) = n^2(14n+9)$

(2)

[4 marks]

D8 **(a)** Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (r+2)(r-1) = \frac{1}{3}n(n-1)(n+4)$$

(3)

(b) Hence, by first writing it in sigma notation, determine the value of

$$18 \times 21 + 19 \times 22 + 20 \times 23 + \cdots + 38 \times 41$$

(2)

[5 marks]

E Exam practice

Exam-style questions in the style of WFM01. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

(a) Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (r+2)(2r-1) = \frac{n}{6}(an^2+bn+c)$$

where a, b and c are integers to be determined.

(4)

(b) Hence determine the value of $\sum_{r=1}^{20} (r+2)(2r-1)$

(2)

(Total for Question E1 is 6 marks)

E2 **(a)** Use the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that, for all positive integers n,

$$\sum_{r=1}^{n} (6r^2-2r+1) = n(2n^2+2n+1)$$

(4)

(b) Hence show that there is exactly one positive integer n for which

$$\sum_{r=1}^{n} (6r^2-2r+1) = 3\sum_{r=1}^{n} r^2 + 237$$

and find this value of n

(3)

(Total for Question E2 is 7 marks)

STRETCH

E3 **(a)** Using the standard summation formulae, show that, for all positive integers n,

$$\sum_{r=1}^{n} r(r-2)(r+2) = \frac{1}{4}n(n+1)(n^2+n-8)$$

(3)

(b) Determine the value of n for which $\sum_{r=1}^{n} r(r-2)(r+2) = 6\sum_{r=1}^{n} r$

(3)

(c) Determine the value of $\sum_{r=10}^{20} r(r-2)(r+2)$

(3)

(Total for Question E3 is 9 marks)

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