



eClassroom

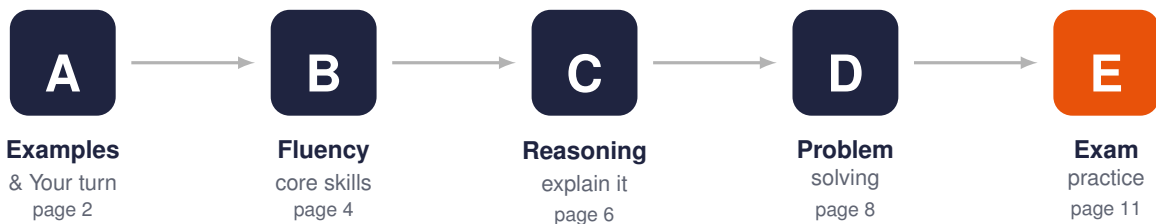
A-LEVEL MATHEMATICS • PURE 3

Functions and Graphs

Worksheet

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Free Functions and Graphs worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: modulus, domain and range • composite and inverse functions • *Stretch:* modulus graphs and transformations.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Modulus graphs with lines, inverses and their domains, composites and transformations.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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Exam questions by topic

Algebra revision engine

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Name

Class

Date



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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Solve $|2x - 3| = 5$

Solve $+(2x - 3) = 5$ and $-(2x - 3) = 5$ separately.

$$2x - 3 = 5 \text{ or } 2x - 3 = -5$$

$$x = 4, \quad x = -1$$

Your turn A1

Solve $|3x + 1| = 2x + 6$, checking each answer.

Worked example A2

$f(x) = 2x - 1$ and $g(x) = x^2 + 3$. Find $fg(2)$ and $gf(x)$

$fg(2) = f(g(2))$: apply g first.

$$f(7) = 13; \quad g(2x - 1) = (2x - 1)^2 + 3$$

$$13, \quad 4x^2 - 4x + 4$$

Your turn A2

$f(x) = 3x + 2$ and $g(x) = x^2 - 1$. Find $gf(1)$ and $fg(x)$

Worked example A3

Find $f^{-1}(x)$ for $f(x) = \frac{x+4}{x-2}$, $x > 2$, and state its domain.

$y(x - 2) = x + 4$, so $x(y - 1) = 2y + 4$; the range of f is $y > 1$

$$f^{-1}(x) = \frac{2x+4}{x-1}, \quad x > 1$$

Your turn A3

Find $f^{-1}(x)$ for $f(x) = \frac{2x+1}{x-3}$, $x > 3$, and state its domain.



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STRETCH

Worked example A4: the range

Find the range of $f(x) = x^2 - 4x + 7$, $x \geq 0$

Complete the square: $(x - 2)^2 + 3$. The vertex $x = 2$ is in the domain.

$$f(x) \geq 3$$

Your turn A4

Find the range of $f(x) = 5 - (x + 1)^2$, $-3 \leq x \leq 2$

Worked example A5: $|f(x)|$ and $f(|x|)$

$P(2, -5)$ lies on $y = f(x)$. Find its images on $y = |f(x)|$ and on $y = f(|x|)$

$|f(x)|$ reflects negative y values; $f(|x|)$ copies the right half into the left.

$$(2, 5); (2, -5) \text{ and } (-2, -5)$$

Your turn A5

$Q(5, -1)$ lies on $y = f(x)$. Find its images on $y = |f(x)|$ and on $y = f(|x|)$

Worked example A6: combined transformations

$P(4, 6)$ lies on $y = f(x)$. Find its image on $y = 2f(x + 1) - 3$

x : subtract 1. y : multiply by 2, then subtract 3.

$$(3, 9)$$

Your turn A6

$Q(-2, 8)$ lies on $y = f(x)$. Find its image on $y = \frac{1}{2}f(2x) + 1$

Worked example A7: a modulus inequality

Solve $|x - 3| < 2x$

Sketch both graphs: the V is below the line to the right of where they meet.

$-(x - 3) = 2x$ gives $x = 1$ (the other branch never meets the line)

$$x > 1$$

Your turn A7

Solve $|2x + 1| > x + 5$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 Solve, where possible,

(a) $|x| = 7$

(b) $|x - 2| = 7$

(c) $|2x - 2| = 7$

(d) $|2x - 2| = -7$

[4 marks]

B2 Find the range of each function.

(a) $f(x) = 2x + 1, 0 \leq x \leq 3$

(b) $f(x) = x^2, -2 \leq x \leq 3$

(c) $f(x) = \frac{1}{x}, x \geq 2$

(d) $f(x) = (x - 1)^2 + 4, x \in \mathbb{R}$

[4 marks]

B3 $f(x) = x + 3, g(x) = 2x^2, h(x) = \frac{1}{x}, x \neq 0$. Find

(a) $fg(1)$

(b) $gf(1)$

(c) $fh(x)$

(d) $hgf(x)$

[4 marks]

B4 Find each inverse function, stating its domain.

(a) $f(x) = 4x - 3, x \in \mathbb{R}$

(b) $f(x) = \frac{x+5}{2}, x \in \mathbb{R}$

(c) $f(x) = \frac{2}{x-1}, x > 1$

(d) $f(x) = \sqrt{x-2}, x \geq 2$

[4 marks]



CORE*continued: transformations and inequalities***B5** $P(2, -6)$ lies on $y = f(x)$. Find the image of P on

(a) $y = f(x - 3)$

(b) $y = 3f(x)$

(c) $y = |f(x)|$

[3 marks]**B6** Solve

(a) $|x - 4| = 3$

(b) $|x - 4| < 3$

(c) $|x - 4| = x$

(d) $|x - 4| > x$

[4 marks]**STRETCH****B7** $P(-3, 4)$ lies on $y = f(x)$. Find the image of P on

(a) $y = f(2x) + 1$

(b) $y = -f(x + 2)$

(c) $y = 2f\left(\frac{1}{2}x\right) - 3$

(d) $y = |f(-x)|$

[4 marks]**B8** $f(x) = 2x + 1$, $x \in \mathbb{R}$, and $g(x) = x^2$, $x \geq 0$

(a) Find $f^{-1}(x)$

(b) Find $g^{-1}(x)$

(c) Find $(fg)^{-1}(x)$ and state its domain.

(d) Find $ff(x)$

[4 marks]**B9** $f(x) = x^2 - 6x + 5$

(a) Find the roots and the vertex of $y = f(x)$

(b) State the coordinates of the maximum point of $y = |f(x)|$ between the roots.

(c) State the coordinates of the minimum points of $y = f(|x|)$

(d) Find the number of solutions of $|f(x)| = 4$

[4 marks]**B10**

(a) Explain why $f(x) = x^2$, $x \in \mathbb{R}$, has no inverse.

(b) $h(x) = (x + 2)^2 - 5$, $x \geq k$. State the least value of k for which h has an inverse.

(c) Using this value of k , find $h^{-1}(x)$

(d) State the domain of h^{-1}

[4 marks]

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

- C1** $f(x) = x + 1$ and $g(x) = 3x^2$. Sam works out $gf(2)$ as $f(g(2)) = 13$
Explain Sam's mistake, and find $gf(2)$ [2 marks]
- C2** For $f(x) = 2x + 3$, Lily writes $f^{-1}(x) = \frac{1}{2x+3}$
Explain Lily's mistake, and find $f^{-1}(x)$ [2 marks]
- C3** $f(x) = 9 - x^2$, $-1 \leq x \leq 2$. Kai says the range is $f(x) \leq 9$
Explain why Kai's answer is incomplete, and give the range. [3 marks]
- C4** To solve $f(x) = x$ for $f(x) = x^3 - 3x$, Jo writes $x^3 = 4x$, divides by x and gives $x = \pm 2$
Explain what Jo has lost. [3 marks]



STRETCH

- C5** Max solves $|x - 1| = 2x + 4$ and gives $x = -5$ and $x = -1$
Explain which answer must be rejected and why. [3 marks]
- C6** $f(x) = \sqrt{x-1} + 2$, $x \geq 1$. Ali writes $f^{-1}(x) = (x-2)^2 + 1$
Explain what is missing from Ali's answer, and why it matters. [3 marks]
- C7** $P(4, 3)$ lies on $y = f(x)$. For $y = f(2x+6)$, Kai says the image of P has x coordinate $\frac{4}{2} - 6 = -4$
Explain Kai's mistake, and find the image of P [3 marks]
- C8** $f(x) = \frac{ax+b}{cx-a}$, where a, b, c are constants with $a^2 + bc \neq 0$, and $x \neq \frac{a}{c}$
- (a) Prove that $ff(x) = x$ (3)
- (b) Hence write down $f^{-1}(x)$ when $f(x) = \frac{3x+1}{2x-3}$ (1)
- (c) Explain what (a) tells you about the graph of $y = f(x)$ (1)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** Figure 1 shows the graph with equation $y = 3|x + 2| - 6$. The point P is the vertex of the graph, and the graph passes through the origin.

(a) State the coordinates of P

(1)

(b) Use algebra to solve the equation $3|x + 2| - 6 = x$, considering each branch of the graph separately and checking each answer in the original equation.

(2)

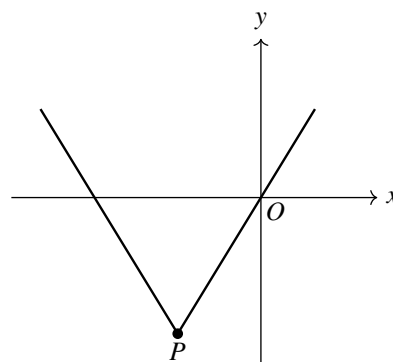


Figure 1

[3 marks]

- D2** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The functions f and g are defined by

$$f(x) = \frac{3x}{x+2} \quad x \geq 0 \qquad g(x) = 5 - x^2 \quad x \in \mathbb{R}$$

(a) Find the value of $gf(2)$

(1)

(b) By writing $f(x) = 3 - \frac{6}{x+2}$, find the range of f , giving both ends and explaining whether each end is included.

(1)

(c) Find $f^{-1}(x)$ and state its domain.

(2)

[4 marks]

- D3** The curve with equation $y = f(x)$ passes through the point $P(5, -2)$, and no other information about f is given. Using only the point P , find the coordinates of the point to which P is mapped when the curve is transformed to the curve with equation

(a) $y = f(x + 3) - 1$

(1)

(b) $y = |f(2x)|$

(1)

(c) $y = -3f(x)$

(1)

[3 marks]



D4 The function f is defined by $f(x) = 7 - \frac{12}{x+3}$, $x \geq 0$

(a) Find the range of f , explaining why the upper end is not included.

(1)

(b) Find $f^{-1}(x)$, stating its domain and explaining how it is linked to your answer to part (a).

(2)

[3 marks]

STRETCH

D5 Figure 2 shows the graph of $y = f(|x|)$, where $f(x) = x^2 - 6x$. The graph passes through the origin and has minimum points A and B

(a) State the coordinates of A and B

(1)

(b) Solve $f(|x|) = 16$, explaining why one root of your quadratic is rejected.

(2)

(c) Using Figure 2, state the values of the constant k for which the equation $f(|x|) = k$ has exactly two solutions.

(1)

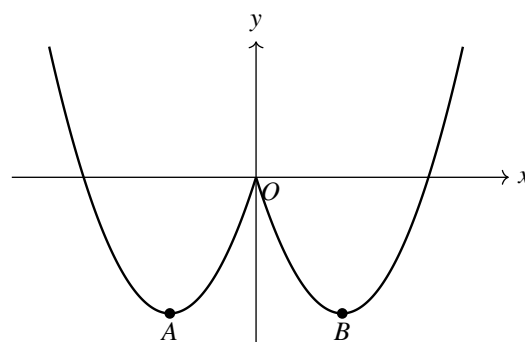


Figure 2

[4 marks]

D6 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The function f is defined by $f(x) = x^2 - 2$, $x \geq 0$

(a) Find an expression for $f^{-1}(x)$ and state its domain and range.

(2)

(b) Explain why the graphs of $y = f(x)$ and $y = f^{-1}(x)$ can only meet on the line $y = x$, and hence solve $f(x) = f^{-1}(x)$

(2)

[4 marks]



- D7** Figure 3 shows the graph with equation $y = |x - 4| + 2$, which has vertex V , and a line with equation $y = mx$, where m is a positive constant.

(a) State the coordinates of V

(1)

(b) Find the value of m for which the line passes through V

(1)

(c) Find the set of values of m for which the line meets the graph at exactly two points, explaining your reasoning with the gradients of the line and of the two branches of the graph.

(2)

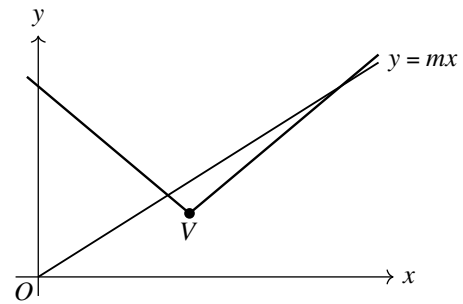


Figure 3

[4 marks]

- D8** The functions f and g are defined by

$$f(x) = 2x + 3 \quad x \in \mathbb{R} \qquad g(x) = \frac{4}{x-1} \quad x > 1$$

(a) Find $fg(x)$, giving your answer in its simplest form.

(1)

(b) State the range of fg

(1)

(c) Solve $fg(x) = 7$

(1)

(d) Find $gf(x)$ as a single fraction, and state the set of values of x for which it is defined, explaining how the domain of g decides this.

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA13. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 4 shows the graph with equation

$$y = |2x - 6| - 4$$

The graph has vertex V , meets the y -axis at A and meets the x -axis at the points B and C , as shown in Figure 4.

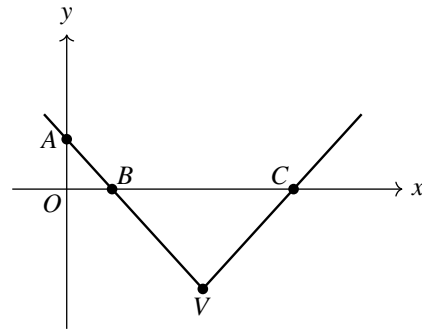


Figure 4

- (a) State the coordinates of V , A , B and C

.....
(3)

- (b) Use algebra to solve the equation $|2x - 6| - 4 = x - 1$, considering each branch of the graph and checking each answer in the original equation.

.....
(2)

(Total for Question E1 is 5 marks)



E2 The function f is defined by

$$f(x) = \frac{3x+2}{x-1} \quad x > 1$$

(a) Find $f^{-1}(x)$, stating its domain. You may find it helpful to write $f(x) = 3 + \frac{5}{x-1}$ first.

.....
(3)

(b) Show that $ff(x) = \frac{ax+b}{cx+d}$, where a, b, c and d are integers to be found.

.....
(2)

The point $P(2, 8)$ lies on the curve with equation $y = f(x)$

(c) Find the point to which P is mapped when the curve is transformed to the curve with equation $y = 2f(x+1) - 5$

.....
(1)

(Total for Question E2 is 6 marks)



STRETCH

E3 The functions f and g are defined by

$$f(x) = x^2 - 4x + 1 \quad x \geq 2$$

$$g(x) = |x - 3| \quad x \in \mathbb{R}$$

Figure 5 shows the graphs of $y = f(x)$ and $y = f^{-1}(x)$, which are reflections of each other in the line $y = x$.

The end points of the graphs are P and Q

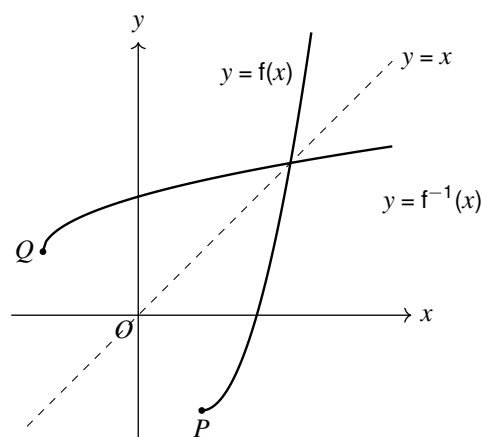


Figure 5

(a) Find the range of f

.....
(1)

(b) Find $f^{-1}(x)$, stating its domain.

.....
(3)

(c) Explain why $fg(x)$ is only defined when $|x - 3| \geq 2$, and solve $fg(x) = 6$

.....
(3)

(d) Explain why the equation $gf(x) = -1$ has no solutions.

.....
(1)
(Total for Question E3 is 8 marks)

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