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INTERNATIONAL A LEVEL MATHEMATICS • MECHANICS 1 (WME01)

Mechanics 1 Practice Paper 3

Worked Solutions

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WME01/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen — the working a full-marks answer shows, with a short note on why each step is taken

M1 A1 — the mark earned on that line: M = method, A = accuracy, B = independent mark

boxed — the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	Three particles and a second collision	12	2
2	A car overtaking a van	12	4
3	A string at an angle to a rough slope	13	5
4	A non-uniform rod hanging from two strings	13	7
5	A pulley and a string jerked taut	12	9
6	A particle under two constant forces	13	11

Question paper & mark scheme | Exam questions by topic | More practice papers

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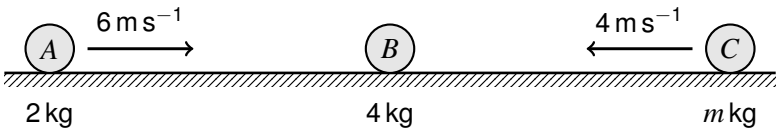

Figure 1

Three particles A, B and C, of masses 2 kg, 4 kg and m kg respectively, move in the same straight line on a smooth horizontal floor, with B between A and C. Particle B is at rest. Particle A moves towards B with speed 6 m s^{-1} and particle C moves towards B with speed 4 m s^{-1}, as shown in Figure 1.

A collides with B. Immediately after this collision, A moves with speed 1 m s^{-1} and its direction of motion is reversed.

(a) Find the speed of B immediately after the collision.

(3)

(b) Find the magnitude of the impulse exerted on B by A in the collision.

(2)

B then collides with C, and the two particles coalesce to form a single particle D

(c) Find the range of values of m for which D moves towards A

(4)

(d) Given that there is a second collision between A and D, find the range of possible values of m

(3)

(a) Take the direction A → B as positive. A rebounds, so its new velocity is −1.

Momentum: $2 \times 6 + 4 \times 0 = 2 \times (-1) + 4v$ M1

$12 = -2 + 4v \Rightarrow 4v = 14$ A1

$v = 3.5$ $\boxed{3.5 \text{ m s}^{-1}}$ A1

(b) Impulse on B = change in momentum of B = $4 \times 3.5 - 0$ M1

$\boxed{14 \text{ N s}}$ A1

(c) C moves towards B: velocity −4. Momentum for B and C:

$4 \times 3.5 + m \times (-4) = (4 + m)v \Rightarrow 14 - 4m = (4 + m)v$ M1

$v = (14 - 4m) / (4 + m)$ A1

D moves towards A when v is negative: $14 - 4m < 0$ M1

$\boxed{m > 3.5}$ A1

(d) A is moving away at 1 m s^{-1}. D only catches A if D's speed towards A is more than 1:

$(4m - 14) / (4 + m) > 1$ ($4 + m > 0$, so multiply through) M1

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Question 1 continued

$4m - 14 > 4 + m \Rightarrow 3m > 18$ A1

$\boxed{m > 6}$ ($m > 3.5$ alone is not enough: D could be too slow) A1

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(Total for Question 1 is 12 marks)

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2. A van moves along a straight horizontal road at a constant speed of $8\,\text{m s}^{-1}$. At time $t = 0$ the van passes a point O, where a car is at rest. At time $t = 5\,\text{s}$ the car sets off from O in the same direction as the van. The car accelerates uniformly at $1\,\text{m s}^{-2}$ until it reaches a speed of $24\,\text{m s}^{-1}$, and then moves at a constant speed of $24\,\text{m s}^{-1}$

(a) On the same axes, sketch speed-time graphs for the motion of the van and the car for $0 \leqslant t \leqslant 35$

(3)

(b) Find the greatest distance between the van and the car before the car overtakes the van.

(3)

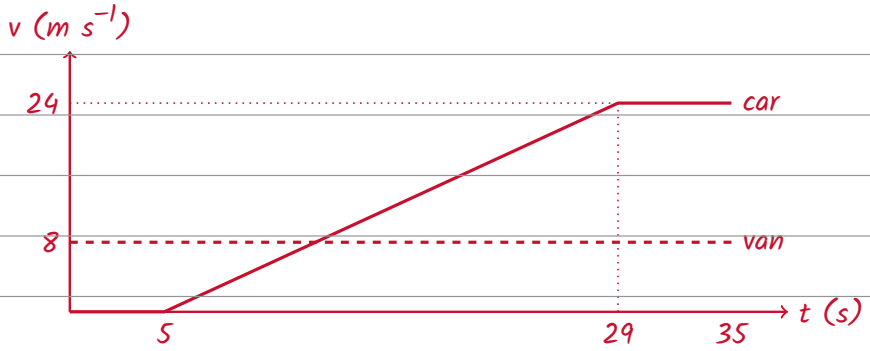
(c) Find the value of t at which the car overtakes the van.

(5)

(d) Find the distance from O at which the car overtakes the van.

(1)

(a) Van: constant 8. Car: 0 until t = 5, then gradient 1, reaching 24 at t = 5 + 24 = 29. B1 B1 B1



(b) The van pulls away until the car is as fast as it: $t - 5 = 8 \Rightarrow t = 13$ M1

Van: $8 \times 13 = 104$ m. Car: $\frac{1}{2} \times 8 \times 8 = 32$ m. Gap $= 104 - 32$ M1

72 m A1

(c) While the car accelerates ($5 \leqslant t \leqslant 29$): car $\frac{1}{2}(t-5)^2$, van $8t$

$\frac{1}{2}(t-5)^2 = 8t$ M1 A1

$t^2 - 10t + 25 = 16t \Rightarrow t^2 - 26t + 25 = 0$ M1

$(t-1)(t-25) = 0 \Rightarrow t = 1$ or $t = 25$ A1

$t = 1$: the car has not set off, reject. $25 < 29$ ✓ $t = 25$ A1

(d) 8×25 200 m from O B1

(Total for Question 2 is 12 marks)

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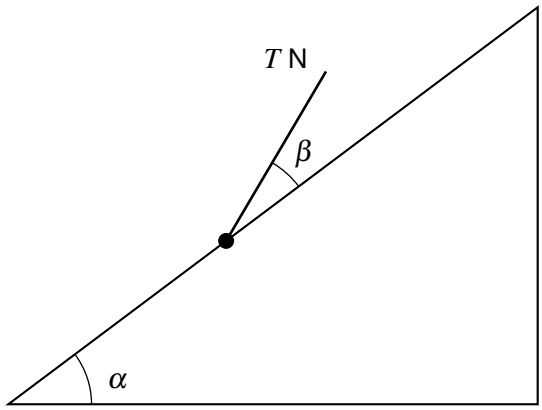

Figure 2

A particle of mass 4 kg is held in equilibrium on a rough plane inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is 0.5

The particle is held by a light string, which lies in the vertical plane containing a line of greatest slope of the plane. The string is inclined at an angle β to the line of greatest slope, where $\tan\beta = \frac{5}{12}$, and the tension in the string is T newtons, as shown in Figure 2.

(a) Find, in terms of T, the magnitude of the normal reaction between the particle and the plane.

(2)

(b) Find the least possible value of T

(4)

(c) Find the greatest possible value of T

(3)

(d) Given that $T = 20$, find the magnitude and direction of the friction acting on the particle.

(2)

The string is now cut.

(e) Find the acceleration of the particle.

(2)

(a) $\sin\alpha = 3/5$, $\cos\alpha = 4/5$; $\sin\beta = 5/13$, $\cos\beta = 12/13$. The string lifts the particle.

Perpendicular to the plane: $R + T\sin\beta = 4g\cos\alpha$ M1

$R = 31.36 - 5T/13$ A1

(b) $\tan\alpha = 0.75 > 0.5$, so without help it slides down. Least T: friction $0.5R$ up the plane.

$T\cos\beta + 0.5R = 4g\sin\alpha$ M1

$12T/13 + 0.5(31.36 - 5T/13) = 23.52$ A1

$12T/13 - 2.5T/13 = 23.52 - 15.68 \Rightarrow 9.5T/13 = 7.84$ M1

$T = 10.728\ldots$ least $T = 10.7$ A1

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Question 3 continued

(c) Greatest T: about to slide up, friction down the plane:

$T\cos\beta = 4g\sin\alpha + 0.5R \Rightarrow 12T/13 = 23.52 + 15.68 - 2.5T/13$ M1 A1

$14.5T/13 = 39.2 \Rightarrow T = 35.14...$ greatest $T = 35.1$ A1

(d) Up the plane: $20 \times 12/13 = 18.46$; down the plane: $4g\sin\alpha = 23.52$ M1

The string alone is too weak, so friction helps it: 5.06 N up the plane A1

(e) No string, so now $R = 4g\cos\alpha = 31.36$: $23.52 - 0.5 \times 31.36 = 4a$ M1

$4a = 7.84$ $1.96\ m\ s^{-2}$ down the plane A1

(Total for Question 3 is 13 marks)

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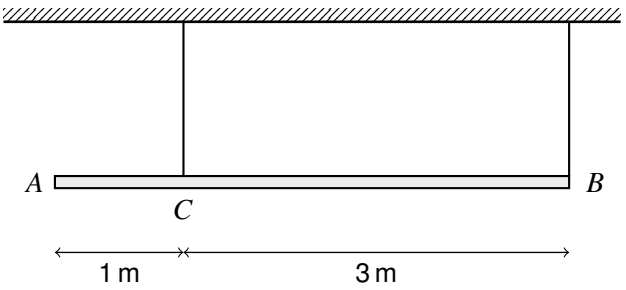

Figure 3

A non-uniform rod AB has mass 4 kg and length 4 m. The rod hangs in equilibrium in a horizontal position from two light inextensible vertical strings. One string is attached to the rod at C, where $AC = 1$ m, and the other string is attached to the rod at B, as shown in Figure 3.

When nothing else is attached to the rod, the tension in the string at C is five times the tension in the string at B

(a) Find the distance of the centre of mass of the rod from A

(4)

A particle of mass 4 kg is now attached to the rod at the point P, where $AP = x$ m. The rod still hangs in equilibrium in a horizontal position.

(b) Given that $x = 3$, find the tension in each string.

(3)

The string at B will break if its tension exceeds 39.2 N. The string at C will not break.

(c) Find the range of possible values of x

(6)

(a) $T_C + T_B = 4g$ and $T_C = 5T_B \Rightarrow 6T_B = 4g \Rightarrow T_B = 2g/3$ M1

G is d m from A. Moments about C: $T_B \times 3 = 4g \times (d - 1)$ M1

$(2g/3) \times 3 = 4g(d - 1) \Rightarrow 2 = 4(d - 1)$ A1

$d = 1.5$ 1.5 m from A A1

(b) G is 0.5 m from C; P is 2 m from C. Moments about C: $3T_B = 4g \times 0.5 + 4g \times 2$ M1

$3T_B = 10g \Rightarrow T_B = 32.67$ $T_B = 32.7$ N A1

$T_C = 8g - T_B = 78.4 - 32.67$ $T_C = 45.7$ N A1

(c) Moments about C: $3T_B = 4g \times 0.5 + 4g(x - 1) = 4g\,x - 2g$ M1

$T_B = g(4x - 2)/3$ A1

P near A pulls the rod round C: the string at B goes slack. Need $T_B \geqslant 0$: M1

$4x - 2 \geqslant 0 \Rightarrow x \geqslant 0.5$ A1

String at B must not break: $g(4x - 2)/3 \leqslant 39.2 = 4g \Rightarrow 4x - 2 \leqslant 12$ M1

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Question 4 continued

$x \leqslant 3.5$ $\boxed{0.5 \leqslant x \leqslant 3.5}$ $(T_C = 8g - T_B > 0$ throughout$)$ A1

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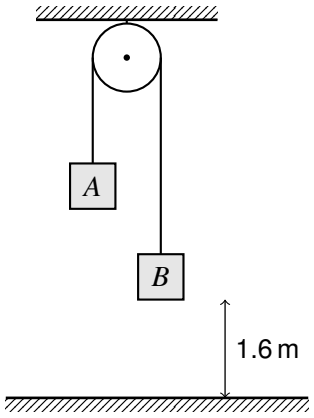

Figure 4

Two particles A and B, of masses 2 kg and 3 kg respectively, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley. The particles hang with the string taut and the hanging parts of the string vertical. B is 1.6 m above horizontal ground, as shown in Figure 4. The system is released from rest. A does not reach the pulley.

(a) Find the acceleration of B and the tension in the string while B is falling.

(4)

(b) Find the speed of B just before it hits the ground.

(2)

B does not rebound. After B hits the ground, A continues to move and the string becomes slack. Later, the string becomes taut again and B is jerked off the ground.

(c) Find the time between B hitting the ground and the string becoming taut again.

(2)

(d) Find the speed with which B starts to rise.

(3)

(e) Find the magnitude of the impulse exerted on B by the string when it becomes taut.

(1)

(a) B falls, A rises, with the same acceleration a. B: $3g - T = 3a$ M1

A: $T - 2g = 2a$ M1

Add: $g = 5a \Rightarrow a = 9.8 \div 5$ $a = 1.96\ m\ s^{-2}$ A1

$T = 2(9.8 + 1.96) = 23.52$ $T = 23.5\ N$ A1

(b) $v^2 = 0 + 2 \times 1.96 \times 1.6 = 6.272$ M1

$v = 2.504...$ $2.50\ m\ s^{-1}$ A1

(c) String slack: A moves freely, up at 2.504, under gravity. It is taut again

when A is back where it was: $0 = 2.504t - 4.9t^2 \Rightarrow t = 2.504 \div 4.9$ M1

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Question 5 continued

$t = 0.5110...$ $\boxed{0.511 \text{ s}}$ A1

(d) A returns down at 2.504. Jerk: momentum conserved, then a common speed w:

$2 \times 2.504 + 3 \times 0 = (2 + 3)w$ M1 A1

$w = 1.0017...$ $\boxed{1.00 \text{ m s}^{-1}}$ A1

(e) Impulse on B $= 3 \times 1.0017 - 0$ $\boxed{3.01 \text{ N s}}$ B1

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(Total for Question 5 is 12 marks)

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6. *In this question,* $\mathbf{i}$ *and* $\mathbf{j}$ *are perpendicular horizontal unit vectors, and position vectors are given relative to a fixed origin* O

A particle P of mass 2 kg moves on a smooth horizontal surface under the action of a constant force $\mathbf{F} = (6\mathbf{i} - 8\mathbf{j})$ N. At time $t = 0$, P passes through O with velocity $(-4\mathbf{i} + 10\mathbf{j})$ m s^{-1}

At time $t = 2$ s, the force $\mathbf{F}$ is removed and replaced by a constant force $\mathbf{G}$. From $t = 2$, P moves in a straight line and comes to instantaneous rest at time $t = 4$ s.

(a) Find the velocity of P at time t seconds, for $0 \leqslant t \leqslant 2$

(2)

(b) Find the least speed of P for $0 \leqslant t \leqslant 2$

(4)

(c) Find $\mathbf{G}$

(3)

(d) Find the position vector of P when it comes to rest.

(4)

(a) $a = F \div 2 = 3i - 4j$. $v = u + at = (-4i + 10j) + (3i - 4j)t$ M1

$v = (3t - 4)i + (10 - 4t)j$ A1

(b) speed$^2 = (3t - 4)^2 + (10 - 4t)^2 = 9t^2 - 24t + 16 + 16t^2 - 80t + 100$ M1

$= 25t^2 - 104t + 116 = 25(t - 2.08)^2 + 7.84$ A1

$t = 2.08$ is after F changes. Speed falls on $0 \leqslant t \leqslant 2$, so least at $t = 2$ M1

speed$^2 = 100 - 208 + 116 = 8 \Rightarrow$ speed $= \sqrt{8}$ 2.83 m s^{-1} A1

(c) At $t = 2$: $v = 2i + 2j$. Straight line, to rest in 2 s, so a is opposite to v: M1

$a = (0 - (2i + 2j)) \div 2 = -i - j$ M1

$G = 2a$ $G = (-2i - 2j)$ N A1

(d) $r = ut + \frac{1}{2}at^2$ at $t = 2$: $2(-4i + 10j) + \frac{1}{2}(3i - 4j) \times 4$ M1

$= -8i + 20j + 6i - 8j = -2i + 12j$ A1

Next 2 s: $(2i + 2j) \times 2 + \frac{1}{2}(-i - j) \times 4 = 4i + 4j - 2i - 2j = 2i + 2j$ M1

$r = -2i + 12j + 2i + 2j$ $14j$ m (due north of O) A1

(Total for Question 6 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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