eClassroom

Vectors in Mechanics

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Mechanics 1

Question 1 — Worked Solution

(a)

$$\mathbf{a} + \mathbf{b} = (3 + 1)\mathbf{i} + (4 - 2)\mathbf{j}$$

$$\therefore = 4\mathbf{i} + 2\mathbf{j}$$

(b)

$$\mathbf{a} - \mathbf{b} = (3 - 1)\mathbf{i} + (4 - (-2))\mathbf{j}$$

$$\therefore = 2\mathbf{i} + 6\mathbf{j}$$

(c)

$$2\mathbf{a} + 3\mathbf{b} = (6 + 3)\mathbf{i} + (8 - 6)\mathbf{j}$$

$$\therefore = 9\mathbf{i} + 2\mathbf{j}$$

Question 2 — Worked Solution

(a)

$$|\mathbf{v}| = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169}$$

$$\therefore = 13 \text{ m s}^{-1}$$

(b) Bearing

The vector (5**i** – 12**j**) has a positive **i**-component (east) and negative **j**-component (south). This lies in the SE quadrant.

$$\text{Angle east of south} = \arctan\left(\frac{5}{12}\right) = 22.6°$$

$$\therefore \text{Bearing} = 180° - 22.6° = 157.4°$$

Question 3 — Worked Solution

(a)

$$\mathbf{R} = \mathbf{F_1} + \mathbf{F_2} = (2 + 4)\mathbf{i} + (5 - 1)\mathbf{j}$$

$$\therefore = (6\mathbf{i} + 4\mathbf{j})\ \text{N}$$

(b)

$$|\mathbf{R}| = \sqrt{6^2 + 4^2} = \sqrt{36 + 16} = \sqrt{52} = \sqrt{4 \times 13}$$

$$\therefore = 2\sqrt{13}\ \text{N}$$

Question 4 — Worked Solution

$$\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{R}$$

$$\mathbf{F_3} = \mathbf{R} - \mathbf{F_1} - \mathbf{F_2}$$

$$= (5\mathbf{i} + 7\mathbf{j}) - (3\mathbf{i} - 2\mathbf{j}) - (-\mathbf{i} + 4\mathbf{j})$$

$$= (5 - 3 + 1)\mathbf{i} + (7 + 2 - 4)\mathbf{j}$$

$$\therefore \mathbf{F_3} = (3\mathbf{i} + 5\mathbf{j}) \text{ N}$$

Question 5 — Worked Solution

(a)

$$|8\mathbf{i} - 6\mathbf{j}| = \sqrt{64 + 36} = \sqrt{100} = 10$$

$$\therefore \hat{\mathbf{u}} = \frac{1}{10}(8\mathbf{i} - 6\mathbf{j}) = \frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}$$

(b)

$$\mathbf{F} = 15 \times \hat{\mathbf{u}} = 15\left(\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}\right)$$

$$\therefore = (12\mathbf{i} - 9\mathbf{j})\ \text{N}$$

Question 6 — Worked Solution

For equilibrium: F1 + F2 + F3 = 0

$$\mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2}) = -((5-3)\mathbf{i} + (2+1)\mathbf{j})$$

$$\therefore \mathbf{F_3} = (-2\mathbf{i} - 3\mathbf{j})\ \text{N}$$

Question 7 — Worked Solution

$$\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$$

$$\mathbf{r}(4) = (3\mathbf{i} - \mathbf{j}) + 4(2\mathbf{i} + 5\mathbf{j})$$

$$= (3\mathbf{i} - \mathbf{j}) + (8\mathbf{i} + 20\mathbf{j})$$

$$\therefore = (11\mathbf{i} + 19\mathbf{j}) \text{ m}$$

Question 8 — Worked Solution

(a)

$$\mathbf{v} = \frac{\mathbf{r}(5) - \mathbf{r}(0)}{5} = \frac{(9\mathbf{i} - 2\mathbf{j}) - (-\mathbf{i} + 3\mathbf{j})}{5} = \frac{10\mathbf{i} - 5\mathbf{j}}{5}$$

$$\therefore = (2\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$$

(b)

$$|\mathbf{v}| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$$

$$\therefore \approx 2.24 \text{ m s}^{-1}$$

(c) Bearing

Velocity (2**i** – **j**): east component = 2, south component = 1.

$$\text{Angle east of north} = 90° - \arctan\left(\frac{1}{2}\right) = 90° - 26.6°$$

$$\therefore \text{Bearing} = 063° \text{ (to nearest degree)}$$

Question 9 — Worked Solution

(a)

Set position vectors equal:

$$(2 + 3t)\mathbf{i} + (1 + 2t)\mathbf{j} = (8 + t)\mathbf{i} + (-5 + 4t)\mathbf{j}$$

i-components: 2+3t = 8+t $\Rightarrow$ 2t = 6 $\Rightarrow$ t = 3

j-components: 1+2(3) = 7 and –5+4(3) = 7 $\Rightarrow$ consistent ✓

Particles meet at t = 3 s.

(b)

$$\mathbf{r} = (2 + 3 \times 3)\mathbf{i} + (1 + 2 \times 3)\mathbf{j} = 11\mathbf{i} + 7\mathbf{j}$$

$$\therefore \text{Position vector} = (11\mathbf{i} + 7\mathbf{j})\ \text{m}$$

Question 10 — Worked Solution

(a)

$$\mathbf{R} = (6-2)\mathbf{i} + (9+3)\mathbf{j} = (4\mathbf{i} + 12\mathbf{j})\ \text{N}$$

$$|\mathbf{R}| = \sqrt{16+144} = \sqrt{160} = \sqrt{16 \times 10}$$

$$\therefore = 4\sqrt{10}\ \text{N}$$

(b)

$$\mathbf{a} = \frac{\mathbf{R}}{m} = \frac{4\mathbf{i} + 12\mathbf{j}}{3}$$

$$\therefore = \left(\frac{4}{3}\mathbf{i} + 4\mathbf{j}\right)\ \text{m s}^{-2}$$

Question 11 — Worked Solution

Position of A at t = 2:

$$\mathbf{r}_A = (10\mathbf{i} + 5\mathbf{j}) + 2(3\mathbf{i} + 4\mathbf{j}) = (16\mathbf{i} + 13\mathbf{j}) \text{ km}$$

Position of B at t = 2:

$$\mathbf{r}_B = (25\mathbf{i} - 3\mathbf{j}) + 2(\mathbf{i} + 6\mathbf{j}) = (27\mathbf{i} + 9\mathbf{j}) \text{ km}$$

Displacement A to B:

$$\mathbf{r}_B - \mathbf{r}_A = (11\mathbf{i} - 4\mathbf{j}) \text{ km}$$

$$|\mathbf{r}_B - \mathbf{r}_A| = \sqrt{121 + 16} = \sqrt{137}$$

$$\therefore \text{Distance} = \sqrt{137} \approx 11.7 \text{ km}$$

Question 12 — Worked Solution

(a)

$$\mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{t} = \frac{(5\mathbf{i} + 9\mathbf{j}) - (-3\mathbf{i} + \mathbf{j})}{4} = \frac{8\mathbf{i} + 8\mathbf{j}}{4}$$

$$\therefore = (2\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-2}$$

(b)

$$|\mathbf{F}| = m|\mathbf{a}| = 2\sqrt{2^2 + 2^2} = 2\sqrt{8} = 4\sqrt{2}$$

$$\therefore \approx 5.66 \text{ N}$$

(c)

$$\mathbf{r} = \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2 = (-3\mathbf{i} + \mathbf{j})(4) + \tfrac{1}{2}(2\mathbf{i} + 2\mathbf{j})(16)$$

$$= (-12\mathbf{i} + 4\mathbf{j}) + (16\mathbf{i} + 16\mathbf{j})$$

$$\therefore = (4\mathbf{i} + 20\mathbf{j}) \text{ m}$$

Question 13 — Worked Solution

At time t: A is at (pt**i** + 6t**j**), B is at ((14–2t)**i** + qt**j**).

(a)

For collision, **j**-components must be equal:

$$6t = qt \Rightarrow q = 6 \quad (t \neq 0)$$

$$\therefore q = 6$$

(b)

Using **i**-components at t = 2:

$$2p = 14 - 2(2) = 10 \Rightarrow p = 5$$

$$\therefore p = 5$$

(c)

$$\text{At } t = 2: \quad \mathbf{r}_A = 5(2)\mathbf{i} + 6(2)\mathbf{j}$$

$$\therefore = (10\mathbf{i} + 12\mathbf{j}) \text{ m}$$

Question 14 — Worked Solution

(a)

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{8\mathbf{i} + 8\mathbf{j}}{4}$$

$$\therefore = (2\mathbf{i} + 2\mathbf{j}) \text{ m s}^{-2}$$

(b)

u = 0 (at rest), t = 5 s:

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} + 2\mathbf{j}) \times 5 = (10\mathbf{i} + 10\mathbf{j}) \text{ m s}^{-1}$$

$$|\mathbf{v}| = \sqrt{100 + 100} = \sqrt{200} = 10\sqrt{2}$$

$$\therefore \approx 14.1 \text{ m s}^{-1}$$

(c)

$$\mathbf{r} = \mathbf{r_0} + \mathbf{u}t + \tfrac{1}{2}\mathbf{a}t^2 = (2\mathbf{i} - 3\mathbf{j}) + \tfrac{1}{2}(2\mathbf{i} + 2\mathbf{j})(25)$$

$$= (2\mathbf{i} - 3\mathbf{j}) + (25\mathbf{i} + 25\mathbf{j})$$

$$\therefore = (27\mathbf{i} + 22\mathbf{j}) \text{ m}$$

Question 15 — Worked Solution

(a)

$$\mathbf{d_1} = (4\mathbf{i} + 3\mathbf{j}) \times 10 = (40\mathbf{i} + 30\mathbf{j}) \text{ m}$$

$$\mathbf{d_2} = (-2\mathbf{i} + 6\mathbf{j}) \times 6 = (-12\mathbf{i} + 36\mathbf{j}) \text{ m}$$

$$\text{Total displacement} = \mathbf{d_1} + \mathbf{d_2} = (28\mathbf{i} + 66\mathbf{j})$$

$$\therefore = (28\mathbf{i} + 66\mathbf{j}) \text{ m}$$

(b)

$$\text{Distance} = \sqrt{28^2 + 66^2} = \sqrt{784 + 4356} = \sqrt{5140}$$

$$\therefore \approx 71.7 \text{ m (3 s.f.)}$$

(c)

$$\text{Average velocity} = \frac{\text{total displacement}}{\text{total time}} = \frac{28\mathbf{i} + 66\mathbf{j}}{16}$$

$$\therefore = \left(\frac{7}{4}\mathbf{i} + \frac{33}{8}\mathbf{j}\right) \text{ m s}^{-1}$$