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A-LEVEL MATHEMATICS • FURTHER PURE 3

Further Coordinate Systems

Worked Solutions

Pearson Edexcel International A Level (WFM03) • Calculator allowed: show every step

Full worked solutions for the free Further Coordinate Systems worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 3 Further Coordinate Systems worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 3 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $144 = 169\left(1 - e^2\right)$, $e = \frac{5}{13}$: **foci** $(\pm 5, 0)$**, directrices** $x = \pm\frac{169}{5}$

A2 $16 = 9\left(e^2 - 1\right)$, $e = \frac{5}{3}$: $(\pm 5, 0)$, $x = \pm\frac{9}{5}$, $y = \pm\frac{4}{3}x$

A3 $\frac{16\sec^2\theta}{16} - \frac{9\tan^2\theta}{9} = \sec^2\theta - \tan^2\theta = 1$, so the point is **on the hyperbola**

STRETCH

A4 $\frac{dy}{dx} = \frac{2\sec^2\theta}{3\sec\theta\tan\theta} = \frac{2}{3\sin\theta}$; simplifying: $2x\sec\theta - 3y\tan\theta = 6$

A5 Normal gradient $\frac{4\sin\theta}{3\cos\theta}$: $4x\sin\theta - 3y\cos\theta = 7\sin\theta\cos\theta$

A6 $e = \frac{5}{4}$, directrix $x = \frac{16}{5}$: $PS = \frac{5}{4}\left(6 - \frac{16}{5}\right)$: $\frac{7}{2}$

A7 $e = 2$, $ae = 4$: $a = 2$, $b^2 = 4(4 - 1)$: $\frac{x^2}{4} - \frac{y^2}{12} = 1$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $a = 3, b = 2$ **(b)** $\frac{\sqrt{5}}{3}$ **(c)** $(\pm\sqrt{5}, 0)$ **(d)** $x = \pm\frac{9}{\sqrt{5}}$

Divide by 36 first to read off a and b

B2 **(a)** $\frac{\sqrt{5}}{2}$ **(b)** $(\pm\sqrt{5}, 0)$ **(c)** $x = \pm\frac{4}{\sqrt{5}}$ **(d)** $y = \pm\frac{1}{2}x$

For a hyperbola $b^2 = a^2\left(e^2 - 1\right)$

B3 **(a)** $\frac{x^2}{25} + \frac{y^2}{9} = 1$ **(b)** $\frac{x^2}{9} + \frac{y^2}{8} = 1$ **(c)** $\frac{x^2}{64} - \frac{y^2}{36} = 1$ **(d)** $\frac{x^2}{9} - \frac{y^2}{36} = 1$

Two facts fix a *and* b

B4 **(a)** $x + 2y = 4$ **(b)** $2x - 3y = 3$ **(c)** 2 **(d)** $y = 2x - 3$

$\frac{xx_1}{a^2} \pm \frac{yy_1}{b^2} = 1$ *is the tangent at* (x_1, y_1)

B5 **(a)** $\frac{x^2}{4} + \frac{y^2}{25} = 1$ **(b)** $\frac{x^2}{9} - \frac{y^2}{4} = 1$ **(c)** $\frac{x^2}{16} - y^2 = 1$

$\cos^2 + \sin^2 = 1$, $\cosh^2 - \sinh^2 = 1$, $\sec^2 - \tan^2 = 1$

B6 **(a)** $5x^2 + 8cx + 4c^2 - 4 = 0$ **(b)** discriminant 0 **(c)** $64c^2 - 20\left(4c^2 - 4\right) = 0$: $c = \pm\sqrt{5}$

(d) $\left(-\frac{4\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right)$

A tangent meets the curve at a repeated root.

STRETCH

B7 **(a)** $\frac{2\cosh t}{3\sinh t}$ **(b)** $2x\cosh t - 3y\sinh t = 6$ **(c)** $(3\operatorname{sech} t, 0)$ **(d)** $(0, -2\operatorname{cosech} t)$

$\cosh^2 t - \sinh^2 t = 1$ *simplifies the tangent.*

B8 **(a)** $e = \frac{5}{7}$: $(\pm 5, 0)$ **(b)** $7 - \frac{5}{7}x$ **(c)** $7 + \frac{5}{7}x$ **(d)** $PS + PS' = 14$

Each distance is e *times the distance to a directrix.*

B9 **(a)** $3x\sin\theta - y\cos\theta = 8\sin\theta\cos\theta$ **(b)** $\left(\frac{8}{3}\cos\theta, 0\right)$ **(c)** $(0, -8\sin\theta)$ **(d)** $\left(\frac{4\sqrt{3}}{3}, 0\right), (0, -4)$

Set $y = 0$ *then* $x = 0$

B10 **(a)** $\left(\cos\theta, \frac{3}{2}\sin\theta\right)$ **(b)** $x^2 + \frac{4y^2}{9} = 1$ **(c)** $\left(1+\cos\theta, \frac{3}{2}\sin\theta\right)$ **(d)** $(x-1)^2 + \frac{4y^2}{9} = 1$, an ellipse centre $(1, 0)$

Make $\cos\theta$ *and* $\sin\theta$ *the subjects, then square and add.*

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 For an ellipse $b^2 = a^2(1-e^2)$: $e^2 = 1 - \frac{9}{25}$: $e = \frac{4}{5}$

C2 $\frac{\sqrt{5}}{3} \approx 0.745 < 1$, which is valid for an ellipse.

C3 A line through O is $y = mx$; she has lost the x: $y = \frac{5\sin\theta}{4\cos\theta}x$

C4 $e = \frac{1}{2}$, $\frac{a}{e} = 12$; directrices are lines: $x = \pm 12$

STRETCH

C5 $\frac{dx}{d\theta} = 4\sec\theta\tan\theta$, not $4\sec\theta$: $\frac{dy}{dx} = \frac{3}{4\sin\theta}$

C6 For a hyperbola $b^2 = a^2(e^2-1)$: $16 = 9(e^2-1)$: $e = \frac{5}{3}$

C7 The midpoint is $\left(\frac{3}{2}\cos\theta, \sin\theta\right)$: $\frac{4x^2}{9} + y^2 = 1$, an ellipse, not a circle

C8 **(a)** $\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$, gradient $-\frac{b^2x_1}{a^2y_1}$; then use $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$, as required

(b) substitute: $(b^2 + a^2m^2)x^2 + 2a^2mcx + a^2(c^2 - b^2) = 0$; discriminant zero gives $c^2 = a^2m^2 + b^2$, as required

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D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** $\frac{x^2}{16/k} - \frac{y^2}{16} = 1$: $\frac{16}{k} + 16 = 25$: $\boxed{k = \frac{16}{9}}$

(b) $a = 3$, $e = \frac{5}{3}$: $\boxed{x = \frac{9}{5}}$

D2 **(a)** $ae \times \frac{a}{e} = a^2 = 16$, $e = \frac{2\sqrt{3}}{4}$: $\boxed{e = \frac{\sqrt{3}}{2} < 1}$

(b) $b^2 = 16 \times \frac{1}{4} = 4$: $\boxed{x^2 + 4y^2 = 16}$

D3 $e = \frac{2a}{9}$, $a^2 - a^2e^2 = 5$: $4a^4 - 81a^2 + 405 = 0$: $\boxed{a^2 = 9,\ e = \frac{2}{3};\ a^2 = \frac{45}{4},\ e = \frac{\sqrt{5}}{3}}$ (both valid)

D4 **(a)** $\frac{dy}{dx} = -\frac{4\cos\theta}{6\sin\theta}$, giving the result

(b) $\boxed{A(6\sec\theta, 0),\ B(0, 4\operatorname{cosec}\theta)}$

STRETCH

D5 **(a)** normal gradient $\frac{2\sin\theta}{\cos\theta}$, giving the result

(b) $N(3\cos\theta, 0)$, area $\frac{1}{2} \times 3\cos\theta \times 2\sin\theta = \frac{3}{2}\sin 2\theta \leqslant \frac{3}{2}$: $\boxed{\frac{3}{2} \text{ at } \theta = \frac{\pi}{4}}$

D6 **(a)** $\boxed{e = \frac{3}{5},\ S(6, 0),\ x = \frac{50}{3}}$

(b) $y^2 = 48$; $PS = \sqrt{1 + 48} = 7$, $PM = \frac{50}{3} - 5 = \frac{35}{3}$; $\frac{3}{5} \times \frac{35}{3} = 7$, as required

D7 **(a)** $\frac{x^2}{4} + (x + c)^2 = 1$, as required

(b) $x = -\frac{4c}{5}$, $y = \frac{c}{5}$: $\boxed{M\left(-\frac{4c}{5}, \frac{c}{5}\right)}$

(c) $\boxed{y = -\frac{1}{4}x}$, through O

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D8 **(a)** tangent gradient $\frac{4}{3\sin\theta}$, normal gradient $-\frac{3\sin\theta}{4}$, giving the result

(b) $N\left(\frac{25}{3}\sec\theta, 0\right)$, $S(5,0)$, $e=\frac{5}{3}$: $SN=\frac{25}{3}\sec\theta-5$, $SP=5\sec\theta-3$, and $\frac{5}{3}SP=SN$, as required

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $\frac{x^2}{9}-\frac{y^2}{4}=1$, $4=9\left(e^2-1\right)$ *M1*

$\boxed{e=\frac{\sqrt{13}}{3}}$ *A1*

(b) $\boxed{\left(\pm\sqrt{13},0\right),\ x=\pm\frac{9}{\sqrt{13}}}$ *B1 B1*

(c) $4x^2-9\left(\frac{2}{3}x+1\right)^2=36$ gives $-12x-9=36$, linear, so one point *M1*

$\boxed{\left(-\frac{15}{4},-\frac{3}{2}\right)}$ *A1*

E2 **(a)** $\frac{dy}{dx}=\frac{3\cos\theta}{-5\sin\theta}$ *M1*

$y-3\sin\theta=-\frac{3\cos\theta}{5\sin\theta}(x-5\cos\theta)$ *M1*

$3x\cos\theta+5y\sin\theta=15\left(\cos^2\theta+\sin^2\theta\right)=15$, as required *A1**

(b) $\boxed{A(0,3\operatorname{cosec}\theta),\ B\left(0,-\frac{16}{3}\sin\theta\right)}$ *B1 B1*

(c) $AB=\frac{3}{\sin\theta}+\frac{16\sin\theta}{3}\geqslant 2\sqrt{16}$, equality when $\sin^2\theta=\frac{9}{16}$ *M1*

$\boxed{\textbf{least } AB=8\textbf{, at } \sin\theta=\frac{3}{4}}$ *A1*

STRETCH

E3 **(a)** $\dfrac{dy}{dx} = -\dfrac{3\cos\theta}{4\sin\theta}$ *M1 A1*

$3x\cos\theta + 4y\sin\theta = 12$, as required *A1**

(b) $T(4\sec\theta, 0)$, $U(0, 3\operatorname{cosec}\theta)$ *M1*

$\boxed{\textbf{midpoint } \left(2\sec\theta, \dfrac{3}{2}\operatorname{cosec}\theta\right)}$ *A1*

(c) $\cos\theta = \dfrac{2}{x}$, $\sin\theta = \dfrac{3}{2y}$ *M1*

$\dfrac{4}{x^2} + \dfrac{9}{4y^2} = 1$ *M1 A1*

$\boxed{9x^2 + 16y^2 = 4x^2y^2}$ *A1*