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A-LEVEL MATHEMATICS • STATISTICS 2

Hypothesis Testing

Worked Solutions

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free Hypothesis Testing worksheet, A-Level and International A-Level Maths

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Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 2 Hypothesis Testing worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $P(X \geqslant 12) = 1 - P(X \leqslant 11) = 1 - 0.9493 = 0.0507 > 0.05$: **do not reject H_0**

A2 $P(X \leqslant 5) = 0.0480 < 0.05$, $P(X \leqslant 6) = 0.1034$: $X \leqslant 5$**, 0.0480**

A3 $H_0: \lambda = 2.5$, $H_1: \lambda > 2.5$; 2 hours: Po(5); $P(X \geqslant 10) = 1 - 0.9682 = 0.0318 < 0.05$: **reject H_0**

STRETCH

A4 $P(X \leqslant 6) = 0.0172$, $P(X \leqslant 7) = 0.0435$; $P(X \geqslant 18) = 0.0212$, $P(X \geqslant 17) = 0.0481$

$X \leqslant 6$ **or** $X \geqslant 18$**, actual significance 0.0384**

A5 5% in each tail: $P(X \leqslant 3) = 0.0424$, $P(X \leqslant 4) = 0.0996$; $P(X \geqslant 14) = 0.0342$, $P(X \geqslant 13) = 0.0638$

$X \leqslant 3$ **or** $X \geqslant 14$**, 0.0766**

A6 Po(60) $\approx$ N(60, 60): $P(Y > 71.5)$, $z = 1.48$, $0.0694 > 0.05$: **do not reject H_0**

A7 $P(X \leqslant 1) = 0.0047 < 0.01$, $P(X \leqslant 2) = 0.0203$: $X \leqslant 1$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $p = 0.5, p > 0.5$ **(b)** $p = 0.1, p > 0.1$ **(c)** $\lambda = 6, \lambda \neq 6$ **(d)** $p = 0.35, p < 0.35$

H_0 *always uses "=";* H_1 *follows what is suspected.*

B2 **(a)** $1 - 0.8982 = 0.1018$: **not significant** **(b)** $1 - 0.9591 = 0.0409$: **reject H_0**

(c) $1 - 0.9861 = 0.0139$: **reject H_0** **(d)** $1 - 0.9961 = 0.0039$: **reject H_0**

For $X \geqslant x$ *use* $1 - P(X \leqslant x - 1)$

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B3 **(a)** $P(X \geqslant 13) = 0.0845$, $P(X \geqslant 12) = 0.1593$: $X \geqslant 13$ **(b)** $P(X \geqslant 14) = 0.0401$: $X \geqslant 14$

(c) $P(X \geqslant 16) = 0.0064$, $P(X \geqslant 15) = 0.0169$: $X \geqslant 16$ **(d)** **0.0401**

A smaller level pushes the region further into the tail.

B4 **(a)** $P(X \leqslant 1) = 0.0404 < 0.05$: **reject H_0** **(b)** $P(X \leqslant 2) = 0.1247$: **do not reject** **(c)** $X \leqslant 1$

(d) **0.0404**

The region and the test by probability always agree.

B5 **(a)** Po(6) **(b)** $P(X \leqslant 2) = 0.0620 > 0.05$: **do not reject H_0** **(c)** $0.0620 < 0.10$: **reject H_0**

The same data can be significant at one level and not at another.

B6 **(a)** $P(X \leqslant 1) = 0.0052$, $P(X \leqslant 2) = 0.0271$: $X \leqslant 1$ **(b)** $P(X \geqslant 11) = 0.0093$, $P(X \geqslant 10) = 0.0338$: $X \geqslant 11$

(c) $0.0052 + 0.0093$: **0.0145** **(d)** **no**, 10 is not in the region

Each tail may hold at most half the level.

STRETCH

B7 **(a)** N(90, 49.5) **(b)** $P(Y > 99.5)$, $z = 1.35$, 0.0885: **do not reject**

(c) $P(Y > 104.5)$, $z = 2.06$, 0.0197: **reject H_0** **(d)** $\dfrac{c - 0.5 - 90}{\sqrt{49.5}} > 1.6449$, $c > 102.07$: $X \geqslant 103$

The region's boundary also needs a continuity correction.

B8 **(a)** Po(120): N(120, 120) **(b)** $P(Y < 105.5)$, $z = -1.32$, 0.0934: **do not reject**

(c) $\dfrac{c + 0.5 - 120}{\sqrt{120}} < -1.6449$, $c < 101.5$: $Y \leqslant 101$ **(d)** $P(Y < 101.5)$, $z = -1.69$: **0.0455**

A large Poisson mean calls for the normal approximation.

B9 **(a)** $P(X \leqslant 2) = 0.0982$, $P(X \leqslant 3) = 0.2340$: $X \leqslant 2$ **(b)** **0.0982** **(c)** $P(X \leqslant 1) = 0.0274$: $X \leqslant 1$

(d) **0.0274**

The actual significance is the probability of the region itself.

B10 **(a)** $H_0 : p = 0.3$, $H_1 : p < 0.3$ **(b)** $P(X \leqslant 3)$: **0.1071**

(c) $0.1071 > 0.05$: **no evidence that fewer than 30% recommend it** **(d)** $P(X \leqslant 2) = 0.0355$: $X \leqslant 2$

The direction comes from what the tester suspects.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $X \geqslant 11$ is the complement of $X \leqslant 10$: $1 - 0.9703$: **0.0297**

C2 A critical region is a set of values of X, not a probability: $X \leqslant 3$

C3 Hypotheses are about the population parameter, using the believed value, not the sample proportion: $H_0 : p = 0.3$, $H_1 : p > 0.3$

C4 A test gives evidence, not certainty: **there is evidence that the coin is biased towards heads**

STRETCH

C5 A two-tailed test at 5% puts 2.5% in each tail: $P(X \leqslant 2) = 0.0090$, $P(X \leqslant 3) = 0.0332$; $P(X \geqslant 13) = 0.0175$, $P(X \geqslant 12) = 0.0442$

$X \leqslant 2$ or $X \geqslant 13$, actual significance 0.0265

C6 3 hours at 2 per hour is Po(6), not Po(2). $P(X \geqslant 11) = 1 - 0.9574 = 0.0426 < 0.05$: **reject H_0: evidence that the rate has increased**

C7 He needs a continuity correction: $P(Y > 59.5)$, $z = 1.9$, $0.0287 > 0.025$: **do not reject H_0**

C8 **(a)** $P(X \leqslant 2) = \dfrac{1 + 12 + 66}{4096} = \dfrac{79}{4096} \approx 0.0193 < 0.025$; $P(X \leqslant 3) = \dfrac{299}{4096} \approx 0.0730$; by symmetry $X \geqslant 10$, as required

(b) $2 \times \dfrac{79}{4096} = \dfrac{79}{2048}$, as required

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $H_0: \lambda = 3.5$, $H_1: \lambda > 3.5$; $P(X \geqslant 8) = 1 - 0.9733 = 0.0267 < 0.05$

Reject H_0: evidence that the advert has increased the rate of sales

D2 **(a)** $P(X \leqslant 1) = 0.0243$, $P(X \leqslant 2) = 0.0913$; $P(X \geqslant 10) = 0.0139$, $P(X \geqslant 9) = 0.0409$: $X \leqslant 1$ **or** $X \geqslant 10$

(b) $0.0243 + 0.0139$: **0.0382**

D3 **(a)** $P(X \leqslant 3) = 0.0212$, $P(X \leqslant 4) = 0.0550$; $P(X \geqslant 15) = 0.0415$, $P(X \geqslant 14) = 0.0739$: $X \leqslant 3$ **or** $X \geqslant 15$

(b) $0.0212 + 0.0415$: **0.0627**

D4 **(a)** $H_0: p = 0.15$, $H_1: p < 0.15$; $P(X = 0) = 0.85^{15} \approx 0.0874 > 0.05$: **do not reject H_0**

(b) Even the most extreme result, 0, has probability above 0.05, so **no critical region exists**

STRETCH

D5 $H_0: p = 0.35$, $H_1: p < 0.35$; $B(200, 0.35) \approx N(70, 45.5)$; $P(Y < 56.5)$, $z = -2.00$, $0.0228 < 0.05$

Reject H_0: evidence that fewer than 35% read a daily newspaper

D6 $H_0: \lambda = 6$, $H_1: \lambda < 6$ per week; 8 weeks: $Po(48) \approx N(48, 48)$; $P(Y < 36.5)$, $z = -1.66$, $0.0485 < 0.05$

Reject H_0: evidence that the rate of complaints has fallen

D7 **(a)** $H_1: p > 0.3$; $P(X \geqslant 11) = 1 - 0.9022 = 0.0978 < 0.1$, $P(X \geqslant 10) = 0.1894$: $X \geqslant 11$

(b) 11 is in the region: **evidence that the player has improved** **(c)** **0.0978**

D8 **(a)** $0.88^n < 0.01$, $n > \dfrac{\ln 0.01}{\ln 0.88} = 36.02$: $n = 37$

(b) 0.88^{37}: **0.0088**

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** Po(4.5) *B1*

Enquiries arrive singly, independently and at a constant rate *B1*

(b) $H_0: \lambda = 4.5$, $H_1: \lambda \neq 4.5$ *B1*

(c) $P(X = 0) = 0.0111$, $P(X \leqslant 1) = 0.0611$ *M1*

$P(X \geqslant 9) = 0.0403$, $P(X \geqslant 8) = 0.0866$ *M1*

$X = 0$ **or** $X \geqslant 9$ *A1*

(d) $0.0111 + 0.0403$ *M1*

0.0514 *A1*

(e) 8 is not in the critical region *M1*

no evidence that the rate of enquiries has changed *A1*

E2 **(a)** $H_0: p = 0.2$, $H_1: p < 0.2$ *B1*

(b) $X \sim B(30, 0.2)$; $P(X \leqslant 2) = 0.0442$ *M1 A1*

$0.0442 < 0.05$: **reject H_0; evidence that fewer students walk** *A1*

(c) $P(X \leqslant 3) = 0.1227 > 0.05$ *M1*

$X \leqslant 2$ *A1*

(d) **0.0442** *B1*

(e) $B(200, 0.2) \approx N(40, 32)$ *B1*

$P(Y < 28.5)$ *M1*

$z = \dfrac{28.5 - 40}{\sqrt{32}} \approx -2.03$; $0.0212 < 0.05$ *M1*

reject H_0: evidence that the proportion has decreased *A1*

STRETCH

E3 **(a)** Under H_0, $X \sim \text{Po}(2t)$; $P(X = 0) = e^{-2t} < 0.05$ *M1*

$t > \dfrac{\ln 20}{2} = 1.50$ *A1*

$\boxed{t = 2}$ *A1*

(b) Po(10): $P(X \leqslant 4) = 0.0293$, $P(X \leqslant 5) = 0.0671$ *M1*

$\boxed{X \leqslant 4}$ *A1*

(c) $\boxed{\textbf{0.0293}}$ *B1*

(d) 4 is in the critical region, so reject H_0 *M1*

$\boxed{\textbf{evidence that the rate of calls has decreased}}$ *A1*

(e) $H_0 : \lambda = 2$, $H_1 : \lambda < 2$ per hour; $\text{Po}(80) \approx \text{N}(80, 80)$ *B1*

$P(Y < 64.5)$ *M1*

$z = \dfrac{64.5 - 80}{\sqrt{80}} \approx -1.73$; $0.0418 > 0.01$ *M1*

$\boxed{\textbf{do not reject } H_0\textbf{: no evidence at 1\% of a decrease}}$ *A1*

(f) The mean, 80, is large *B1*

so the Poisson is close to symmetric and the normal fits well *B1*