



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Integration

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Integration worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Integration worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $4 \sin x - \ln |x| + \tan x + c$

A2 $\frac{1}{3} \sin(3x+1) + c; \frac{(5-2x)^5}{5} \div (-2): -\frac{1}{10}(5-2x)^5 + c$

A3 $[\ln(2x-1)]_1^3 = \ln 5 - \ln 1: \ln 5$

STRETCH

A4 $\cos x$ is the derivative of $2 + \sin x$: $\ln(2 + \sin x) + c$

A5 $\frac{d}{dx} e^{x^3} = 3x^2 e^{x^3}: \frac{1}{3} e^{x^3} + c$

A6 $\tan^2 x = \sec^2 x - 1: \tan x - x + c$

A7 $[2e^{\frac{x}{2}}]_0^2 = 2e - 2: 2(e - 1)$



B**Fluency solutions**

One mark for each correct part (each indefinite integral needs +c). The notes in italics explain why each sequence was chosen.

CORE

B1 (a) $\frac{1}{3}e^{3x} + c$ (b) $6\ln|x| + c$ (c) $-\frac{1}{2}\cos 2x + c$ (d) $\frac{1}{5}\tan 5x + c$

Each inside coefficient becomes a divisor.

B2 (a) $\frac{1}{18}(3x+2)^6 + c$ (b) $\frac{1}{4}\ln|4x-3| + c$ (c) $-e^{2-x} + c$ (d) $-\frac{3}{2}\ln|1-2x| + c$

A negative coefficient of x gives a negative factor.

B3 (a) $\frac{1}{3}$ (b) $\frac{1}{3}\left(-\cos\frac{\pi}{2} + \cos 0\right) = \frac{1}{3}$ (c) 2 (d) $\frac{1}{2}(4-1) = \frac{3}{2}$

Substitute both limits, even when one gives 0 or 1

B4 (a) $\ln|x^3+1| + c$ (b) $\frac{1}{2}\ln(x^2+4) + c$ (c) $\frac{\sin x}{\cos x}: -\ln|\cos x| + c = \ln|\sec x| + c$ (d) $\ln(e^x+1) + c$

Adjust by a constant when the numerator is a multiple of the derivative.

B5 (a) $\frac{1}{2}x + \frac{1}{4}\sin 2x + c$ (b) $\frac{1}{2}\sin 2x: -\frac{1}{4}\cos 2x + c$ (c) $\sec^2 x: \tan x + c$

Rewrite with an identity until each term is standard.

B6 (a) $x + 2\ln|x+1| + c$ (b) $2x + \frac{1}{x}: x^2 + \ln|x| + c$ (c) $1 + 2\ln 2 - 0: 1 + 2\ln 2$ (d) $x + \frac{1}{x+1}: \frac{1}{2}x^2 + \ln|x+1| + c$

An improper fraction must be divided before it can be integrated.

STRETCH

B7 (a) $\frac{1}{20}(2x^2-1)^5 + c$ (b) $-\frac{1}{4}\cos^4 x + c$ (c) $e^{\sin x} + c$ (d) $\sqrt{x^2+9} + c$

Guess the function, differentiate it, then fix the constant.



- B8** (a) $\frac{1}{2}x - \frac{1}{12}\sin 6x + c$ (b) $1 - \sin 2x$: $x + \frac{1}{2}\cos 2x + c$ (c) $\operatorname{cosec}^2 x - 1$: $-\cot x - x + c$ (d) $\cos x - \sin x$: $\sin x + \cos x + c$
In (d), $\cos 2x = (\cos x - \sin x)(\cos x + \sin x)$ cancels the denominator.

- B9** (a) $\ln k = 2$: $k = e^2$ (b) $\frac{1}{2}(e^{2k} - 1) = 4$, $e^{2k} = 9$: $k = \ln 3$
 (c) $\sin a = \frac{1}{2}$: $a = \frac{\pi}{6}$ (d) $\ln(b - 1) = \ln 3$: $b = 4$

Integrate in terms of the unknown limit, then solve.

- B10** (a) $x = \ln 4$ (b) $[4x - e^x]_0^{\ln 4} = (4\ln 4 - 4) - (0 - 1)$: $4\ln 4 - 3$ (c) 1 (d) $\ln 2$
Find where the curve crosses the axis first: that gives a limit.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** Sam forgot to divide by the coefficient 2: $\frac{1}{2}\ln|2x+3| + c$

- C2** $\sin^2 x$ is not of the form $f'(x)[f(x)]^n$; use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$: $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$

- C3** The integral of $\sin$ is $-\cos$, and $+c$ is missing: $-\frac{1}{4}\cos 4x + c$

- C4** The curve is below the axis there, so the integral is negative; an area is positive: $\text{area} = 2$

STRETCH

- C5** $e^0 = 1$, not 0, so the lower limit gives $\frac{1}{2}$: $\frac{1}{2}(e^2 - 1)$

- C6** The x outside is part of the chain-rule derivative $2x$, not an extra factor to integrate: $\frac{1}{10}(x^2 + 1)^5 + c$



C7 $\int \frac{1}{x} dx = \ln|x| + c$, which is defined for negative x : $\ln 1 - \ln 3 = \boxed{-\ln 3}$

C8 (a) $\ln x + x \cdot \frac{1}{x} - 1 = \ln x$ **as required**

(b) $[x \ln x - x]_1^e = (e - e) - (0 - 1)$: **1** (c) $\ln x \geq 0$ on $[1, e]$, so the area of the region is exactly **1**

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\frac{d}{dx} (1 - x^2)^4 = -8x(1 - x^2)^3$, so the integral is $\left[-\frac{1}{8} (1 - x^2)^4 \right]_0^1 = 0 - \left(-\frac{1}{8} \right)$: **$\frac{1}{8}$**

D2 (a) $(x+2)(2x-1) = 2x^2 + 3x - 2$, remainder 1: **$A = 2, B = -1, C = 1$**

(b) $[x^2 - x + \ln(x+2)]_0^2 = (2 + \ln 4) - \ln 2$: **$2 + \ln 2$** , so $p = 2, q = 2$

D3 $2 \cos^2 x = 1 + \cos 2x$: $\left[x + \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{3}} = \frac{\pi}{3} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$: **$\frac{\pi}{3} + \frac{\sqrt{3}}{4}$**

D4 $\left[\frac{3}{2} \ln(2x+1) \right]_0^4 = \frac{3}{2} \ln 9$: **$3 \ln 3$**

STRETCH

D5 (a) $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos 2x \times 1$ **as required**

(b) $\left[\frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{4}} = \frac{1}{2}$; the identity replaces the fourth powers by a single standard function.

D6 (a) $[2 \ln(2x+k)]_1^2 = 2 \ln(4+k) - 2 \ln(2+k) = 2 \ln \frac{4+k}{2+k}$ **as required**

(b) $\frac{4+k}{2+k} = \frac{5}{3}$, so $12 + 3k = 10 + 5k$: **$k = 1$**

D7 (a) $e^{2x} = e^2$ gives $x = 1$ **as required**

(b) $\int_0^1 (e^2 - e^{2x}) dx = \left[e^2 x - \frac{1}{2} e^{2x} \right]_0^1 = \frac{1}{2} e^2 + \frac{1}{2}$: **$\frac{1}{2} (e^2 + 1)$**



D8 (a) $\tan x (1 + \tan^2 x) = \tan x \sec^2 x$ as required

(b) $\frac{d}{dx} \left(\frac{1}{2} \tan^2 x \right) = \tan x \sec^2 x; \left[\frac{1}{2} \tan^2 x \right]_0^{\frac{\pi}{4}} : \frac{1}{2}$

(c) $[\tan x - x]_0^{\frac{\pi}{4}} : 1 - \frac{\pi}{4}$

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $\cos^2 3x = \frac{1}{2}(1 + \cos 6x)$ M1

$\frac{1}{2}x + \frac{1}{12} \sin 6x + c$ A1

(b) $\frac{d}{dx} (2x^3 + 5)^5 = 30x^2 (2x^3 + 5)^4$ M1

$k(2x^3 + 5)^5$ with $k = \frac{1}{30}$ dM1

$\frac{1}{30} (2x^3 + 5)^5 + c$ A1

E2 (a) $(x^2 + 1)(x + 2) = x^3 + 2x^2 + x + 2$ M1

remainder $2x$: $A = 1, B = 2, C = 2$ A1 A1

(b) $\left[\frac{1}{2}x^2 + 2x + \ln(x^2 + 1) \right]_0^1$ M1 A1

$\frac{5}{2} + \ln 2$ A1



STRETCH

- E3** (a) $e^x = 4 - 3e^{-x}$; multiply by e^x : $e^{2x} - 4e^x + 3 = 0$ as required M1 A1*
- (b) $(e^x - 1)(e^x - 3) = 0$ M1
- $x = 0, x = \ln 3$ A1
- (c) $\int_0^{\ln 3} (4 - 3e^{-x} - e^x) dx$ M1
- $= [4x + 3e^{-x} - e^x]_0^{\ln 3}$ A1
- $= (4\ln 3 + 1 - 3) - (0 + 3 - 1)$ dM1
- $4\ln 3 - 4$ A1

