



eClassroom

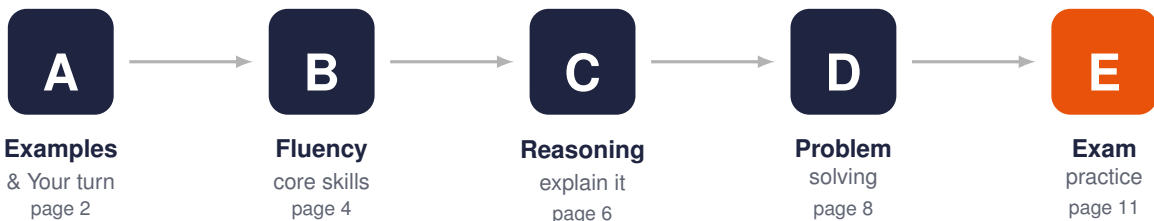
A-LEVEL MATHEMATICS • PURE 1

Radians

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Radians worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: convert between degrees and radians, using exact values • find arc lengths, perimeters and sector areas with $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ • find the area of a segment • sketch and use trigonometric graphs in radians • *Stretch:* find r and θ from given conditions, solve composite area problems exactly and prove results.

Which questions are for me?

CORE The methods every A-level student must secure.
Start here.

STRETCH Finding the radius and angle from given information, overlapping circles and lunes, and trigonometric graphs in radians.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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Name

Class

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Convert 150° to radians, and $\frac{7\pi}{4}$ radians to degrees.

$180^\circ = \pi$ radians, so multiply by $\frac{\pi}{180}$ one way and by $\frac{180}{\pi}$ the other.

$$150 \times \frac{\pi}{180} = \frac{5\pi}{6}; \quad \frac{7\pi}{4} \times \frac{180}{\pi} = 315$$

$$\frac{5\pi}{6}; 315^\circ$$

Your turn A1

Convert 210° to radians, and $\frac{5\pi}{12}$ radians to degrees.

Worked example A2

A sector has radius 6 cm and angle 1.2 radians. Find its arc length and its area.

With θ in radians: $s = r\theta$ and $A = \frac{1}{2}r^2\theta$

$$s = 6 \times 1.2; \quad A = \frac{1}{2} \times 36 \times 1.2$$

$$7.2 \text{ cm}; 21.6 \text{ cm}^2$$

Your turn A2

A sector has radius 9 cm and angle $\frac{2\pi}{3}$ radians. Find its exact arc length and its exact area.

Worked example A3

A chord of a circle of radius 8 cm subtends an angle of $\frac{\pi}{3}$ at the centre. Find the area of the minor segment.

Segment = sector – triangle = $\frac{1}{2}r^2(\theta - \sin \theta)$

$$\frac{1}{2} \times 64 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right) = \frac{32\pi}{3} - 16\sqrt{3}$$

$$5.80 \text{ cm}^2$$

Your turn A3

A chord of a circle of radius 10 cm subtends an angle of 1.4 radians at the centre. Find the area of the minor segment.



STRETCH

Worked example A4: radius and angle from two facts

A sector has area 30 cm^2 and arc length 12 cm . Find r and θ

$$\frac{1}{2}r^2\theta = \frac{1}{2}r(r\theta) = \frac{1}{2}r \times 12$$

$$6r = 30, \text{ so } r = 5, \text{ and } \theta = \frac{12}{5}$$

$$r = 5 \text{ cm}, \quad \theta = 2.4$$

Your turn A4

A sector has area 48 cm^2 and perimeter 28 cm . Find the two possible pairs of values of r and θ

Worked example A5: a chord

Points A and B lie on a circle with centre O and radius 7 cm , and angle $AOB = 1.8$ radians. Find the length of the chord AB .

Halve the isosceles triangle OAB : $AB = 2 \times 7 \sin 0.9$

$$AB = 11.0 \text{ cm}$$

Your turn A5

Points A and B lie on a circle with centre O and radius 12 cm , and angle $AOB = 2.2$ radians. Find the length of the chord AB .

Worked example A6: a translated graph

For $y = \sin\left(x + \frac{\pi}{6}\right)$, $0 \leq x \leq 2\pi$, find the exact y intercept and the x intercepts.

$$\frac{\pi}{6} \text{ to the left: } \sin \frac{\pi}{6} = \frac{1}{2}; \quad x + \frac{\pi}{6} = \pi \text{ or } 2\pi$$

$$\left(0, \frac{1}{2}\right); \quad x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

Your turn A6

For $y = \cos\left(x - \frac{\pi}{4}\right)$, $0 \leq x \leq 2\pi$, find the exact y intercept and the x intercepts.

Worked example A7: exact solutions

Solve $2 \sin x = 1$ for $0 \leq x \leq 2\pi$

$\sin x = \frac{1}{2}$: one solution from the exact value, the other from the symmetry of the graph about $x = \frac{\pi}{2}$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

Your turn A7

Solve $2 \cos x = -1$ for $0 \leq x \leq 2\pi$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 Convert each angle to radians, giving exact answers in terms of π

(a) 45°

(b) 120°

(c) 330°

(d) 72°

[4 marks]

B2 Convert each angle to degrees, giving your answer to 1 decimal place where necessary.

(a) $\frac{\pi}{9}$

(b) $\frac{5\pi}{3}$

(c) 1.2

(d) $\frac{3\pi}{8}$

[4 marks]

B3 A sector has radius 6 cm. Find its arc length when the angle is

(a) $\frac{\pi}{3}$

(b) 1.5

(c) $\frac{3\pi}{4}$

(d) 150°

[4 marks]

B4 A sector has radius 4 cm. Find its area when the angle is

(a) $\frac{\pi}{4}$

(b) 0.6

(c) $\frac{5\pi}{6}$

(d) $\frac{3\pi}{2}$

[4 marks]



CORE

*continued: perimeters and segments***B5** A circle has radius 5 cm. Find the perimeter of**(a)** a sector of angle 1.2**(b)** a semicircle**(c)** the major sector left when the sector in (a) is removed**[3 marks]****B6** A circle has radius 6 cm. Find the area of the minor segment cut off by a chord that subtends, at the centre, an angle of**(a)** $\frac{\pi}{2}$ **(b)** $\frac{\pi}{6}$ **(c)** 2**(d)** π **[4 marks]**

STRETCH

B7 For each sector, find the missing values.**(a)** arc length 15 cm and radius 6 cm: find θ **(b)** area 40 cm² and $\theta = 0.8$: find r **(c)** arc length 8 cm and area 24 cm²: find r and θ **(d)** perimeter 30 cm and radius 8 cm: find θ **[4 marks]****B8** Write down the period of**(a)** $y = 2 \sin x$ **(b)** $y = \sin 2x$ **(c)** $y = \tan x$ **(d)** $y = \cos \frac{x}{3}$ **[4 marks]****B9** State the number of solutions of each equation for $0 \leq x \leq 2\pi$ **(a)** $\sin x = 0.4$ **(b)** $\cos 2x = 0.5$ **(c)** $\tan x = -1$ **(d)** $\sin \left(x - \frac{\pi}{4} \right) = 1$ **[4 marks]****B10** Find the exact value of**(a)** $\sin \frac{\pi}{3}$ **(b)** $\cos \frac{3\pi}{4}$ **(c)** $\tan \frac{7\pi}{6}$ **(d)** $\sin \frac{5\pi}{3}$ **[4 marks]**

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam finds the arc length of a sector with radius 5 cm and angle 70° as $5 \times 70 = 350$ cm
Explain Sam's mistake, and find the correct arc length. [2 marks]
- C2** Lily says the perimeter of a sector with radius 6 cm and angle 1.5 radians is 9 cm.
Explain what Lily has forgotten, and find the correct perimeter. [2 marks]
- C3** A sector has radius r and its arc length is also r
Show that the angle of the sector is exactly 1 radian, find its area in terms of r , and explain why 1 radian is about 57.3° [3 marks]
- C4** The points A , O and B lie on a straight line, and the point C lies on a circle with centre O and radius 4 cm.
Angle $AOC = 2.1$ radians. Kai says angle $COB = 2\pi - 2.1$
Explain Kai's mistake, and find the arc length CB of the sector COB [3 marks]



STRETCH

- C5** A badge is made from the triangle OAB , with $OA = 6$ cm and $AB = 4$ cm, joined along $OB = 5$ cm to the sector OBC , which has radius 5 cm and angle 0.8 radians. Mo gives the perimeter of the badge as 24 cm. Explain Mo's mistake, and find the correct perimeter. [3 marks]
- C6** A chord of a circle with radius r subtends an angle θ radians at the centre. Prove that the area of the minor segment is $\frac{1}{2}r^2(\theta - \sin \theta)$ [3 marks]
- C7** Show that $x = \frac{\pi}{6}$ is a solution of the equation $\sin x = \frac{3x}{\pi}$
By considering the graphs of $y = \sin x$ and $y = \frac{3x}{\pi}$, explain why the equation has exactly three solutions, and write down the other two. [3 marks]
- C8** A chord AB of a circle with centre O and radius r subtends an angle θ radians at O
- (a) Show that the length of the chord is $2r \sin \frac{\theta}{2}$ (2)
- (b) A sector of radius 10 cm has an arc of length 12 cm. Find the length of its chord, and explain why it is shorter than the arc. (1)
- (c) Find the exact value of θ for which the chord AB is the same length as the radius. (2)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** A fan blade is cut from a sheet of metal in the shape of the sector OAB shown in Figure 1. The sector is part of a circle with centre O and radius 8 cm. The angle AOB is 0.9 radians, and the chord AB is shown dashed.

(a) Find the length of the arc AB .

(1)

(b) Find the area of the sector OAB .

(1)

(c) The segment cut off by the chord AB is removed. Find the area of this segment, giving your answer in cm^2 to 3 significant figures.

(1)

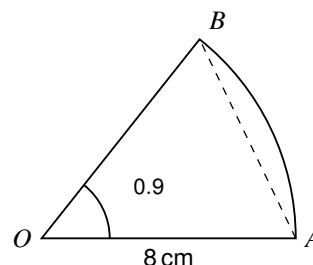


Diagram NOT accurately drawn

Figure 1

[3 marks]

- D2** Figure 2 shows the region R , shaded, that is cleaned by a windscreen wiper. The region is bounded by two arcs of circles with centre O and by the straight lines AB and CD .

The arc BC has radius 6 cm, the arc AD has radius 10 cm, $AB = CD = 4$ cm and angle $AOD = 1.4$ radians.

(a) Find the area of R .

(2)

(b) Find the perimeter of R .

(2)

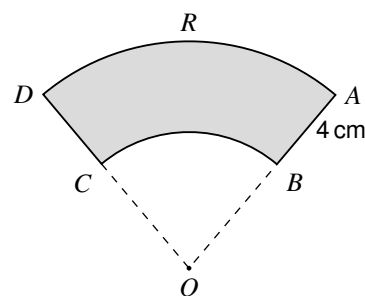


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Figure 2

[4 marks]



D3 Figure 3 shows the curve with equation

$$y = 2 \cos \left(x - \frac{\pi}{3} \right), \quad 0 \leq x \leq 2\pi, \quad \text{where } x \text{ is in radians.}$$

The curve is the graph of $y = \cos x$ after a stretch and a translation, and it has a maximum point at P , as shown in Figure 3.

(a) Find the exact coordinates of the point where the curve crosses the y -axis.

(1)

(b) Write down the exact coordinates of P .

(1)

(c) Find the exact x coordinates of the points where the curve crosses the x -axis.

(1)

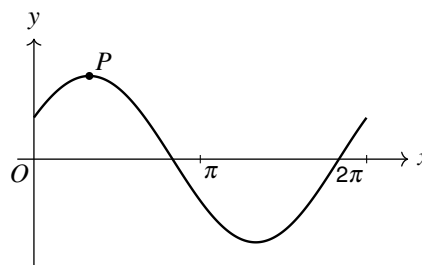


Figure 3

[3 marks]

D4 Figure 4 shows a flower bed in the shape of the sector OAB of a circle with centre O and radius 12 cm on a scale plan. The angle AOB is θ radians. The area of the sector on the plan is 90 cm^2 , and the whole edge of the flower bed is to be lined with stones.

(a) Find the value of θ .

(1)

(b) Find the perimeter of the sector on the plan.

(2)

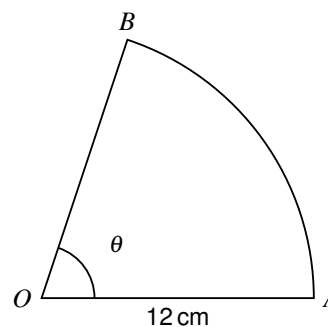


Diagram NOT accurately drawn

Figure 4

[3 marks]

STRETCH

D5 Figure 5 shows the sector OAB of a circle with centre O and radius 9 m, with angle $AOB = 1.1$ radians. The point C lies on OB , and AC is perpendicular to OB .

The region R , shown shaded, is bounded by AC , CB and the arc AB . It is to be planted with grass, and a low fence is to be put along its edge.

(a) Find the area of R , in m^2 to 3 significant figures.

(2)

(b) Find the perimeter of R , in metres to 3 significant figures.

(2)

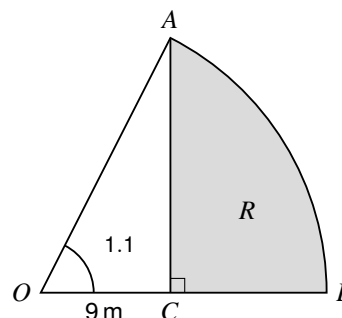


Diagram NOT accurately drawn

Figure 5

[4 marks]



- D6** On a scale map, the range of each of two radio transmitters is a circle of radius 5 cm, as shown in Figure 6. The circles have centres O and Q , where $OQ = 6$ cm, and they meet at the points A and B . The region common to both circles is shown shaded.
- (a) Show that angle $AOB = 1.855$ radians, to 4 significant figures.

(2)

- (b) Find the area of the shaded region, in cm^2 to 3 significant figures.

(2)

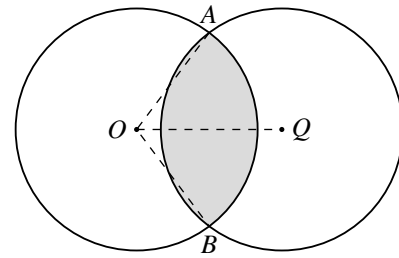


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Figure 6

[4 marks]

- D7** Figure 7 shows the curve with equation $y = \tan\left(x + \frac{\pi}{3}\right)$ for $0 \leq x \leq 2\pi$, where x is in radians.

The curve is the graph of $y = \tan x$ after a translation, and its asymptotes are shown as dashed lines.

- (a) Write down the equations of the two asymptotes.

(1)

- (b) Write down the exact coordinates of the point where the curve crosses the y -axis.

(1)

- (c) Find the exact values of x , in the interval $0 \leq x \leq 2\pi$, where the curve crosses the x -axis, showing your working.

(2)

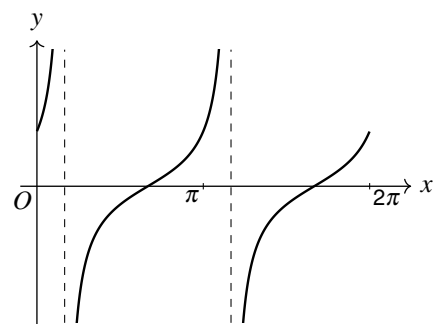


Figure 7

[4 marks]

- D8** Figure 8 shows the sector OAB of a circle with centre O and radius r cm. The angle AOB is θ radians. The minor segment cut off by the chord AB is shown shaded; it is used as a decorative panel. The perimeter of the sector is twice the length of the arc AB , and the area of the sector is 49 cm^2

- (a) Show that $\theta = 2$

(2)

- (b) Find the value of r .

(1)

- (c) Find the area of the shaded segment, giving your answer in cm^2 to 3 significant figures.

(2)

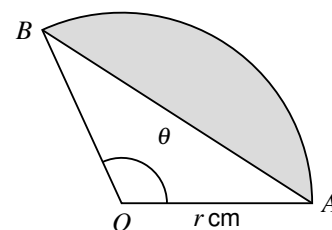


Diagram NOT accurately drawn

Figure 8

[5 marks]



E

Exam practice

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** A gardener plans to turf a lawn and to put an edging strip all the way around it. Figure 9 shows the plan of the lawn $OABC$. The lawn consists of the sector OAB of a circle with centre O , joined along OB to the triangle OBC .

Given that

- $OA = OB = 12$ m
- angle $AOB = 0.9$ radians
- $BC = 5$ m and angle OBC is a right angle

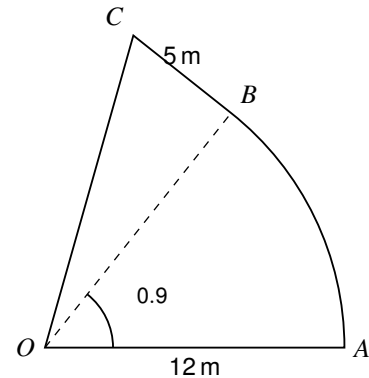


Diagram NOT accurately drawn

Figure 9

- (a) find the area of the lawn, giving your answer in m^2 ,

.....
(2)

- (b) find the length of edging strip needed, which is the perimeter of the lawn. The line OB is not part of the perimeter.

.....
(3)

(Total for Question E1 is 5 marks)



E2 Figure 10 shows part of the curve C with equation

$$y = a \cos(x + b) \quad 0 \leq x \leq 2\pi$$

where a and b are positive constants, $0 < b < \pi$, and x is measured in radians.

The curve crosses the y -axis at the point $(0, 1)$ and

has a maximum point at $\left(\frac{5\pi}{3}, 2\right)$

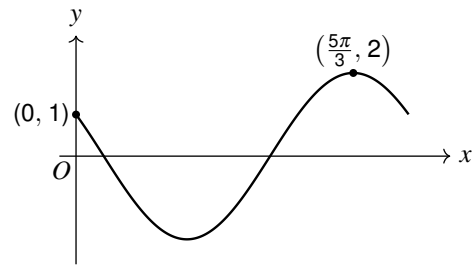


Figure 10

(a) Find the value of a and the exact value of b , showing how each value follows from the information given.

.....

(3)

(b) Find the exact x coordinates of the points where C crosses the x -axis in the given interval.

.....

(2)

(c) State the number of solutions of the equation $a \cos(x + b) = 1.5$ for $0 \leq x \leq 2\pi$, giving a reason for your answer.

(1)

(Total for Question E2 is 6 marks)



STRETCH

E3 A logo is designed from a sector and a semicircle.

Figure 11 shows the sector OAB of a circle with

centre O and radius 6 cm, with angle $AOB = \frac{2\pi}{3}$

radians. A semicircle is drawn on the chord AB as diameter, on the opposite side of AB to O

The region L , shown shaded, is inside the semicircle and outside the sector.

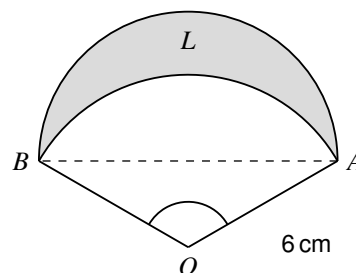


Diagram NOT accurately drawn

Figure 11

(a) Use the cosine rule to show that $AB = 6\sqrt{3}$ cm.

(2)

(b) Find the exact area of the minor segment cut off by the chord AB , giving your answer in the form $p\pi - q\sqrt{3}$

(3)

(c) Hence show that the area of the region L is $\frac{3\pi}{2} + 9\sqrt{3}$ cm²

(3)

(Total for Question E3 is 8 marks)

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