eClassroom

A-LEVEL MATHEMATICS • STATISTICS 2

Binomial Distributions

Worksheet

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Free Binomial Distributions worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 9

Tap a section to jump straight to it

By the end of this worksheet you can: state a binomial model with its conditions • find probabilities from the formula and the tables • use the mean and variance • find an unknown n or p • *Stretch:* swapping to failures, conditional probabilities, least values of n and binomial models inside binomial models.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Counting failures, unknown n and p, conditional probabilities, least values of n and two-stage models.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

Full worked solutions | Exam questions by topic | Statistics 2 exam questions | Formula sheet builder

Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: the formula

$X \sim \text{B}(10, 0.3)$. Find $\text{P}(X = 2)$

$\text{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$

$\binom{10}{2}(0.3)^2(0.7)^8 = 45(0.09)(0.057648)$

0.2335

Your turn A1

$X \sim \text{B}(12, 0.25)$. Find $\text{P}(X = 3)$

Worked example A2: the tables

$X \sim \text{B}(20, 0.35)$. Find $\text{P}(X > 8)$ and $\text{P}(4 \leqslant X \leqslant 9)$

The tables give $\text{P}(X \leqslant x)$

$1 - \text{P}(X \leqslant 8) = 1 - 0.7624$; $\text{P}(X \leqslant 9) - \text{P}(X \leqslant 3) = 0.8782 - 0.0444$

0.2376, 0.8338

Your turn A2

$X \sim \text{B}(25, 0.2)$. Find $\text{P}(X \geqslant 7)$ and $\text{P}(3 < X < 8)$

Worked example A3: mean and variance

$X \sim \text{B}(40, 0.15)$. Find the mean and variance of X, and $\text{P}(X >$ mean$)$

$np = 6$, $np(1-p) = 5.1$; $1 - \text{P}(X \leqslant 6) = 1 - 0.6067$

6, 5.1, 0.3933

Your turn A3

$X \sim \text{B}(30, 0.4)$. Find the mean and variance of X, and $\text{P}(X <$ mean$)$

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

STRETCH

Worked example A4: when $p > 0.5$

75% of seeds germinate. 20 seeds are planted. Find the probability that at least 17 germinate.

Count the failures: $Y \sim \text{B}(20, 0.25)$

$\text{P}(X \geqslant 17) = \text{P}(Y \leqslant 3)$

0.2252

Your turn A4

90% of components pass a test. 15 are tested. Find the probability that at most 12 pass.

Worked example A5: n and p from the moments

$X \sim \text{B}(n, p)$ has mean 7.5 and variance 5.625. Find n and p

Divide: $\dfrac{np(1-p)}{np} = 1 - p$

$1 - p = \dfrac{5.625}{7.5} = 0.75,\ p = 0.25,\ n = \dfrac{7.5}{0.25}$

$n = 30,\ p = 0.25$

Your turn A5

$X \sim \text{B}(n, p)$ has mean 4.8 and variance 2.88. Find n and p

Worked example A6: the least n

Each item is defective with probability 0.08, independently. Find the least n for which P(at least one defective in n) > 0.95

$1 - 0.92^n > 0.95,\ n > \dfrac{\ln 0.05}{\ln 0.92} = 35.9$

$n = 36$

Your turn A6

Each item is defective with probability 0.15, independently. Find the least n for which P(at least one defective in n) > 0.99

Worked example A7: a conditional probability

$X \sim \text{B}(10, 0.4)$. Find $\text{P}(X = 4 \mid X \geqslant 3)$

$\dfrac{\text{P}(X = 4)}{\text{P}(X \geqslant 3)}$, *as* $X = 4$ *lies inside* $X \geqslant 3$

$\dfrac{0.2508}{1 - 0.1673}$

0.3012

Your turn A7

$X \sim \text{B}(15, 0.3)$. Find $\text{P}(X \leqslant 2 \mid X \leqslant 5)$

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 $X \sim \text{B}(15, 0.4)$. Use the tables to find

(a) $\text{P}(X \leqslant 5)$

(b) $\text{P}(X < 5)$

(c) $\text{P}(X \geqslant 5)$

(d) $\text{P}(X > 5)$

[4 marks]

B2 $X \sim \text{B}(8, p)$. Use the formula to find $\text{P}(X = 2)$ when

(a) $p = 0.1$

(b) $p = 0.25$

(c) $p = 0.5$

(d) $p = 0.75$

[4 marks]

B3 $X \sim \text{B}(20, 0.45)$. Find

(a) $\text{P}(6 \leqslant X \leqslant 10)$

(b) $\text{P}(6 < X \leqslant 10)$

(c) $\text{P}(6 \leqslant X < 10)$

(d) $\text{P}(6 < X < 10)$

[4 marks]

B4 For each binomial distribution, find what is asked.

(a) B(50, 0.2): the mean and the variance

(b) B(n, 0.5) with mean 12: n

(c) B(30, p) with variance 6.3: p

(d) B(36, p) with standard deviation 3: p

[4 marks]

B5 State, with a reason, whether each variable can be modelled by a binomial distribution. If it can, give the distribution.

(a) The number of heads in 10 tosses of a fair coin.

(b) The number of aces in 5 cards dealt from a standard pack of 52 cards.

(c) The number of rolls of a fair dice needed to obtain the first six. **[3 marks]**

B6 $X \sim \text{B}(25, 0.3)$. Find

(a) the largest a with $\text{P}(X \leqslant a) < 0.1$

(b) the smallest b with $\text{P}(X \geqslant b) < 0.05$

(c) the smallest c with $\text{P}(X \leqslant c) > 0.9$

(d) the smallest d with $\text{P}(X > d) < 0.2$

[4 marks]

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

STRETCH

B7 On a railway line, 80% of trains arrive on time, independently. 20 trains are observed. Find the probability that

(a) all 20 are on time

(b) at least 18 are on time

(c) fewer than 14 are on time

(d) between 14 and 17 (inclusive) are on time

[4 marks]

B8 Find the unknown in each case.

(a) B(n, 0.2): the least n with P($X = 0$) < 0.01

(b) B(2, p) with P($X = 0$) = 0.36: p

(c) B(n, p) with mean 3.2 and variance 2.56: n and p

(d) B(5, p) with P($X = 5$) = P($X = 4$): p

[4 marks]

B9 $X \sim$ B(12, 0.3)

(a) Find P($X \geqslant 4$)

(b) Find P($X = 4 \mid X \geqslant 4$)

(c) Two independent observations of X are made. Find the probability that exactly one of them is at least 4

(d) Ten independent observations are made. Find the expected number that are at least 4

[4 marks]

B10 $X \sim$ B(20, 0.35) and $Y = 3X - 5$

(a) Find E(Y)

(b) Find Var(Y)

(c) Find P($Y >$ E(Y))

(d) Find P($Y \leqslant$ E(Y) $- \sigma_Y$), where σ_Y is the standard deviation of Y

[4 marks]

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 $X \sim \text{B}(20, 0.3)$. Amir writes $\text{P}(X \geqslant 8) = 1 - \text{P}(X \leqslant 8) = 0.1133$. Explain his error, and find $\text{P}(X \geqslant 8)$

[2 marks]

C2 15% of a firm's parcels are delayed. Asked for a suitable model for the number delayed out of 30 parcels, Bea writes "binomial". Explain why this does not score, and give the model. **[2 marks]**

C3 70% of students pass a test, independently. Cal wants the probability that at least 12 of 15 students pass, but says the tables cannot be used because they stop at $p = 0.5$

Explain how the tables can be used, and find the probability. **[3 marks]**

C4 $X \sim \text{B}(n, p)$ has mean 8 and standard deviation 2. Dev writes $np(1-p) = 2$ and finds $p = 0.75$ and $n = 10.7$

Explain his error, and find n and p **[3 marks]**

STRETCH

C5 $X \sim \text{B}(10, 0.25)$. Asked for the probability that $X = 3$ given that $X \geqslant 2$, Eve gives $\text{P}(X = 3) = 0.2503$

Explain her error, and find the probability. **[3 marks]**

C6 $X \sim \text{B}(2, p)$, where $p > 0.5$, and $\text{P}(X = 1) = 0.42$. Fay solves $2p(1-p) = 0.42$ and gives $p = 0.3$

Explain her error, and find p **[3 marks]**

C7 Gus models the number of red cards among 5 cards dealt from a standard pack of 52 cards as B(5, 0.5)

Explain why this model is not valid, and describe how the dealing could be changed so that it is.

[3 marks]

C8 $X \sim \text{B}(n, p)$, where $n \geqslant 2$ and $0 < p < 1$, and $\text{P}(X = 0) = \text{P}(X = 1)$

(a) Show that $p = \dfrac{1}{n+1}$

(2)

(b) Given also that $\text{Var}(X) = 0.81$, show that $n = 9$, and find $\text{P}(X \geqslant 2)$

(3)

[5 marks]

D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 A machine fills bags of flour, and 15% of the bags it fills are underweight, independently of each other. A quality inspector selects a random sample of 25 bags. Find the probability that the number of underweight bags in the sample is within one standard deviation of its mean. **[3 marks]**

D2 A test consists of 20 multiple-choice questions, each with 4 options, only one of which is correct. Each correct answer gains 3 marks and each incorrect answer loses 1 mark. Ravi answers every question by choosing one option at random.

(a) Show that Ravi's expected total score is zero.

(2)

(b) Find the probability that Ravi scores at least 10 marks.

(2)

[4 marks]

D3 Seeds of a certain plant germinate independently with probability 0.85. A gardener sows trays of 10 seeds, and a tray is said to have failed if fewer than 8 of its seeds germinate. The gardener sows 6 trays.

Find the probability that exactly 2 of the 6 trays fail. **[3 marks]**

D4 The number of left-handed students in a group, X, is modelled by B(n, p). The mean of X is 3.6 and its standard deviation is 1.8

(a) Find the value of n and the value of p

(2)

(b) Find P$(X = 0)$

(1)

[3 marks]

STRETCH

D5 Components are produced on a production line and each component is defective with probability 0.05, independently of all others. An inspector takes a random sample of n components.

Find the least value of n for which the probability that the sample contains at least 2 defective components is greater than 0.9 **[4 marks]**

D6 The random variable $X \sim$ B(40, 0.35). Given that P$(X > k) = 0.0699$, where k is an integer,

(a) find the value of k

(2)

(b) find P$(X > k \mid X > 12)$

(2)

[4 marks]

D7 An archer hits the target with probability p on each shot, independently. The probability that the archer hits the target at least once in 2 shots is 0.36

(a) Find the value of p

(2)

(b) The archer takes 20 shots. Find the probability that the archer hits the target more than 5 times.

(2)

[4 marks]

D8 A large batch of items, 10% of which are defective, is checked as follows. A random sample of 10 items is taken. If it contains no defective items the batch is accepted; if it contains 2 or more the batch is rejected. If it contains exactly 1, a second random sample of 10 items is taken and the batch is accepted only if this second sample contains no defective items.

(a) Find the probability that the batch is accepted after the first sample.

(1)

(b) Find the probability that the batch is accepted.

(2)

(c) Find the expected number of items inspected.

(2)

[5 marks]

E Exam practice

Full-size exam questions in the style of WST02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 A cafe accepts payment by cash or by card. Past records from the cafe show that 35% of its customers pay by card. On a particular morning, a random sample of 30 customers is taken, and the random variable X represents the number of these 30 customers who pay by card.

(a) State a suitable model for the distribution of X, and a necessary assumption for the model to be valid.

(2)

(b) Find $P(X = 10)$

(2)

(c) Find the probability that at least 13 of the 30 customers pay by card.

(2)

(d) Find $P(7 < X \leqslant 14)$

(2)

The probability that fewer than r of the 30 customers pay by card is less than 0.05

(e) Find the largest possible value of r

(3)

(Total for Question E1 is 11 marks)

E2 A factory produces electronic components, and packs them into boxes of n components chosen at random from its production. The number of faulty components in a box, X, is modelled by B(n, p), where p is the probability that a component is faulty. The mean number of faulty components in a box is 3 and the variance of the number of faulty components in a box is 2.7

(a) Find the value of n and the value of p

(3)

(b) Find the probability that a box contains at most 2 faulty components.

(1)

(c) Find the probability that a box contains exactly 4 faulty components, given that it contains at least 2 faulty components.

(3)

(d) Find the probability that a box contains at least 27 components that are not faulty.

(2)

A customer buys 5 of these boxes, and the numbers of faulty components in the boxes are independent.

(e) Find the probability that at least one of the 5 boxes contains more than 5 faulty components.

(2)

(Total for Question E2 is 11 marks)

STRETCH

E3 A company sends notifications to the users of its mobile phone app. Each notification sent to a user is opened by the user with probability p, independently of all other notifications. The probability that none of 4 notifications sent to a user is opened is 0.0256

(a) Show that $p = 0.6$

(3)

(b) A user receives 25 notifications. Find the probability that more than 18 of them are opened.

(3)

(c) Find the least number of notifications that must be sent to a user for the probability that at least one is opened to exceed 0.999

(3)

The company decides to send notifications to a user in sets of 10, with the sets independent of each other. A set is described as *successful* if at least 7 of its 10 notifications are opened by the user.

(d) Show that the probability that a set is successful is 0.3823, and find the probability that exactly 5 of 12 sets are successful.

(3)

(e) Find the least number of sets that must be sent for the probability that at least one set is successful to exceed 0.99

(2)

(Total for Question E3 is 14 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Statistics 2 Chapter 1, Binomial Distributions, with a full mark scheme. Open the booklet

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then practise real exam questions on the **Statistics 2 exam questions page**.