



eClassroom

A-LEVEL MATHEMATICS • PURE 4

Coordinate Geometry

Worksheet

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Free Coordinate Geometry worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: find points and parameters on a curve • eliminate t to find a Cartesian equation • use $\sin^2 t + \cos^2 t = 1$ and other identities • find where a parametric curve meets a line or the axes • *Stretch:* state the domain and range correctly, use double-angle and sec, tan and cot identities, and sketch the part of the curve that is traced.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Double-angle and sec, tan and cot forms, ranges with a turning point inside the interval, and intersections that need an identity.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1: eliminating t

A curve has parametric equations $x = 2t + 1$, $y = t^2 - 3$. Find a Cartesian equation.

Make t the subject of the simpler equation, then substitute.

$$t = \frac{x-1}{2}, \text{ so } y = \left(\frac{x-1}{2}\right)^2 - 3$$

$$y = \frac{1}{4}(x-1)^2 - 3$$

Your turn A1

A curve has parametric equations $x = 3 - t$, $y = t^2 + 2t$. Find a Cartesian equation in the form $y = ax^2 + bx + c$

Worked example A2: $\sin^2 t + \cos^2 t = 1$

A curve has parametric equations $x = 3\cos t$, $y = 2\sin t$. Find a Cartesian equation.

Make $\cos t$ and $\sin t$ the subjects, then square and add.

$$\cos t = \frac{x}{3}, \sin t = \frac{y}{2}: \frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Your turn A2

A curve has parametric equations $x = 2 + 3\cos t$, $y = 3\sin t - 1$. Find a Cartesian equation, and describe the curve.

Worked example A3: meeting a line

The curve $x = t^2$, $y = 2t$ meets the line $y = x - 3$. Find the points where they meet.

Substitute x and y into the line, and solve for t

$$2t = t^2 - 3 \Rightarrow (t-3)(t+1) = 0, \text{ so } t = 3 \text{ or } t = -1$$

$$(9, 6) \text{ and } (1, -2)$$

Your turn A3

The curve $x = t + 1$, $y = t^2$ meets the line $y = 2x + 1$. Find the points where they meet.



STRETCH

Worked example A4: domain and range

$x = t^2 - 1$, $y = 2t + 3$, for $-1 \leq t \leq 2$. Find the range of values of x and of y

y is linear in t , but x has a minimum at $t = 0$, inside the interval.

$t = -1, 0, 2$ give $x = 0, -1, 3$

y runs from 1 to 7

$$-1 \leq x \leq 3, \quad 1 \leq y \leq 7$$

Your turn A4

$x = 3t - 2$, $y = t^2 + 1$, for $-2 \leq t \leq 1$. Find the range of values of x and of y

Worked example A5: double-angle forms

$x = \sin t$, $y = \cos 2t$. Find a Cartesian equation and the possible values of x

Write $\cos 2t$ in terms of $\sin t$: $\cos 2t = 1 - 2\sin^2 t$

$y = 1 - 2x^2$, and $-1 \leq \sin t \leq 1$

$$y = 1 - 2x^2, \quad -1 \leq x \leq 1$$

Your turn A5

$x = 2 \cos t$, $y = \cos 2t + 1$. Find a Cartesian equation and the possible values of x

Worked example A6: sec, tan, cosec and cot

$x = 2 \sec t$, $y = 3 \tan t$. Find a Cartesian equation.

Use $\sec^2 t - \tan^2 t = 1$

$$\sec t = \frac{x}{2}, \quad \tan t = \frac{y}{3}: \quad \frac{x^2}{4} - \frac{y^2}{9} = 1$$

$$\frac{x^2}{4} - \frac{y^2}{9} = 1$$

Your turn A6

$x = \cot t$, $y = 2 \operatorname{cosec} t + 1$. Find a Cartesian equation.

Worked example A7: axes and sketching

$x = t^2 - 4$, $y = t + 1$. Find where the curve meets the axes, and its vertex.

$x = 0$: $t = \pm 2$, so $(0, 3)$, $(0, -1)$; $y = 0$: $t = -1$, so $(-3, 0)$

x is least when $t = 0$: the vertex is $(-4, 1)$, opening to the right.

$$(0, 3), (0, -1), (-3, 0); \text{ vertex } (-4, 1)$$

Your turn A7

$x = 2t + 1$, $y = t^2 - 2t - 3$. Find where the curve meets the axes, and its lowest point.



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 A curve has parametric equations $x = t^2 + 1$, $y = 3t - 2$

(a) Find the coordinates of the point where $t = 2$

(b) Find the value of t at the point $(10, 7)$

(c) Show that $(5, -8)$ lies on the curve.

(d) Show that $(2, 2)$ does not lie on the curve.

[4 marks]

B2 Find a Cartesian equation for each curve.

(a) $x = t + 3$, $y = 2t - 1$

(b) $x = 2t$, $y = t^2 - 1$

(c) $x = \frac{1}{t}$, $y = t + 2$

(d) $x = \sqrt{t}$, $y = t - 4$

[4 marks]

B3 Use $\sin^2 t + \cos^2 t = 1$ to find a Cartesian equation for each curve.

(a) $x = 4 \cos t$, $y = 4 \sin t$

(b) $x = 2 \cos t$, $y = 5 \sin t$

(c) $x = 3 + \cos t$, $y = 2 \sin t$

(d) $x = \sin t + \cos t$, $y = \sin t - \cos t$

[4 marks]

B4 Find the points where

(a) $x = t^2$, $y = 4t$ meets the line $y = 2x$

(b) $x = 2t$, $y = \frac{1}{t}$ meets the line $y = \frac{x}{8}$

(c) $x = t + 1$, $y = t^2$ meets the x -axis and the y -axis

(d) $x = 3 \cos t$, $y = 3 \sin t$ meets the line $y = x$

[4 marks]



CORE

continued: domain and range

- B5** A curve has parametric equations $x = 2\cos t$, $y = \sin t$, $0 \leq t \leq \pi$
- (a) Find the coordinates of the end points of the curve.
 - (b) Find the range of values of y
 - (c) Find a Cartesian equation, and state which part of this curve is traced. [3 marks]
- B6** A curve has parametric equations $x = t^2 - 2t$, $y = t + 1$, $0 \leq t \leq 3$
- (a) Find the range of values of x
 - (b) Find the range of values of y
 - (c) Find a Cartesian equation in the form $x = y^2 + by + c$
 - (d) Explain why the end values of t alone do not give the range of x [4 marks]

STRETCH

- B7** Find a Cartesian equation for each curve.
- | | |
|--------------------------------------|---|
| (a) $x = \cos t$, $y = \cos 2t$ | (b) $x = \sin t + \cos t$, $y = \sin 2t$ |
| (c) $x = 1 + \cos 2t$, $y = \sin t$ | (d) $x = \cos t$, $y = \cos 3t$ |
- [4 marks]
- B8** Find a Cartesian equation for each curve.
- | | |
|---|---|
| (a) $x = \sec t$, $y = \tan t$ | (b) $x = 2\tan t$, $y = \cos^2 t$ |
| (c) $x = \operatorname{cosec} t$, $y = \cot t + 1$ | (d) $x = \sec t + \tan t$, $y = \sec t - \tan t$ |
- [4 marks]
- B9**
- (a) Find where $x = t^2$, $y = t^3$ meets the line $y = 2x$
 - (b) Find where $x = 2\cos t$, $y = 2\sin t$ meets $y = x + 2$, and the value of t at each point, $0 \leq t < 2\pi$
 - (c) Find the exact points where $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$ meets the line $y = 1$
 - (d) Find the value of m for which the line $y = mx + 1$ meets $x = t^2$, $y = 2t$ at exactly one point. [4 marks]
- B10** A ball moves so that $x = 20t$ and $y = 15t - 5t^2$, in metres, for $0 \leq t \leq 3$
- (a) Find a Cartesian equation of the path.
 - (b) Find the horizontal distance the ball travels before it lands.
 - (c) Find the greatest height of the ball.
 - (d) State the domain of the Cartesian equation in this model. [4 marks]



C

Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** For $x = t - 1$, $y = t^2 + 2t$, Ali writes the Cartesian equation $y = t^2 + 2(x + 1)$
Explain why this is not a Cartesian equation, and give a correct one. [2 marks]
- C2** For $y = 4t - t^2$, $0 \leq t \leq 3$, Bina substitutes $t = 0$ and $t = 3$ and says that $0 \leq y \leq 3$
Explain her mistake and give the correct range of y [2 marks]
- C3** Chen says that $x = 3 + 2\cos t$, $y = 1 + 2\sin t$ is a circle with centre $(-3, -1)$ and radius 4
Find his two errors, and give the correct centre and radius, with a Cartesian equation. [3 marks]
- C4** To find where $x = t^2$, $y = 2t$ meets the line $x = 4$, Dan solves $t^2 = 4$ to get $t = 2$, and gives the point $(4, 4)$
Explain what Dan has missed, and give the complete answer. [3 marks]

STRETCH

- C5** For $x = 2\sin t$, $y = \cos 2t$, Eva finds $y = 1 - \frac{x^2}{2}$ and says the curve is the whole parabola.
Explain why she is wrong, and state the domain and range of the curve. [3 marks]
- C6** Show that $x = t^2$, $y = t^4$ and $x = s$, $y = s^2$ give the same Cartesian equation, and explain why they are not the same curve. [3 marks]
- C7** For $x = t + \frac{1}{t}$, $y = t - \frac{1}{t}$, Fin writes $x^2 = t^2 + \frac{1}{t^2}$ and $y^2 = t^2 - \frac{1}{t^2}$
Correct his work, and find a Cartesian equation of the curve. [3 marks]
- C8** (a) Show that, for every real value of t , the point with coordinates $\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)$ lies on the circle $x^2 + y^2 = 1$ (3)
- (b) Show that the point $(0, -1)$ is never reached by these equations, for any real value of t (2)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** Figure 1 shows the curve C with parametric equations

$$x = 1 + 2\cos^2 t, \quad y = 3\sin^2 t, \quad 0 \leq t \leq \frac{\pi}{2}$$

where t is a parameter. The ends of C are the points A and B , where A lies on the x -axis.

(a) Using a suitable identity, show that C is part of the straight line with equation $3x + 2y = 9$

(2)

(b) Find the exact length of C , giving your answer as a surd.

(1)

[3 marks]

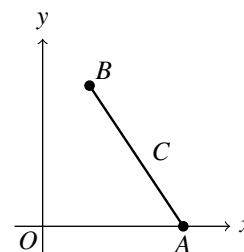


Figure 1

- D2** Figure 2 shows the curve C with parametric equations

$$x = 1 + 2\cos t, \quad y = 2 + 2\sin t, \quad 0 \leq t < 2\pi$$

where t is a parameter. The curve touches the x -axis, and meets the y -axis at the points P and Q

(a) Find a Cartesian equation for C , and state the coordinates of its centre and its radius.

(2)

(b) Using algebra, find the exact coordinates of P and Q , giving your answers in surd form.

(2)

[4 marks]

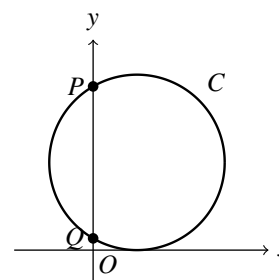


Figure 2

- D3** Figure 3 shows the curve C with parametric equations

$$x = 2t, \quad y = \frac{6}{t}, \quad t > 0$$

where t is a parameter. The line l with equation $x + y = 8$ meets C at the points A and B

Using algebra, find the coordinates of A and B , showing your working clearly, and hence find the exact length of AB , giving your answer as a simplified surd.

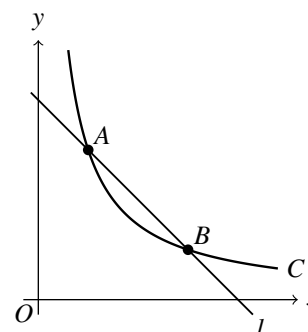


Figure 3

[3 marks]



- D4** Figure 4 shows the curve C with parametric equations

$$x = t^2 + 2t, \quad y = 3 - t, \quad a \leq t \leq b$$

where t is a parameter and a and b are constants. Both ends of C lie on the line with equation $x = 3$

- (a) Find the value of a and the value of b

(2)

- (b) Find the range of values of y for points on C , giving your answer as an inequality.

(1)

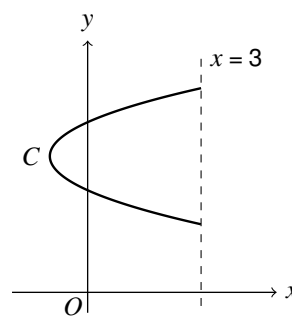


Figure 4

[3 marks]

STRETCH

- D5** Figure 5 shows the curve C with parametric equations

$$x = \tan^2 t, \quad y = 2 \sec t, \quad 0 \leq t < \frac{\pi}{2}$$

where t is a parameter. The curve starts at a point A on the y -axis.

- (a) Using a suitable trigonometric identity, show that all points on C satisfy the equation $y^2 = 4(x + 1)$

(2)

- (b) The curve C is only part of the curve with equation $y^2 = 4(x + 1)$. State the domain and the range of C , giving each as an inequality.

(2)

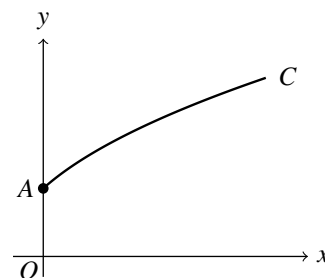


Figure 5

[4 marks]

- D6** Figure 6 shows the curve C with parametric equations

$$x = 1 - \cos 2t, \quad y = 2 \sin t, \quad 0 \leq t \leq \frac{\pi}{2}$$

where t is a parameter. The line $y = x$ meets C at the origin O and at the point P

- (a) Using a suitable double-angle formula, find a Cartesian equation for C in the form $x = f(y)$

(2)

- (b) Using algebra, find the coordinates of P and the value of t at P , showing your working clearly.

(2)

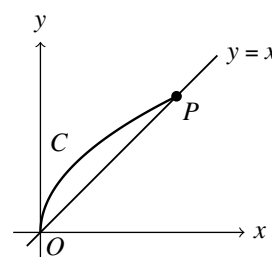


Figure 6

[4 marks]



D7 A curve C has parametric equations

$$x = \frac{t}{1+t}, \quad y = \frac{t^2}{1+t}, \quad t > -1$$

where t is a parameter.

(a) Show that a Cartesian equation of C is $y = \frac{x^2}{1-x}$, and state the set of possible values of x

(2)

(b) The line with equation $y = 2x$ meets C at the origin and at the point R . Find the coordinates of R and the value of t at R . You must show all stages of your working.

(2)

[4 marks]

D8 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 7 shows the curve C with parametric equations

$$x = 4 \cos t, \quad y = 3 \sin t, \quad 0 \leq t < 2\pi$$

The line l with equation $4y = 3x + 12$ meets C at the points A and B

(a) Find a Cartesian equation for C

(1)

(b) Show that, at A and B , $\sin t - \cos t = 1$, and hence find the values of t at A and B

(3)

(c) Find the area of triangle OAB

(1)

[5 marks]

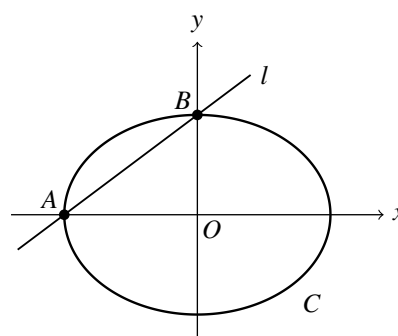


Figure 7



E**Exam practice**

Exam-style questions in the style of WMA14. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** Figure 8 shows a sketch of the curve C with parametric equations

$$x = 5 \cos t, \quad y = 3 \sin t, \quad 0 \leq t < 2\pi$$

The line with equation $x = 4$ meets C at the points P and Q , as shown in Figure 8

- (a) Find a Cartesian equation for the curve C , giving your answer in the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are positive integers to be found.

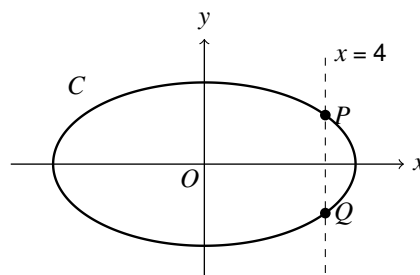


Figure 8

(2)

- (b) Using the parametric equations, find the exact coordinates of P and Q , and the exact length of PQ , giving your answers as fractions.

(3)

(Total for Question E1 is 5 marks)

- E2** Figure 9 shows a sketch of the curve C with parametric equations

$$x = 2t + 3, \quad y = t^2 - 4t, \quad -1 \leq t \leq 5$$

where t is a parameter.

- (a) Show that a Cartesian equation of C can be written in the form $y = ax^2 + bx + c$, where a , b and c are constants to be found.

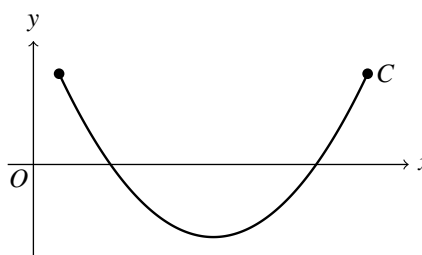


Figure 9

(3)

- (b) State the domain of C , the set of values of x for points on C , giving your answer as an inequality.

(1)

- (c) Find the range of C , the set of values of y for points on C . You must make your method clear.

(2)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 10 shows the curve C with parametric equations

$$x = 3t^2, \quad y = 6t, \quad t \in \mathbb{R}$$

The line l with equation $y = x - 12$ meets C at the points A and B , where A is above the x -axis.

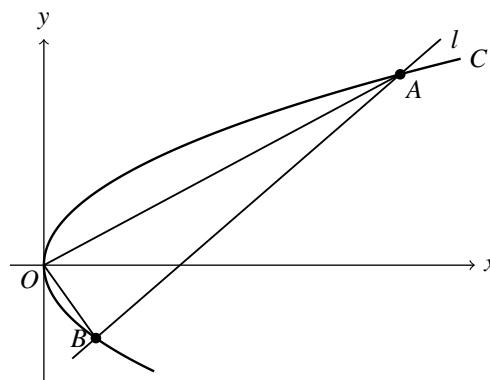


Figure 10

(a) Find a Cartesian equation for C

(1)

(b) Show that the values of t at A and B satisfy $t^2 - 2t - 4 = 0$

(2)

(c) Find the exact coordinates of A and B

(2)

(d) Using your answer to part (b), or otherwise, prove that the angle AOB is a right angle.

(3)

(Total for Question E3 is 8 marks)

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