



eClassroom

A-LEVEL MATHEMATICS • MECHANICS 1

Vectors in Mechanics

Worked Solutions

Pearson Edexcel International A Level (WME01) & A Level (9MA0) • Calculator and statistical tables allowed: take $g = 9.8 \text{ m s}^{-2}$ and give answers to 2 or 3 s.f.

Full worked solutions for the free Vectors in Mechanics worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Mechanics 1 Vectors in Mechanics worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dm** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $\sqrt{25+144}$: **13**; $180^\circ - \tan^{-1} \frac{12}{5}$: **112.6°**

A2 $|12\mathbf{i} - 5\mathbf{j}| = 13$; $3(12\mathbf{i} - 5\mathbf{j})$: **$(36\mathbf{i} - 15\mathbf{j}) \text{ km h}^{-1}$**

A3 $(-4+10)\mathbf{i} + (7+15)\mathbf{j}$: **$6\mathbf{i} + 22\mathbf{j}$**

STRETCH

A4 $\frac{9\mathbf{i} + 6\mathbf{j}}{3}$: **$3\mathbf{i} + 2\mathbf{j}$**

A5 $\mathbf{r}_A = 7\mathbf{i} + 9\mathbf{j}$, $\mathbf{r}_B = 4\mathbf{i} + 5\mathbf{j}$; $\overrightarrow{AB} = -3\mathbf{i} - 4\mathbf{j}$: **5**

A6 $1+2t=7$ gives $t=3$; $8-9=-1 \neq -2$: **no**

A7 Resultant $(4+p)\mathbf{i} + 4\mathbf{j}$; $\frac{4+p}{1} = \frac{4}{2}$: **$p = -2$**

B**Fluency solutions**

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 (a) $\sqrt{64+225}$: **17 ms^{-1}** (b) $90^\circ + \tan^{-1} \frac{15}{8}$: **152°**

(c) **$\frac{1}{17}(8\mathbf{i} - 15\mathbf{j})$** (d) **$(16\mathbf{i} - 30\mathbf{j}) \text{ ms}^{-1}$**

A sketch shows the vector points south of east, so the bearing is more than 90°

B2 (a) **$(-2\mathbf{i} + 3\mathbf{j}) \text{ N}$** (b) $\sqrt{13}$: **3.61 N** (c) $\frac{-2\mathbf{i} + 3\mathbf{j}}{0.5}$: **$(-4\mathbf{i} + 6\mathbf{j}) \text{ ms}^{-2}$** (d) **$7.21 \text{ ms}^{-2}$**

$\mathbf{F} = m\mathbf{a}$ works on whole vectors.



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B3 (a) 10 km h^{-1} (b) $(10+6t)\mathbf{i} + (8t-4)\mathbf{j}$ (c) $t = 1.5$: $(19\mathbf{i} + 8\mathbf{j})\text{ km}$ (d) $8t - 4 = 0$: $12:30$

Due east of O means the $\mathbf{j}$ -component is 0

B4 (a) $\frac{20\mathbf{i} - 15\mathbf{j}}{5}$: $(4\mathbf{i} - 3\mathbf{j})\text{ ms}^{-1}$ (b) 5 ms^{-1} (c) $(4\mathbf{i} + 9\mathbf{j}) - 3(4\mathbf{i} - 3\mathbf{j})$: $-8\mathbf{i} + 18\mathbf{j}$

(d) $\mathbf{r} = (4t - 8)\mathbf{i} + (18 - 3t)\mathbf{j}$

The interval is 5 s, not 8 s

CORE

B5 (a) $(8-4t)\mathbf{i} + (4-2t)\mathbf{j}$ (b) both components are 0 when $t = 2$ (c) $8\mathbf{i} + 3\mathbf{j}$

A collision needs $\vec{AB} = \mathbf{0}$: both components at the same time.

B6 (a) $-6 + 2t = 0$: $t = 3$ (b) $13\mathbf{j}$ (c) $t = 5$, $4 + 15 = 19$: **yes** (d) $t = 8$, $4 + 24 = 28 \neq 26$: **no**

Use one component to find t , then test the other.

STRETCH

B7 (a) $3(p+2) = 2(2p-1)$: $p = 8$ (b) $10\mathbf{i} + 15\mathbf{j}$ (c) 18.0 (d) $\tan^{-1} \frac{10}{15}$: 034°

Parallel: the components are in the same ratio, not equal.

B8 (a) $2(2\mathbf{i} + \mathbf{j})$: $(4\mathbf{i} + 2\mathbf{j})\text{ N}$ (b) $a + 3 = 4$, $b - 5 = 2$: $a = 1$, $b = 7$ (c) $\sqrt{50}$: 7.07 N (d) 81.9°

Find the resultant from $m\mathbf{a}$ first, then subtract the known force.

B9 (a) $(8\mathbf{i} + 6\mathbf{j})\text{ km}$ (b) $+(-3\mathbf{i} + 15\mathbf{j})$: $(5\mathbf{i} + 21\mathbf{j})\text{ km}$ (c) $\sqrt{466}$: 21.6 km (d) $\tan^{-1} \frac{5}{21}$: 013°

The second stage starts from the end of the first.

B10 (a) $5\mathbf{i} + 5\mathbf{j}$ (b) 7.07 ms^{-1} (c) $-1 + 2t = 0$: $t = 0.5$ (d) $|\mathbf{i} + 2\mathbf{j}|$: 2.24 ms^{-2}

Parallel to $\mathbf{i}$ means the $\mathbf{j}$ -component of velocity is 0



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 Components are squared, added and square-rooted: $\sqrt{9+16} = 5$

C2 Divide by the time interval, $7 - 2 = 5$: $2\mathbf{i} - \mathbf{j}$

C3 Parallel means a multiple, not equal: $\frac{p-6}{4} = \frac{p+9}{1}$, so $p-6 = 4p+36$: $p = -14$

C4 $-3\mathbf{i} - 3\mathbf{j}$ points south-west, not north-east: 225°

STRETCH

C5 Both components must agree at the same t . $\mathbf{j}$: $3+t = 2+2t$ gives $t = 1 \neq 2$: they do not collide

C6 $|9\mathbf{i} + 12\mathbf{j}| = 15 \text{ km h}^{-1}$, not m s^{-1} : $\frac{15000}{3600} = 4.17 \text{ m s}^{-1}$

C7 $3\mathbf{i} - 4\mathbf{j}$ has magnitude 5, so it must be made a unit vector first: $\frac{20}{5}(3\mathbf{i} - 4\mathbf{j}) = 12\mathbf{i} - 16\mathbf{j}$

C8 (a) $(5-3t)^2 + (4t-5)^2 = 25 - 30t + 9t^2 + 16t^2 - 40t + 25 = 25t^2 - 70t + 50$

(b) $25(t-1.4)^2 + 1$: least distance 1, when $t = 1.4$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $\frac{(15\mathbf{i} - \mathbf{j}) - (3\mathbf{i} + 7\mathbf{j})}{4}$: $(3\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$

(b) $(3\mathbf{i} + 7\mathbf{j}) + (t-2)(3\mathbf{i} - 2\mathbf{j})$: $(3t-3)\mathbf{i} + (11-2t)\mathbf{j}$



D2 (a) $4(2\mathbf{i} - \mathbf{j}) = 8\mathbf{i} - 4\mathbf{j}$; $3 + p = 8$, $8 + q = -4$: $p = 5$, $q = -12$

(b) $\sqrt{25 + 144}$: 13 N ; $\tan^{-1} \frac{12}{5}$: 67.4° below $\mathbf{i}$

D3 (a) $76 = 36 + X^2 + 2(6)X \cos 60^\circ$, so $X^2 + 6X - 40 = 0$: $X = 4$ (reject -10)

(b) $\frac{\sin \phi}{4} = \frac{\sin 120^\circ}{\sqrt{76}}$: 23.4°

D4 $1 + 8t = 25$ gives $t = 3$; $6(3) - 6 = 12 \checkmark$: A passes P at 15:00, half an hour after B

STRETCH

D5 (a) $\frac{(3\mathbf{i} + 8\mathbf{j}) - (5\mathbf{i} + 4\mathbf{j})}{2}$: $\mathbf{a}_B = -\mathbf{i} + 2\mathbf{j}$; $\mathbf{v}_B(0) = 6\mathbf{i} + 2\mathbf{j}$

(b) $\mathbf{v}_B = (6 - t)\mathbf{i} + (2 + 2t)\mathbf{j}$; $6 - t = 0$: $t = 6$, speed 14 km h^{-1}

D6 (a) $\vec{SB} = (18 - 6t)\mathbf{i} + (8t - 8)\mathbf{j}$; $SB^2 = 100t^2 - 344t + 388$; least when $t = \frac{344}{200}$: $t = 1.72$

(b) $\sqrt{388 - \frac{344^2}{400}} = \sqrt{92.16}$: 9.6 km

D7 (a) $\mathbf{i}$: $2t = 8 - t$ gives $t = \frac{8}{3}$; $\mathbf{j}$: $5 + t = 1 + 3t$ gives $t = 2$: different times, so they never collide

(b) B due east of A : $\mathbf{j}$ -components equal, $t = 2$; $\mathbf{r}_A = 4\mathbf{i} + 7\mathbf{j}$, $\mathbf{r}_B = 6\mathbf{i} + 7\mathbf{j}$: 2 m

D8 (a) $12 - 4t = 0$: $t = 3$, $\vec{AB} = 2\mathbf{j}$: $t = 3$, 2 m

(b) Equal positive components: $12 - 4t = 8 - 2t$, $t = 2$, $\vec{AB} = 4\mathbf{i} + 4\mathbf{j}$: $t = 2$, 5.66 m

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.



CORE Exam questions for everyone

- E1** (a) $\mathbf{v} = \frac{(16\mathbf{i} + 14\mathbf{j}) - (4\mathbf{i} - 2\mathbf{j})}{2} = 6\mathbf{i} + 8\mathbf{j}$ M1 A1
- $\mathbf{r}_A = (4 + 6t)\mathbf{i} + (8t - 2)\mathbf{j}$ A1*
- (b) $10 \text{ km h}^{-1} = \frac{10000}{3600}$ M1
- 2.78 ms^{-1} A1
- (c) $\mathbf{r}_B = (31 - 6t)\mathbf{i} + (7 + 10t)\mathbf{j}$; equate positions at times t_1 and t_2 M1
- $4 + 6t_1 = 31 - 6t_2$ and $8t_1 - 2 = 7 + 10t_2$: $t_1 = 3$, $t_2 = 1.5$ M1 A1
- $\mathbf{P} = 22\mathbf{i} + 22\mathbf{j}$; **1.5 hours between them** A1
-
- E2** (a) $\mathbf{a}_A = 2\mathbf{i} - \mathbf{j}$ M1
- $\sqrt{5} = 2.24 \text{ km h}^{-2}$ A1
- (b) $\frac{(3\mathbf{i} + 8\mathbf{j}) - (6\mathbf{i} + 2\mathbf{j})}{3}$ M1
- $\mathbf{a}_B = -\mathbf{i} + 2\mathbf{j}$ A1
- (c) $(6\mathbf{i} + 2\mathbf{j}) - (-\mathbf{i} + 2\mathbf{j})$ M1
- $7\mathbf{i} \text{ km h}^{-1}$ A1
- (d) $(1 + 2t)^2 + (4 - t)^2 = (7 - t)^2 + (2t)^2$ M1 A1
- $5t^2 - 4t + 17 = 5t^2 - 14t + 49$: $t = 3.2$ (15:12) A1
- (e) $\sqrt{7.4^2 + 0.8^2}$: 7.44 km h^{-1} B1



STRETCH

- E3** (a) $\mathbf{r}_A = (2+5t)\mathbf{i} + (3+2t)\mathbf{j}$, $\mathbf{r}_B = (12-t)\mathbf{i} + (6t-7)\mathbf{j}$ M1
- $\overrightarrow{AB} = (10-6t)\mathbf{i} + (4t-10)\mathbf{j}$ A1*
- (b) $\mathbf{i}: t = \frac{5}{3}$; $\mathbf{j}: t = 2.5$; **never both zero** M1 A1
- (c) $AB^2 = 52t^2 - 200t + 200$ M1 A1
- least at $t = \frac{200}{104} = \frac{25}{13} \approx 1.92$ (13:55) M1
- 2.77 km** A1
- (d) $\overrightarrow{AB} = -1.54\mathbf{i} - 2.31\mathbf{j}$; $180^\circ + \tan^{-1} \frac{1.54}{2.31}$ M1
- 214°** A1
- (e) $52t^2 - 200t + 200 = 9$; roots differ by $\frac{\sqrt{272}}{52} \approx 0.317$ h M1
- 19 minutes** A1

