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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	2	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	28	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	54	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	80	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	106	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	132	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	158	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	184	

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CHAPTER 1 • KEY SKILLS

1 Complex Numbers

Specification FP1: Complex numbers • 10 questions • 80 marks • about 96 minutes

1 Arithmetic

$i^2 = -1$. Add and subtract real and imaginary parts; multiply out brackets and replace i^2 by -1

2 Conjugate and division

$z = a + bi$ has conjugate $z^* = a - bi$. To divide, multiply top and bottom by the conjugate of the denominator.

3 Equating parts

Two complex numbers are equal when their real parts are equal and their imaginary parts are equal: two equations.

4 Argand diagram

Plot $a + bi$ at (a, b): real numbers on the real axis, conjugate pairs reflected in it.

5 Modulus

$|a + bi| = \sqrt{a^2 + b^2}$, always positive. Give it exactly (surd form) unless told otherwise.

6 Argument

Sketch first, then use $\tan^{-1}\frac{|b|}{|a|}$ and adjust for the quadrant:
$-\pi < \arg z \leqslant \pi$, in radians.

7 Modulus–argument form

$z = r(\cos\theta + i\sin\theta)$. $|zw| = |z||w|$ and $\arg(zw) = \arg z + \arg w$

8 Quadratic equations

Use the formula or complete the square; a negative discriminant gives the conjugate pair $p \pm qi$

9 Cubics and quartics

Real coefficients: complex roots come in conjugate pairs. Multiply $(z - \alpha)(z - \alpha^*)$ to get a real quadratic factor.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Arithmetic	•			•	•		•	•	•	
2 Conjugate and division	•			•				•		
3 Equating parts				•	•		•	•		
4 Argand diagram			•			•	•			•
5 Modulus		•	•	•				•	•	
6 Argument		•						•	•	•
7 Modulus–argument form		•							•	
8 Quadratic equations			•				•		•	•
9 Cubics and quartics						•	•		•	
Interleaved: simultaneous equations and quadratics (P1)			•	•	•		•	•		•
Interleaved: factor theorem and algebraic division (P2)						•	•		•	
Interleaved: trigonometry, exact values and areas (P1)		•				•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Show the algebra: a calculator answer alone scores nothing when working is required. Sketch an Argand diagram before finding any argument, and give final answers as complex numbers in the form asked for.

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1. The complex numbers z_1 and z_2 are given by $z_1 = 3 - 2i$ and $z_2 = 1 + 4i$. Without using a calculator, and showing your working,

(a) find $z_1 z_2$ in the form $a + bi$

(2)

(b) find $\frac{z_1}{z_2}$ in the form $a + bi$, where a and b are rational numbers,

(3)

(c) find $z_1 + z_2^*$

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The complex number $z = -3 + 3\sqrt{3}\,\mathrm{i}$

(a) Find $|z|$.

(1)

(b) Find $\arg z$, giving your answer as an exact multiple of π.

(2)

(c) Write z in the form $r(\cos\theta + \mathrm{i}\sin\theta)$

(1)

The complex number $w = 1 - \mathrm{i}$

(d) Find the exact values of $|zw|$ and $\arg(zw)$.

(2)

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Question 2 continued

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3. **(a)** Solve the equation $z^2 + 6z + 13 = 0$, giving your answers in the form $p + qi$

(3)

(b) Show the roots on an Argand diagram.

(1)

(c) Find the exact modulus of each root.

(1)

(d) Explain why the roots of any quadratic equation with real coefficients and negative discriminant have equal moduli.

(1)

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Question 3 continued

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4. **(a)** Find the complex number z which satisfies $2z + iz^* = 5 + 4i$

(4)

(b) Find the exact value of $|z - 3|$.

(2)

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Question 4 continued

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5. **(a)** Given that $(a+b\text{i})^2 = 5 - 12\text{i}$, where a and b are real, find the possible values of a and b.

(5)

(b) Hence write down the solutions of $z^2 = 5 - 12\text{i}$

(1)

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6.

$$f(z) = z^3 - 5z^2 + 17z - 13$$

Given that $2 + 3i$ is a root of the equation $f(z) = 0$,

(a) write down another complex root of the equation,

(1)

(b) find the quadratic factor of $f(z)$ with these two roots,

(2)

(c) solve completely the equation $f(z) = 0$,

(2)

(d) show all the roots of $f(z) = 0$ on a single Argand diagram.

(2)

The points representing the roots are the vertices of a triangle.

(e) Find the area of this triangle.

(3)

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Question 6 continued

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Question 6 continued

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7.

$$f(z) = z^4 + pz^2 + qz + 50$$

where p and q are real constants. Given that $1 + 2i$ is a root of $f(z) = 0$,

(a) find the value of p and the value of q,

(4)

(b) find the other three roots of $f(z) = 0$,

(4)

(c) show all four roots on a single Argand diagram.

(2)

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Question 7 continued

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8.

$$w = \frac{k + 6\text{i}}{2 - \text{i}}$$

where k is a real constant.

(a) Express w in the form $a + b\text{i}$, where a and b are in terms of k.

(3)

Given that $\arg w = \frac{\pi}{4}$

(b) find the value of k,

(3)

(c) find the exact value of $|w|$.

(1)

(d) Find the possible values of the real constant c for which $|w + c\text{i}| = 10$

(3)

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Question 8 continued

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9. The complex number $z = 1 + \sqrt{3}\mathrm{i}$

(a) Find $|z|$ and $\arg z$.

(2)

(b) Find z^2 in the form $a + b\mathrm{i}$, and show that $|z^2| = |z|^2$ and $\arg(z^2) = 2\arg z$

(3)

(c) Show that $z^3 = -8$

(2)

(d) Hence, or otherwise, find all the roots of the equation $z^3 + 8 = 0$

(3)

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Question 9 continued

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10. The equation $z^2 - 2kz + k^2 + 16 = 0$, where k is a real constant, has roots z_1 and z_2, where $\text{Im}(z_1) > 0$

(a) Show that the roots are not real for any value of k.

(2)

(b) Find z_1 and z_2 in terms of k.

(2)

(c) Given that $\arg z_1 = \frac{\pi}{4}$, find the value of k.

(2)

The points P and Q represent z_1 and z_2 on an Argand diagram, and O is the origin. Given instead that $k > 0$ and triangle OPQ is equilateral,

(d) find the exact value of k,

(3)

(e) find the exact area of triangle OPQ.

(1)

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TOTAL FOR CHAPTER 1 IS 80 MARKS

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CHAPTER 2 • KEY SKILLS

2 Roots of Quadratic Equations

Specification FP1: Roots of quadratic equations • 10 questions • 80 marks • about 96 minutes

1 Sum and product

For $ax^2+bx+c=0$ with roots α, β: $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$. Watch the sign of the sum.

2 Sum of squares

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

3 Sum of cubes

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

4 Reciprocals

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ and $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

5 Difference of roots

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$; then $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$

6 Forming equations

New roots with sum S and product P: $x^2-Sx+P=0$. Multiply to integer coefficients and keep "= 0".

7 Without solving

Work only with $\alpha+\beta$ and $\alpha\beta$: finding the roots themselves earns no marks.

8 Unknown coefficients

Turn a condition on the roots (one is twice the other, they differ by 2) into equations using the sum and product.

9 Complex roots

Real coefficients give conjugate roots, so $\alpha\beta=\alpha\alpha^*=|\alpha|^2$. Symmetric sums can be negative.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Sum and product	•	•	•	•	•	•	•	•	•	•
2 Sum of squares	•	•		•		•		•	•	•
3 Sum of cubes	•					•				
4 Reciprocals		•					•			
5 Difference of roots		•						•		•
6 Forming equations			•	•		•	•	•	•	•
7 Without solving	•	•	•	•		•	•	•		
8 Unknown coefficients					•			•	•	•
9 Complex roots						•			•	
Interleaved: complex numbers (Chapter 1)						•			•	
Interleaved: algebraic fractions and surds (P1)		•		•	•		•			•
Interleaved: discriminant and quadratics (P1)					•	•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write down $\alpha+\beta$ and $\alpha\beta$ first. Rewrite every expression in terms of them before substituting, and finish a new equation with integer coefficients and "= 0".

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1. The roots of the equation $2x^2 - 5x + 4 = 0$ are α and β. Without solving the equation,

(a) write down the values of $\alpha + \beta$ and $\alpha\beta$,

(1)

(b) find the value of $\alpha^2 + \beta^2$,

(2)

(c) find the value of $\alpha^3 + \beta^3$.

(3)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The roots of the equation $x^2 + 3x - 7 = 0$ are α and β, where $\alpha > \beta$. Without solving the equation, find the exact value of

(a) $\frac{1}{\alpha} + \frac{1}{\beta}$

(2)

(b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(2)

(c) $\alpha - \beta$

(2)

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Question 2 continued

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3. Let α and β denote the two solutions of $x^2 - 4x + 2 = 0$. A second quadratic has solutions $2\alpha + 1$ and $2\beta + 1$. Do not solve either equation.

(a) Calculate $(2\alpha + 1) + (2\beta + 1)$.

(2)

(b) Calculate $(2\alpha + 1)(2\beta + 1)$.

(2)

(c) Hence write the second quadratic in the form $x^2 + px + q = 0$, stating the integers p and q.

(2)

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Question 3 continued

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4. α and β are the roots of $3x^2 + 2x + 5 = 0$

(a) Show, without finding α and β, that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{26}{15}$

(3)

(b) Deduce an equation with integer coefficients whose roots are the ratios $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

(3)

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5. The equation $x^2 - kx + 18 = 0$, where k is a constant, has one root which is twice the other.

(a) Find the possible values of k.

(4)

(b) For the positive value of k, state the roots of the equation.

(1)

(c) Find the values of k for which the equation has equal roots.

(1)

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6. The roots of the equation $x^2 + 2x + 6 = 0$ are α and β. Without solving the equation,

(a) write down the values of $\alpha + \beta$ and $\alpha\beta$,

(1)

(b) show that $\alpha^2 + \beta^2 = -8$,

(2)

(c) find the value of $\alpha^3 + \beta^3$,

(2)

(d) find a quadratic equation, with integer coefficients, whose roots are α^3 and β^3

(3)

(e) Explain how $\alpha^2 + \beta^2$ can be negative, and state the roots α and β.

(2)

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7. Two numbers α and β satisfy $2\alpha^2 - 3\alpha + 4 = 0$ and $2\beta^2 - 3\beta + 4 = 0$, with $\alpha \neq \beta$. Two new numbers are defined by

$$u = \alpha + \frac{1}{\beta} \quad \text{and} \quad v = \beta + \frac{1}{\alpha}$$

(a) State the values of $\alpha + \beta$ and $\alpha\beta$.

(1)

(b) Show that $u + v = \frac{9}{4}$

(3)

(c) Find the value of uv.

(3)

(d) Hence obtain an equation of the form $ax^2 + bx + c = 0$, with integers a, b and c, whose solutions are u and v.

(3)

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Question 7 continued

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Question 7 continued

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8. For a constant c, the numbers α and β, with $\alpha > \beta$, satisfy $x^2 - 5x + c = 0$, and the sum of their squares is 13

(a) Determine c.

(3)

(b) Write down an equation, in the form $x^2 + px + q = 0$, satisfied by both α^2 and β^2

(3)

(c) Using only $\alpha + \beta$ and $\alpha\beta$, calculate $\alpha^3 - \beta^3$.

(4)

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. The equation $x^2 - 4x + k = 0$, where k is a real constant, has non-real roots α and β. Given that $|\alpha| = 5$

(a) explain why $\alpha\beta = |\alpha|^2$, and hence find the value of k,

(3)

(b) find α and β in the form $p \pm qi$,

(2)

(c) find a quadratic equation, with integer coefficients, whose roots are α^2 and β^2,

(3)

(d) verify that the product of the roots in part (c) is equal to $\left|\alpha^2\right|^2$.

(2)

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10. The roots of the equation $x^2 - (k+2)x + 2k + 1 = 0$, where k is a real constant, are α and β.

(a) Show that $\alpha^2 + \beta^2 = k^2 + 2$

(3)

(b) Hence state the least possible value of $\alpha^2 + \beta^2$.

(1)

(c) Given that the roots differ by 2, find the exact possible values of k.

(3)

(d) Find the values of k for which the equation has equal roots.

(2)

(e) For $k = 4$, find a quadratic equation whose roots are $\alpha + \beta$ and $\alpha\beta$.

(1)

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TOTAL FOR CHAPTER 2 IS 80 MARKS

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CHAPTER 3 • KEY SKILLS

3 Numerical Solutions of Equations

Specification FP1: Numerical solution of equations • 10 questions • 80 marks • about 96 minutes

1 Change of sign

If f is continuous on $[a, b]$ and $f(a)$, $f(b)$ have opposite signs, there is a root in (a, b). State all three parts.

2 Interval bisection

Evaluate f at the midpoint; keep the half with the sign change. Each step halves the width.

3 Linear interpolation

Join $(a, f(a))$ to $(b, f(b))$:

$\alpha \approx a + \dfrac{|f(a)|}{|f(a)| + |f(b)|}(b - a)$, by similar triangles.

4 Newton–Raphson

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$: a minus sign. Show $f(x_n)$ and $f'(x_n)$ before the new value.

5 Differentiating powers

$\frac{d}{dx}x^{\frac{3}{2}} = \frac{3}{2}x^{\frac{1}{2}}$ and $\frac{d}{dx}\left(\frac{2}{x}\right) = -\frac{2}{x^2}$.

Rewrite roots and fractions as powers first.

6 Checking accuracy

To show $\alpha = 1.83$ to 2 d.p., evaluate $f(1.825)$ and $f(1.835)$, show a sign change and conclude "so $\alpha = 1.83$ to 2 d.p.".

7 When methods fail

A sign change across a discontinuity need not mean a root.

Newton–Raphson fails if $f'(x_0) = 0$ and is unreliable near a turning point.

8 Working shown

Give each iterate to the accuracy asked; with no working the method marks are lost.

9 Comparing methods

Interpolation uses the sizes of $f(a)$ and $f(b)$; bisection ignores them.

Newton–Raphson usually converges fastest.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Change of sign	•	•		•	•	•	•	•		•
2 Interval bisection	•						•			
3 Linear interpolation		•					•			•
4 Newton–Raphson			•	•		•		•	•	•
5 Differentiating powers			•	•		•		•	•	•
6 Checking accuracy					•	•				
7 When methods fail						•		•		
8 Working shown	•	•	•	•	•	•	•	•	•	•
9 Comparing methods		•					•		•	
Interleaved: differentiation of powers (P1)			•	•		•		•	•	•
Interleaved: exponentials and graphs of functions (P1, P2)							•	•		
Interleaved: tangents and quadratics (P1)						•		•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Every sign-change answer needs the two values, "sign change", "continuous" and a conclusion. In Newton–Raphson show $f(x_0)$, $f'(x_0)$ and the substitution, not just the answer.

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1.

$$f(x) = x^3 - 2x - 5$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

(2)

(b) Use interval bisection twice to find an interval of width 0.25 that contains α.

(4)

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Question 1 continued

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2.

$$f(x) = x^2 - 3x - 1$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[3, 4]$.

(2)

(b) Use linear interpolation once on the interval $[3, 4]$ to find an approximation to α.

(3)

(c) Given that $\alpha = \frac{3+\sqrt{13}}{2}$, state whether your approximation is an over-estimate or an under-estimate.

(1)

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3.

$$f(x) = x^3 + 2x - 7$$

(a) Find $f'(x)$.

(1)

(b) Taking $x_0 = 1.5$ as a first approximation to a root of $f(x) = 0$, apply the Newton–Raphson process once to find x_1, giving your answer to 3 decimal places.

(3)

(c) Apply the process a second time to find x_2, giving your answer to 3 decimal places.

(2)

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Question 3 continued

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4.

$$f(x) = x^{\frac{3}{2}} + \frac{2}{x} - 6, \qquad x > 0$$

(a) Differentiate $f(x)$ with respect to x.

(2)

(b) The curve $y = f(x)$ crosses the x-axis close to $x = 3$. Starting from 3, carry out one step of the Newton–Raphson process, stating the new estimate correct to 3 decimal places.

(3)

(c) Explain why the curve crosses the x-axis between $x = 3$ and $x = 3.1$

(1)

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Question 4 continued

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5. The equation $x^3 + x - 8 = 0$ has a single real root α.

(a) Show that α lies in the interval [1, 2].

(2)

(b) Show that $\alpha = 1.83$ correct to 2 decimal places.

(3)

(c) Explain why checking the values of $x^3 + x - 8$ at $x = 1.83$ alone is not enough.

(1)

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Question 5 continued

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6.

$$f(x) = x^3 - 4x^2 + 2$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [3.5, 4].

(2)

(b) Using $x_0 = 3.5$ as a first approximation, apply the Newton–Raphson process once to find a second approximation to α, giving your answer to 3 decimal places.

(4)

(c) Given that $\alpha = 3.87$ to 2 decimal places, show that this is correct.

(3)

(d) Explain why $x_0 = \frac{8}{3}$ would not be a suitable first approximation.

(1)

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Question 6 continued

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Question 6 continued

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7.

$$f(x) = 2^x - x - 3$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

(2)

(b) Use interval bisection twice to find an interval of width 0.25 containing α.

(3)

(c) Use linear interpolation once on the interval $[2, 3]$ to find an approximation to α, giving your answer to 3 decimal places.

(3)

(d) Given that $\alpha = 2.445$ to 3 decimal places, comment on the accuracy of each method after the work done.

(2)

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Question 7 continued

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8.

$$g(x) = x + \frac{3}{x-2}, \qquad x \neq 2$$

(a) Show that g(1) and g(3) have opposite signs.

(2)

(b) Show that the equation $g(x) = 0$ has no real roots.

(3)

(c) Explain why your answers to parts (a) and (b) do not contradict each other.

(1)

$$h(x) = x^3 - 3x + 1$$

(d) Explain why the Newton–Raphson process cannot be used with $x_0 = 1$ to find a root of $h(x) = 0$

(2)

(e) Apply the Newton–Raphson process once with $x_0 = 0.5$, giving your answer to 3 decimal places.

(2)

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9. The Newton–Raphson process is used to find $\sqrt{10}$, as a root of $f(x) = x^2 - 10 = 0$

(a) Show that the process gives the iteration formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{10}{x_n}\right)$

(3)

(b) Starting with $x_0 = 3$, find x_1 and x_2, giving your answers to 5 decimal places.

(3)

(c) Explain, with reference to the tangent to the curve $y = f(x)$ at $x = x_n$, how the formula for x_{n+1} arises.

(2)

(d) Find the error in x_2, and comment on how quickly the process converges.

(2)

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10.

$$f(x) = x^3 - kx + 1$$

where k is a constant. Given that f(2) = 1

(a) find the value of k.

(1)

(b) By considering values of f(x) at suitable integers, show that f(x) = 0 has three real roots, and give an interval of width 1 containing each.

(3)

(c) Taking $x_0 = 1.8$, apply the Newton–Raphson process twice to approximate the largest root, giving your answers to 4 decimal places.

(4)

(d) Use linear interpolation once on [0, 1] to approximate the root in that interval.

(2)

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TOTAL FOR CHAPTER 3 IS 80 MARKS

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CHAPTER 4 • KEY SKILLS

4 Coordinate Systems

Specification FP1: Coordinate systems • 10 questions • 80 marks • about 96 minutes

1 Parametric to Cartesian

Eliminate t: $x = at^2$, $y = 2at$ gives $y^2 = 4ax$; $x = ct$, $y = \frac{c}{t}$ gives $xy = c^2$

2 The parabola

$y^2 = 4ax$: focus $S(a, 0)$, directrix $x = -a$, general point $(at^2, 2at)$

3 Focus and directrix

Every point P on the parabola is as far from S as from the directrix: $PS = x_P + a$

4 Rectangular hyperbola

$xy = c^2$: general point $\left(ct, \frac{c}{t}\right)$; the axes are asymptotes.

5 Gradients

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$: $\frac{1}{t}$ on $y^2 = 4ax$ and $-\frac{1}{t^2}$ on $xy = c^2$. Differentiate; do not use t as the gradient.

6 Tangents and normals

Use $y - y_1 = m(x - x_1)$; the normal has gradient $-\frac{1}{m}$. Rearrange into the form asked for.

7 Meeting the curve again

Substitute the line into the curve (or the parameters into the line) and solve the quadratic with working; one root is already known.

8 Lengths and areas

Exact lengths by Pythagoras (surds). Triangle area $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$, or use a right angle.

9 General results

Work with parameters p, q, t throughout, and simplify fully to reach a given answer.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Parametric to Cartesian	•	•					•			
2 The parabola	•	•	•		•		•	•		•
3 Focus and directrix	•				•			•		
4 Rectangular hyperbola		•		•		•	•		•	
5 Gradients			•	•		•	•	•	•	•
6 Tangents and normals			•	•		•	•	•	•	•
7 Meeting the curve again		•				•	•			•
8 Lengths and areas				•		•	•		•	•
9 General results			•	•	•	•		•	•	•
Interleaved: differentiation and straight lines (P1)			•	•		•	•	•	•	•
Interleaved: quadratics and simultaneous equations (P1)		•			•	•	•			•
Interleaved: surds and algebraic manipulation (P1)						•		•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Sketch the curve and the lines first. Find gradients by differentiation, show the quadratic when a line meets a curve again, and give lengths exactly.

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1. The parabola C has equation $y^2 = 12x$

(a) Write down the coordinates of the focus S of C and the equation of its directrix.

(2)

(b) The point P on C has y-coordinate 6. Find the distance PS.

(2)

(c) Write down parametric equations for C in the form $x = at^2$, $y = 2at$

(1)

(d) Find the coordinates of the points on C whose distance from the directrix is 15

(1)

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Question 1 continued

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2. The curve C_1 has parametric equations $x = 5t^2$, $y = 10t$, and the curve C_2 has parametric equations $x = 5s$, $y = \frac{10}{s}$, $s \neq 0$

(a) Find a Cartesian equation for each curve.

(3)

(b) Find, using algebra, the coordinates of the point where C_1 and C_2 meet.

(3)

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Question 2 continued

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3. The point $P\left(at^2, 2at\right)$, where $t \neq 0$, lies on the parabola with equation $y^2 = 4ax$, where a is a positive constant.

(a) Show that the tangent to the parabola at P has equation $ty = x + at^2$

(4)

(b) Find the equation of the tangent to the parabola $y^2 = 12x$ at the point where $t = 2$

(2)

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Question 3 continued

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4. The point $P\left(3t, \frac{3}{t}\right)$, $t \neq 0$, lies on the rectangular hyperbola H with equation $xy = 9$

(a) Show that the tangent to H at P has equation $x + t^2y = 6t$

(4)

(b) The tangent at the point where $t = 3$ meets the x-axis at A and the y-axis at B. Find the area of triangle OAB.

(2)

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Question 4 continued

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5. The point $P\left(2t^2, 4t\right)$ lies on the parabola $y^2 = 8x$, which has focus S.

(a) Show that $PS = 2\left(t^2 + 1\right)$

(3)

(b) Given that $PS = 10$, find the possible coordinates of P.

(3)

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Question 5 continued

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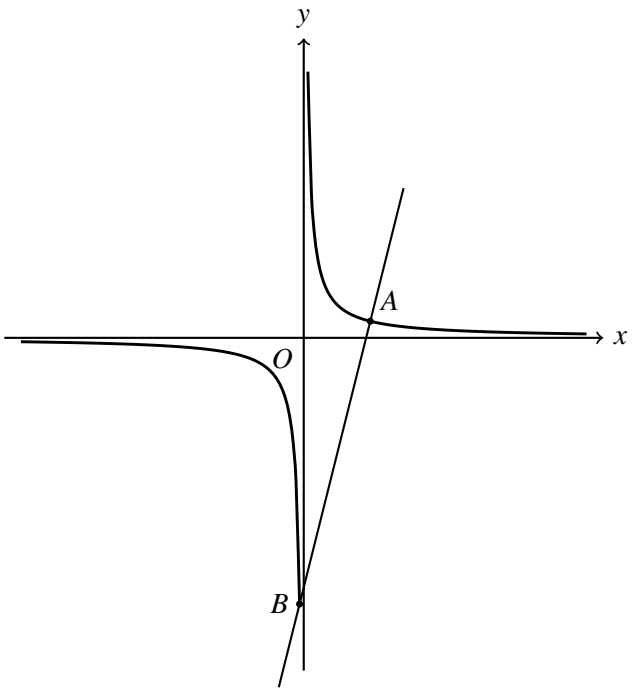

Figure 1

The rectangular hyperbola H has equation $xy = 16$. The point $P\left(4t, \frac{4}{t}\right)$, $t \neq 0$, lies on H.

(a) Show that the normal to H at P has equation $t^3x - ty = 4t^4 - 4$

(5)

The normal to H at the point A, where $t = 2$, meets H again at the point B, as shown in Figure 1.

(b) Find the coordinates of B.

(3)

(c) Find the exact length of AB, giving your answer in the form $k\sqrt{17}$

(2)

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Question 6 continued

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Question 6 continued

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7. The parabola C has equation $y^2 = 16x$ and the rectangular hyperbola H has equation $xy = 32$

(a) Using algebra, show that C and H meet at exactly one point, P, and find its coordinates.

(3)

(b) Find an equation of the normal to C at P, in the form $x + y = k$

(3)

(c) The normal meets C again at the point Q. Find the coordinates of Q.

(3)

(d) Find the exact length of PQ.

(1)

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Question 7 continued

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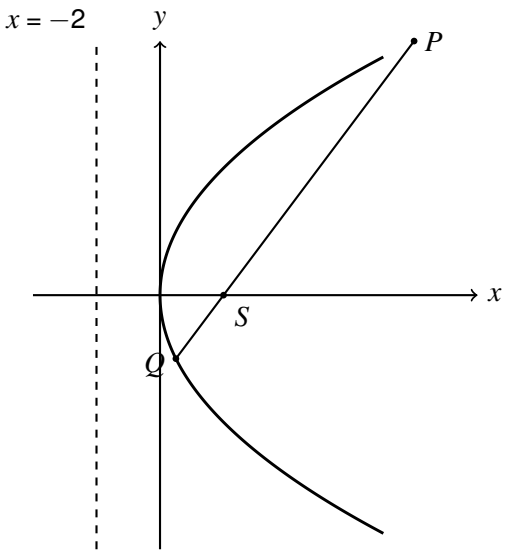

Figure 2

The parabola C has equation $y^2 = 8x$ and focus S. The points $P\left(2p^2, 4p\right)$ and $Q\left(2q^2, 4q\right)$, where $p \neq q$, lie on C, and the chord PQ passes through S, as shown in Figure 2.

(a) Show that $pq = -1$

(4)

(b) Show that the tangents to C at P and Q are perpendicular.

(2)

(c) Show that the tangents to C at P and Q meet on the directrix of C.

(4)

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Question 8 continued

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9. The points $P\left(4p, \frac{4}{p}\right)$ and $Q\left(4q, \frac{4}{q}\right)$, where p, q are non-zero and $p \neq q$, lie on the rectangular hyperbola $xy = 16$

(a) Show that the gradient of the chord PQ is $-\frac{1}{pq}$

(3)

(b) The tangents to the hyperbola at P and Q meet at R. Given that the tangent at P has equation $x + p^2y = 8p$, find the coordinates of R in terms of p and q.

(4)

(c) For $p = 1$ and $q = 3$, find the area of triangle PQR.

(3)

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Question 9 continued

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10. The point $P\left(at^2, 2at\right)$, $t \neq 0$, lies on the parabola $y^2 = 4ax$

(a) Show that the normal to the parabola at P has equation $y + tx = 2at + at^3$

(3)

(b) The normal meets the parabola again at the point with parameter s. Show that $s = -t - \frac{2}{t}$

(4)

(c) For $a = 1$ and $t = 1$, find the coordinates of the point Q where the normal meets the parabola again, and the area of triangle OPQ, where O is the origin.

(3)

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Question 10 continued

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TOTAL FOR CHAPTER 4 IS 80 MARKS

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CHAPTER 5 • KEY SKILLS

5 Matrices

Specification FP1: Matrix algebra • 10 questions • 80 marks • about 96 minutes

1 Order

An $m \times n$ matrix times an $n \times p$ matrix gives $m \times p$. The inner numbers must match, or the product is undefined.

2 Multiplication

Row by column. In general $\mathbf{AB} \neq \mathbf{BA}$, so keep the order exactly as written.

3 Identity

$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ with $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$. Write $5\mathbf{I}$, not 5, inside matrix equations.

4 Determinant

$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$, with care over minus signs. Singular means determinant 0.

5 Inverse

$\mathbf{A}^{-1} = \dfrac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$: swap the leading diagonal, change the signs of the other diagonal, divide by the determinant.

6 Matrix equations

$\mathbf{AX} = \mathbf{B}$ gives $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$; $\mathbf{XA} = \mathbf{B}$ gives $\mathbf{X} = \mathbf{BA}^{-1}$. The inverse goes on the same side as $\mathbf{A}$.

7 Products

$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ and $\det(\mathbf{AB}) = \det\mathbf{A}\det\mathbf{B}$

8 Simultaneous equations

Write as $\mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$. If $\mathbf{A}$ is singular there are no solutions or infinitely many.

9 Unknown entries

Multiply out with the unknowns, then equate corresponding entries and check every equation.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Order	•									
2 Multiplication	•		•		•	•	•	•	•	
3 Identity							•		•	
4 Determinant	•	•		•		•	•			•
5 Inverse		•	•	•		•	•	•	•	•
6 Matrix equations			•			•		•		
7 Products	•					•	•			
8 Simultaneous equations				•			•			•
9 Unknown entries		•			•	•		•	•	•
Interleaved: quadratics and simultaneous equations (P1)		•		•	•	•				•
Interleaved: algebraic proof and generalising patterns (P1, P2)							•	•	•	
Interleaved: fractions and surds (P1)		•		•					•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the determinant out with its brackets before simplifying. When solving a matrix equation, decide which side the inverse goes on before multiplying.

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1.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 3 & 4 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{pmatrix}$$

(a) Find $\mathbf{AB}$.

(3)

(b) State the order of the matrix $\mathbf{BA}$.

(1)

(c) Explain why $\mathbf{A}^2$ is not defined.

(1)

(d) Find $\det(\mathbf{AB})$.

(1)

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Question 1 continued

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2. A 2×2 matrix $\mathbf{M}$ has k and $k-5$ on its leading diagonal, 3 in the top right and 2 in the bottom left, where k is a real number.

(a) Show that $\det \mathbf{M} = (k-6)(k+1)$, and state the values of k for which $\mathbf{M}$ has no inverse.

(3)

(b) Find the inverse of $\mathbf{M}$ when $k = 2$

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$.

(2)

(b) The matrix $\mathbf{X}$ satisfies $\mathbf{XA} = \mathbf{B}$. Find $\mathbf{X}$.

(3)

(c) Explain why $\mathbf{A}^{-1}\mathbf{B}$ is not the answer to part (b).

(1)

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Question 3 continued

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4.

$$3x - 2y = 7, \qquad 5x + 4y = 19$$

(a) Write these equations as a single matrix equation.

(1)

(b) Find the inverse of the matrix of coefficients.

(2)

(c) Use your answer to part (b) to solve the equations.

(3)

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Question 4 continued

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5.

$$\mathbf{A} = \begin{pmatrix} a & 2 \\ 1 & b \end{pmatrix}, \qquad \mathbf{A}^2 = \begin{pmatrix} 11 & 8 \\ 4 & 3 \end{pmatrix}$$

(a) Find the value of a and the value of b.

(4)

(b) With $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, show that $\mathbf{AB} \neq \mathbf{BA}$

(2)

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Question 5 continued

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6.

$$\mathbf{A} = \begin{pmatrix} p & 5 \\ 2 & p+3 \end{pmatrix}, \quad \text{where } p \text{ is a constant.}$$

(a) Find the values of p for which $\mathbf{A}$ is singular.

(3)

Given that $p = 3$,

(b) find $\mathbf{A}^{-1}$,

(3)

(c) find the matrix $\mathbf{B}$ such that $\mathbf{AB} = \begin{pmatrix} -2 & 11 \\ -4 & 10 \end{pmatrix}$

(3)

(d) verify that $\det(\mathbf{AB}) = \det\mathbf{A} \times \det\mathbf{B}$.

(1)

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Question 6 continued

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Question 6 continued

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7.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

(a) Show that, for any non-singular 2×2 matrices $\mathbf{A}$ and $\mathbf{B}$, $(\mathbf{AB})\left(\mathbf{B}^{-1}\mathbf{A}^{-1}\right) = \mathbf{I}$

(2)

(b) Find $\mathbf{A}^{-1}$ and $\mathbf{B}^{-1}$.

(2)

(c) Hence find $(\mathbf{AB})^{-1}$, and check your answer by finding $\mathbf{AB}$.

(3)

(d) Solve $\mathbf{AB}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(3)

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Question 7 continued

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8.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

(a) Find $\mathbf{M}^2$ and $\mathbf{M}^3$, and suggest a form for $\mathbf{M}^n$, where n is a positive integer.

(3)

(b) Find $\mathbf{M}^{-1}$, and show that it fits your form with $n = -1$

(3)

(c) Find the matrix $\mathbf{X}$ such that $\mathbf{MX} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

(4)

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Question 8 continued

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9.

$$\mathbf{A} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$$

(a) Show that $\mathbf{A}^2 - 5\mathbf{A} + 6\mathbf{I} = \mathbf{0}$

(3)

(b) Hence show that $\mathbf{A}^{-1} = \frac{1}{6}(5\mathbf{I} - \mathbf{A})$

(3)

(c) Find $\mathbf{A}^{-1}$, and check your answer using the formula for the inverse of a 2×2 matrix.

(2)

(d) Express $\mathbf{A}^3$ in the form $p\mathbf{A} + q\mathbf{I}$, where p and q are integers.

(2)

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10. The simultaneous equations

$$kx + 2y = 4, \qquad 8x + ky = 8$$

where k is a constant, are written as $\mathbf{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$

(a) Find the values of k for which $\mathbf{M}$ is singular.

(2)

(b) For $k = 2$, use $\mathbf{M}^{-1}$ to solve the equations.

(4)

(c) For each of the values of k in part (a), determine whether the equations have no solutions or infinitely many solutions, giving a reason.

(4)

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Question 10 continued

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TOTAL FOR CHAPTER 5 IS 80 MARKS

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CHAPTER 6 • KEY SKILLS

6 Transformations Using Matrices

Specification FP1: Matrix transformations • 10 questions • 80 marks • about 96 minutes

1 Columns are images

The first column is the image of $\begin{pmatrix}1\\0\end{pmatrix}$, the second the image of $\begin{pmatrix}0\\1\end{pmatrix}$. The origin never moves.

2 Standard matrices

Rotation by θ anticlockwise about O: $\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$. Reflection in $y=x$: $\begin{pmatrix}0&1\\1&0\end{pmatrix}$; in $y=-x$: $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$

3 Full descriptions

Rotation: centre O, angle and direction. Reflection: the mirror line. Enlargement: centre O and scale factor.

4 Images

Multiply the matrix by each position vector; for a line, transform a general point $(t, mt+c)$

5 Combined transformations

$\mathbf{P}$ followed by $\mathbf{Q}$ is $\mathbf{QP}$: the first transformation is on the right.

6 Area scale factor

Image area = $|\det\mathbf{M}|\times$ original area. Do not divide by it or square it. A negative determinant includes a reflection.

7 Inverse transformation

$\mathbf{M}^{-1}$ maps images back to the original points.

8 Finding a matrix

From images of points, or from $\mathbf{CA}=\mathbf{B}$, giving $\mathbf{C}=\mathbf{BA}^{-1}$

9 Lines of points

Points with $\mathbf{M}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$ do not move: for a reflection they form the mirror line.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Columns are images	•							•		
2 Standard matrices	•	•		•		•	•		•	
3 Full descriptions		•		•			•	•	•	
4 Images	•		•		•	•				•
5 Combined transformations				•		•	•		•	
6 Area scale factor			•		•	•	•	•	•	•
7 Inverse transformation					•		•			•
8 Finding a matrix						•	•	•		
9 Lines of points					•			•		•
Interleaved: matrix algebra and inverses (Chapter 5)			•	•	•	•	•	•	•	•
Interleaved: exact trigonometric values (P1)	•	•						•	•	
Interleaved: coordinate geometry and areas (P1)			•		•	•		•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Check a matrix by finding the images of $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ on a quick sketch before describing it. Write the first transformation on the right, and use the determinant for areas.

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1. Write down the 2×2 matrix that represents

(a) a rotation through 90° anticlockwise about the origin,

(1)

(b) a reflection in the line $y = -x$,

(1)

(c) an enlargement with centre O and scale factor 3,

(1)

(d) a stretch parallel to the x-axis with scale factor 4,

(1)

(e) a rotation through 45° anticlockwise about the origin, giving the entries exactly.

(1)

(f) Find the image of the point (2, 5) under the transformation in part (b).

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Describe fully the single geometrical transformation represented by each of these matrices.

(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(2)

(b) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(1)

(c) $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$

(2)

(d) $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$

(1)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3.

$$\mathbf{M} = \begin{pmatrix} 3 & -5 \\ 2 & 4 \end{pmatrix}$$

(a) Find $\det \mathbf{M}$.

(1)

(b) A region R of area 6 is transformed by $\mathbf{M}$. Find the area of its image.

(1)

(c) The triangle with vertices $(0,0)$, $(2,0)$ and $(0,3)$ is transformed by $\mathbf{M}$. Find the coordinates of the images of the vertices.

(2)

(d) Use your answer to part (c) to verify your answer to part (a).

(2)

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Question 3 continued

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4. The transformation P is a reflection in the x-axis. The transformation Q is a rotation through 90° anticlockwise about the origin.

(a) Find the matrix representing the transformation P followed by Q.

(4)

(b) Describe fully this combined transformation as a single transformation.

(2)

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Question 4 continued

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5.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$$

(a) The point A is mapped by $\mathbf{M}$ to the point (5, 8). Find the coordinates of A.

(3)

(b) Explain why $\mathbf{M}$ does not change the area of any shape.

(1)

(c) Find the equation of the image of the line $y = x$ under $\mathbf{M}$.

(2)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6.

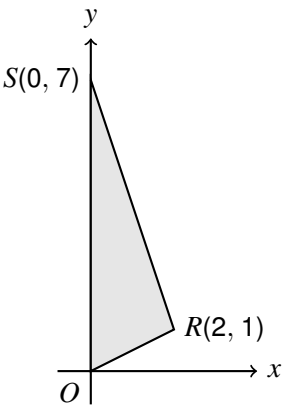

Figure 1

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ -1 & 3 \end{pmatrix}, \quad \text{where } p \text{ is a constant.}$$

The triangle ORS has vertices $O(0, 0)$, $R(2, 1)$ and $S(0, 7)$, as shown in Figure 1. The transformation represented by $\mathbf{A}$ maps R to the point $R'(12, 1)$.

(a) Find the value of p.

(2)

(b) Find the area of triangle ORS.

(2)

(c) Hence, using your answer to part (b), find the area of the image of triangle ORS under $\mathbf{A}$.

(2)

(d) Find the coordinates of the image of S.

(1)

(e) Find the matrix representing the transformation $\mathbf{A}$ followed by a rotation through 90° anticlockwise about the origin.

(3)

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Question 6 continued

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7.

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) A shape of area 4 is transformed by $\mathbf{A}$. Find the area of its image.

(2)

(b) Describe fully the single geometrical transformation represented by $\mathbf{B}$.

(2)

(c) Find the matrix $\mathbf{C}$ such that $\mathbf{CA} = \mathbf{B}$

(4)

(d) Describe, in terms of transformations, what the equation $\mathbf{CA} = \mathbf{B}$ means.

(2)

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Question 7 continued

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8. A transformation T maps the point (1, 0) to (0.6, 0.8) and the point (0, 1) to (0.8, -0.6). T is represented by the matrix $\mathbf{M}$.

(a) Write down $\mathbf{M}$.

(1)

(b) Show that $\mathbf{M}^2 = \mathbf{I}$, and explain what this means geometrically.

(3)

(c) Find $\det \mathbf{M}$, and explain what its value tells you about T.

(2)

(d) T is a reflection. By finding the points that are not moved by T, find the equation of its mirror line.

(4)

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9. The matrix $\mathbf{R}$ represents a rotation through 60° anticlockwise about the origin, and $\mathbf{M} = 2\mathbf{R}$

(a) Write down $\mathbf{R}$, giving the entries exactly.

(1)

(b) Show that $\mathbf{M} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$, and describe the transformation $\mathbf{M}$ as a combination of two transformations.

(3)

(c) Show that $\mathbf{M}^3 = -8\mathbf{I}$, and describe the transformation $\mathbf{M}^3$ geometrically.

(3)

(d) Write down the area scale factor of $\mathbf{M}^3$.

(1)

(e) Find the smallest positive integer n for which $\mathbf{M}^n = k\mathbf{I}$ with $k > 0$, and state the value of k.

(2)

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10.

$$\mathbf{T} = \begin{pmatrix} k & 2 \\ 1 & k \end{pmatrix}, \quad \text{where } k \text{ is a real constant.}$$

(a) Find the values of k for which $\mathbf{T}$ maps the whole plane onto a straight line.

(2)

(b) For the positive value of k found in part (a), find the equation of that line.

(3)

Given instead that $k = 3$

(c) find the area of the image of the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 2)$,

(2)

(d) find the point whose image under $\mathbf{T}$ is $(8, 5)$.

(3)

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 6 IS 80 MARKS

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CHAPTER 7 • KEY SKILLS

7 Series

Specification FP1: Series • 10 questions • 80 marks • about 96 minutes

1 Standard results

$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$,

$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$,

$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

2 Constants

$\sum_{r=1}^{n} 5 = 5n$, not 5. Constant multiples come outside: $\sum 3r^2 = 3\sum r^2$

3 Splitting sums

The sum of a sum is the sum of the separate sums. Expand any brackets in r first.

4 Factorising

Take out common factors such as $\frac{1}{6}n(n+1)$ straight away; it makes the remaining bracket simple.

5 Between limits

$\sum_{r=a}^{b} = \mathrm{f}(b) - \mathrm{f}(a-1)$: subtract the sum up to $a-1$, not up to a.

6 From $n+1$ to $2n$

$\mathrm{f}(2n) - \mathrm{f}(n)$. Substitute $2n$ into your formula rather than starting again.

7 Solving for n

Set the formula equal to the value and solve; n must be a positive integer.

8 Given answers

Show the substitution, the common factor and the expanded bracket before the final form.

9 Patterns

Results such as $\sum r^3 = (\sum r)^2$ and products of consecutive integers link the formulae together.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Standard results	●	●	●	●	●	●	●	●	●	●
2 Constants	●				●		●			
3 Splitting sums	●	●		●	●	●	●		●	●
4 Factorising	●	●		●	●	●	●		●	●
5 Between limits			●			●			●	
6 From $n+1$ to $2n$					●		●			
7 Solving for n	●					●		●		●
8 Given answers	●	●		●	●	●	●	●	●	●
9 Patterns								●		●
Interleaved: algebraic manipulation and factorising (P1)	●	●		●	●	●	●		●	●
Interleaved: quadratic and polynomial equations (P1, P2)	●					●		●	●	●
Interleaved: arithmetic series (P2)	●							●		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Expand, split, substitute the standard results, then factor out the common fraction and bracket before simplifying. For sums between limits, write $\mathrm{f}(b) - \mathrm{f}(a-1)$ before evaluating.

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1.

(a) Show that $\sum_{r=1}^{n}(4r-1) = n(2n+1)$

(3)

(b) Hence find the value of $\sum_{r=1}^{20}(4r-1)$

(1)

(c) Find the value of n for which $\sum_{r=1}^{n}(4r-1) = 465$

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2.

(a) Use the standard results for $\sum r$ and $\sum r^2$ to show that $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$

(4)

(b) Hence evaluate $\sum_{r=1}^{10} r(r+1)$

(2)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. Using the standard results, find the exact value of

(a) $\sum_{r=10}^{30} r^2$

(3)

(b) $\sum_{r=10}^{30} r^3$

(3)

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Question 3 continued

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4.

(a) Show that $\sum_{r=1}^{n}\left(r^3 - r\right) = \frac{1}{4}(n-1)n(n+1)(n+2)$

(4)

(b) Hence find the value of $\sum_{r=1}^{5}\left(r^3 - r\right)$, and check your answer by direct calculation.

(2)

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Question 4 continued

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5.

(a) Show that $\sum_{r=1}^{n}\left(3r^2-r+2\right)=n\left(n^2+n+2\right)$

(4)

(b) Hence show that $\sum_{r=n+1}^{2n}\left(3r^2-r+2\right)=n\left(7n^2+3n+2\right)$

(2)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6.

(a) Show that $\sum_{r=1}^{n} r^2(2r-1) = \frac{1}{6}n(n+1)\left(3n^2+n-1\right)$

(5)

(b) Hence find the value of $\sum_{r=11}^{20} r^2(2r-1)$

(3)

(c) Find the smallest value of n for which $\sum_{r=1}^{n} r^2(2r-1) > 10000$

(2)

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Question 6 continued

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Question 6 continued

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7.

(a) Show that $\sum_{r=1}^{n}(r+1)(r+4) = \frac{1}{3}n(n+4)(n+5)$

(4)

(b) Hence show that $\sum_{r=n+1}^{2n}(r+1)(r+4) = \frac{1}{3}n(n+1)(7n+20)$

(4)

(c) Hence find the value of $\sum_{r=11}^{20}(r+1)(r+4)$

(2)

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Question 7 continued

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8.

(a) Use the standard results to show that $\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$

(2)

(b) Find the value of n for which $\sum_{r=1}^{n} r = 210$

(3)

(c) Hence write down the value of $\sum_{r=1}^{n} r^3$ for this value of n.

(1)

(d) By considering $\sum_{r=1}^{40} r^3$, find the value of $\sum_{r=1}^{20} (2r-1)^3$

(4)

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Question 8 continued

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9.

(a) Show that $\sum_{r=1}^{n}\left(r^2+kr\right)=\frac{1}{6}n(n+1)(2n+1+3k)$, where k is a constant.

(4)

(b) Given that $\sum_{r=1}^{10}\left(r^2+kr\right)=660$, find the value of k.

(3)

(c) For this value of k, find the value of $\sum_{r=11}^{20}\left(r^2+kr\right)$

(3)

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10. **(a)** Show that $\sum_{r=1}^{n} r(r+1)(r+2) = \frac{1}{4}n(n+1)(n+2)(n+3)$

(5)

(b) Find the value of n for which $\sum_{r=1}^{n} r(r+1)(r+2) = 756$, explaining why there is only one value.

(2)

(c) Using this result and the result $\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$, suggest a formula for $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$, and check it for $n = 1$ and $n = 2$

(3)

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 7 IS 80 MARKS

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CHAPTER 8 • KEY SKILLS

8 Proof

Specification FP1: Proof by induction • 10 questions • 80 marks • about 96 minutes

1 Four parts

Basis ($n = 1$), assumption ($n = k$), inductive step ($n = k+1$), conclusion. Every part earns marks.

2 The basis case

Show both sides for $n = 1$ with working, e.g. "LHS = 1, RHS = $\frac{1}{3}(1)(1)(3) = 1$".

3 Sums

Add the $(k+1)$th term to the assumed sum, then factorise towards the formula with $k+1$ in place of n.

4 Divisibility

Write f$(k+1)$ in terms of f(k), e.g. f$(k+1)$ = f$(k)+5\times 6^k$, so it is a sum of multiples.

5 Matrix powers

$\mathbf{M}^{k+1} = \mathbf{M}^k\mathbf{M}$ (or $\mathbf{M}\mathbf{M}^k$). Multiply out and simplify each entry to the $n = k+1$ form.

6 Recurrence relations

Substitute the assumed u_k into $u_{k+1} = \ldots$ and simplify to the formula with $n = k+1$.

7 Index laws

$6^{k+1} = 6\times 6^k$, $5^{2(k+1)} = 25\times 5^{2k}$, $2^{k+3} = 2\times 2^{k+2}$

8 The conclusion

"If true for $n = k$ then true for $n = k+1$; true for $n = 1$; so true for all positive integers n."

9 Using the result

After the proof, substitute into the formula: do not multiply matrices out again.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Four parts	•	•	•	•	•	•	•	•	•	•
2 The basis case	•	•	•	•	•	•	•	•	•	•
3 Sums	•				•			•		
4 Divisibility		•					•			
5 Matrix powers			•			•				•
6 Recurrence relations				•					•	
7 Index laws		•	•	•		•	•	•	•	
8 The conclusion	•	•	•	•	•	•	•	•	•	•
9 Using the result						•	•	•	•	•
Interleaved: series (Chapter 7)	•				•			•		
Interleaved: matrices and transformations (Chapters 5, 6)			•			•				•
Interleaved: indices, logarithms and sequences (P1, P2)		•		•			•	•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Show the $n = 1$ case on both sides, state the assumption for $n = k$, show the algebra to the $n = k+1$ form, and finish with the full conclusion every time.

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1. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

(6)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Use mathematical induction to show that $6^n + 4$ is a multiple of 5 for every positive integer n.

(6)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3.

$$\mathbf{M} = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$$

Prove by induction that, for $n \in \mathbb{Z}^+$,

$$\mathbf{M}^n = \begin{pmatrix} 3^n & 0 \\ 3^n - 1 & 1 \end{pmatrix}$$

(6)

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Question 3 continued

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(Total for Question 3 is 6 marks)

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4. A sequence of numbers is defined by

$$u_1 = 2, \qquad u_{n+1} = 3u_n - 2, \quad n \geqslant 1$$

Prove by induction that, for $n \in \mathbb{Z}^+$, $u_n = 3^{n-1} + 1$

(6)

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Question 4 continued

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(Total for Question 4 is 6 marks)

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5. A student claims that, for all positive integers n,

$$1+2+3+\cdots+n=\frac{1}{2}\left(n^2+n+2\right)$$

(a) Show that, if the claim is true for $n=k$, then it is true for $n=k+1$

(3)

(b) Show that the claim is false.

(2)

(c) Explain why part (a) does not prove the claim.

(1)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6.

$$\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$$

(a) Prove by induction that, for $n \in \mathbb{Z}^+$, $\mathbf{M}^n = \begin{pmatrix} 2^n & 1-2^n \\ 0 & 1 \end{pmatrix}$

(5)

(b) Hence write down $\mathbf{N} = \mathbf{M}^5$

(1)

The matrix $\mathbf{B} = \mathbf{NT}$, where $\mathbf{T} = \begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$. A region R is transformed by $\mathbf{B}$ onto a region of area 480

(c) Find the area of R.

(3)

(d) Show that the formula in part (a) also gives the correct matrix when $n = 0$

(1)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7.

$$f(n) = 5^{2n} - 1$$

(a) Show that $f(k+1) = 25f(k) + 24$, and hence prove by induction that $f(n)$ is divisible by 24 for all $n \in \mathbb{Z}^+$

(6)

(b) Hence find the remainder when 5^{2n+1} is divided by 24

(2)

(c) Determine whether $5^n - 1$ is divisible by 24 for all $n \in \mathbb{Z}^+$

(2)

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Question 7 continued

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Question 7 continued

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8.

(a) Prove by induction that, for $n \in \mathbb{Z}^+$, $\sum_{r=1}^{n} r \times 2^r = 2 + (n-1)2^{n+1}$

(6)

(b) Hence find the value of $\sum_{r=1}^{10} r \times 2^r$

(2)

(c) Hence find the value of $\sum_{r=11}^{20} r \times 2^r$

(2)

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Question 8 continued

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9. A sequence is defined by

$$u_1 = 5, \qquad u_{n+1} = 2u_n + 3, \quad n \geqslant 1$$

(a) Find u_2 and u_3.

(2)

(b) Prove by induction that, for $n \in \mathbb{Z}^+$, $u_n = 2^{n+2} - 3$

(5)

(c) Find the smallest value of n for which $u_n > 1000$

(3)

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Question 9 continued

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10.

$$\mathbf{M} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$$

(a) Prove by induction that, for $n \in \mathbb{Z}^+$, $\mathbf{M}^n = \begin{pmatrix} 2n+1 & -2n \\ 2n & 1-2n \end{pmatrix}$

(5)

(b) Show that $\det(\mathbf{M}^n) = 1$ for all n, and state what this means for the areas of shapes transformed by $\mathbf{M}^n$

(2)

(c) Show that the point (1, 1) is mapped to itself by $\mathbf{M}^n$ for every positive integer n, and find the image of the point (1, 0) under $\mathbf{M}^{10}$

(3)

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TOTAL FOR CHAPTER 8 IS 80 MARKS

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