eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2

Statistics 2

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WST02/01) • Calculator and Statistical Tables allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)

		Qs	Marks	Page	My score
1	**Binomial Distributions** Conditions, probabilities and tables, mean and variance, unknown n and p	10	80	3	
2	**Poisson Distributions** Probabilities, modelling conditions, scaling the rate, adding Poisson variables	10	80	10	
3	**Approximations** Poisson and normal approximations, continuity corrections	10	80	17	
4	**Continuous Random Variables** Probability density and cumulative distribution functions, mean, variance, median, mode	10	81	22	
5	**Continuous Uniform Distribution** Rectangular distribution, mean and variance, modelling	10	80	29	
6	**Sampling and Sampling Distributions** Populations and samples, statistics, sampling distributions	10	80	36	
7	**Hypothesis Testing** Critical regions, significance levels, binomial and Poisson tests	10	80	44	

Question booklet Free courses More worksheets

How to use this mark scheme

The same marking rules Pearson examiners use for WST02.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

Probabilities, tables and conclusions. Probabilities read from the statistical tables are given to 4 decimal places; other answers to 3 significant figures (awrt means “answers which round to”). A distribution must be stated with its parameters, e.g. B(20, 0.3), not just “binomial”. A hypothesis test conclusion must refer to the context and must not claim certainty.


★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

1 Chapter 1: Binomial Distributions

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$X \sim \text{B}(12, 0.3)$ *Oct 2024 report: "binomial" on its own scores B0; both parameters are needed.*	B1 (1)
(b)	$\binom{12}{4}(0.3)^4(0.7)^8$ 0.2311 (awrt 0.231)	M1 A1 (2)
(c)	$1 - \text{P}(X \leqslant 4) = 1 - 0.7237$ 0.2763 *Jan 2014 report:* $1 - \text{P}(X \leqslant 5)$ *is the common error.*	M1 A1 (2)
(d)	$\text{P}(X \leqslant 5) - \text{P}(X \leqslant 1) = 0.8822 - 0.0850 = 0.7972$	B1 (1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$X \sim \text{B}(20, 0.9)$; failures $Y = 20 - X \sim \text{B}(20, 0.1)$ $\text{P}(X \geqslant 18) = \text{P}(Y \leqslant 2)$ 0.6769 *Oct 2019 report: using* $\text{B}(20, 0.1)$ *with the original inequality is the common error.*	M1 M1 A1 (3)
(b)	$0.9^{20} = 0.1216$	B1 (1)
(c)	$\text{P}(X \leqslant 16) = \text{P}(Y \geqslant 4) = 1 - \text{P}(Y \leqslant 3)$ $1 - 0.8670 = 0.1330$	M1 A1 (2)
		Total for Question 2: 6 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3

6 marks

Part	Scheme	Marks
(a)	$np = 6$ and $np(1-p) = 4.2$	M1
	$1-p = \frac{4.2}{6} = 0.7$, $\boxed{p = 0.3}$	A1
	$\boxed{n = 20}$	A1
		(3)
(b)	$P(X \leqslant 3) = \boxed{0.1071}$	B1
		(1)
(c)	$6 + \sqrt{4.2} = 8.05$, so $P(X \geqslant 9) = 1 - P(X \leqslant 8)$	M1
	$1 - 0.8867 = \boxed{0.1133}$	A1
		(2)
	Total for Question 3: 6 marks	

Question 4

6 marks

Part	Scheme	Marks
(a)	$\boxed{0.6331}$	B1
		(1)
(b)	$P(X \leqslant 3) - P(X \leqslant 2) = 0.3823 - 0.1673$ (or the formula)	M1
	$\boxed{0.2150}$	A1
		(2)
(c)	$\frac{P(X = 3 \cap X \leqslant 4)}{P(X \leqslant 4)}$	M1
	$= \frac{P(X = 3)}{P(X \leqslant 4)} = \frac{0.2150}{0.6331}$	M1
	$\boxed{0.340}$	A1
	Oct 2019 report: the conditional probability was often not identified, and $P(X = 3)$ alone scores no marks.	
		(3)
	Total for Question 4: 6 marks	

Question 5

6 marks

Part	Scheme	Marks
(a)	P(at least one) = $1 - 0.95^n$	M1
	$1 - 0.95^n > 0.9 \Rightarrow 0.95^n < 0.1$ *	A1*
		(2)
(b)	$n \ln 0.95 < \ln 0.1$	M1
	$n > \dfrac{\ln 0.1}{\ln 0.95} = 44.9$ (the inequality reverses: $\ln 0.95 < 0$)	A1
	$\boxed{n = 45}$	A1
		(3)
(c)	Components are defective independently, with the same probability 0.05 each	B1
		(1)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$H \sim \text{B}(5, 0.4)$: $\text{P}(H \geqslant 3) = 1 - \text{P}(H \leqslant 2)$	M1
	$= 1 - 0.6826$	M1
	$= 0.3174$ *	A1*
		(3)
(b)	$W \sim \text{B}(8, 0.3174)$	B1
	$\binom{8}{3}(0.3174)^3(0.6826)^5$	M1
	$\boxed{0.265}$	A1
	Jan 2014 report: the second binomial must use the probability from part (a); 0.3174 is not in the tables.	
		(3)
(c)	$(0.6826)^3(0.3174)$	M1
	$\boxed{0.101}$	A1
		(2)
(d)	Mean $8 \times 0.3174 = \boxed{2.54}$	B1
	Variance $8 \times 0.3174 \times 0.6826 = \boxed{1.73}$	B1
		(2)
		Total for Question 6: 10 marks

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7

10 marks

Part	Scheme	Marks
(a)	$0.6 \times 0.05 + 0.4 \times 0.175$ $0.03 + 0.07 = 0.1$ *	M1 A1* (2)
(b)	$X \sim \text{B}(20, 0.1)$: $1 - \text{P}(X \leqslant 3)$ $1 - 0.8670 =$ $\boxed{0.1330}$	M1 A1 (2)
(c)	$\frac{0.03}{0.1}$ $\boxed{0.3}$	M1 A1 (2)
(d)	Mean $50 \times 0.1 =$ $\boxed{5}$ Standard deviation $\sqrt{50 \times 0.1 \times 0.9} =$ $\boxed{2.12}$	B1 B1 (2)
(e)	$D \sim \text{B}(50, 0.1)$: $\text{P}(D \leqslant 5)$ $\boxed{0.6161}$	M1 A1 (2)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$\binom{7}{2}p^2(1-p)^5 = 2\binom{7}{1}p(1-p)^6$	M1
	$21p^2(1-p)^5 = 14p(1-p)^6$	A1
	Divides by $7p(1-p)^5$ (non-zero): $3p = 2(1-p)$	M1
	$p = 0.4$ *	A1*
		(4)
(b)	$1 - \text{P}(X \leqslant 3) = 1 - 0.7102$	M1
	$\boxed{0.2898}$	A1
		(2)
(c)	$\dfrac{\text{P}(X = 5)}{\text{P}(X \geqslant 4)} = \dfrac{0.07741}{0.2898}$	M1
	$\boxed{0.267}$	A1
		(2)
(d)	$\text{E}(X) = \boxed{2.8}$	B1
	$\text{Var}(X) = 7 \times 0.4 \times 0.6 = \boxed{1.68}$	B1
		(2)
	Total for Question 8: 10 marks	

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9

10 marks

Part	Scheme	Marks
(a)	$X \sim \text{B}(5, 0.3)$: $1 - \text{P}(X \leqslant 2) = 1 - 0.8369$ $\boxed{0.1631}$	**M1** **A1** **(2)**
(b)	$0.1631k = 0.5$ $\boxed{k = 3.07}$	**M1** **A1** **(2)**
(c)	Expected profit per game $0.50 - 3 \times 0.1631$ $= 0.0107$ $\times 200 = \boxed{£2.14}$ *Oct 2024 report: write the expectation statement, not only the arithmetic.*	**M1** **A1** **A1** **(3)**
(d)	Takings £5; loss if $3W > 5$, i.e. $W \geqslant 2$, where $W \sim \text{B}(10, 0.1631)$ $1 - \left(0.8369^{10} + 10(0.1631)(0.8369)^{9}\right) = \boxed{0.503}$ A loss is (slightly) more likely than not over 10 games, even though the expected profit is positive: the profit only shows over many games	**M1** **A1** **B1** **(3)**
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$np = 6$, $np(1-p) = 3.6$	**M1**
	$\boxed{p = 0.4}$	**A1**
	$\boxed{n = 15}$	**A1**
		(3)
(b)	$\dfrac{\binom{15}{r+1}p^{r+1}q^{14-r}}{\binom{15}{r}p^{r}q^{15-r}} = \dfrac{15-r}{r+1}\cdot\dfrac{p}{q}$	**M1**
	$\dfrac{p}{q} = \dfrac{2}{3}$, giving $\dfrac{2(15-r)}{3(r+1)}$ *	**A1***
	Ratio > 1 when $30-2r > 3r+3$, i.e. $r < 5.4$	**M1**
	Probabilities increase up to $r = 6$ and decrease after, so the mode is 6 *	**A1***
		(4)
(c)	$\binom{15}{6}(0.4)^6(0.6)^9 = \boxed{0.2066}$	**B1**
		(1)
(d)	$P(4 \leqslant Y \leqslant 8) = P(Y \leqslant 8) - P(Y \leqslant 3) = 0.9050 - 0.0905$	**M1**
	$\boxed{0.8145}$	**A1**
		(2)
		Total for Question 10: 10 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2 Chapter 2: Poisson Distributions

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$X \sim \text{Po}(4)$ *Jan 2023 scheme: Po or Poisson with 4; "P(4)" is not accepted.*	B1 (1)
(b)	e.g. pairs of *shoes* are sold singly / independently / randomly / at a constant rate *Oct 2024 report: "constant probability" is not a Poisson condition; the context is required.*	B1 (1)
(c)	$\dfrac{e^{-4}4^6}{6!}$ 0.1042	M1 A1 (2)
(d)	$1 - \text{P}(X \leqslant 5) = 1 - 0.7851$ 0.2149	M1 A1 (2)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$Y \sim \text{Po}(3.5)$: $\text{P}(Y \leqslant 3) - \text{P}(Y \leqslant 2) = 0.5366 - 0.3208$ 0.2158	M1 A1 (2)
(b)	Po(7): $1 - \text{P}(\leqslant 9) = 1 - 0.8305$ 0.1695	M1 A1 (2)
(c)	Po(0.7): $e^{-0.7}$ 0.4966 *Oct 2024 report: the rate "7 every 10 minutes" must be scaled to the interval asked about.*	M1 A1 (2)
		Total for Question 2: 6 marks

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3

6 marks

Part	Scheme	Marks
(a)	$\sum fx = 152$, mean = $\boxed{1.52}$	B1
	$\sum fx^2 = 380$; variance = $\frac{380}{100} - 1.52^2$	M1
	$\boxed{1.49}$ (1.50 with divisor 99)	A1
		(3)
(b)	The mean and variance are close, as they would be equal for a Poisson distribution	B1
		(1)
(c)	$1 - \mathrm{P}(X \leqslant 2) = 1 - 0.8088 = 0.1912$	M1
	Observed proportion $\frac{20}{100} = 0.2$, close to 0.1912, so the model fits well	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$T \sim \mathrm{Po}(2.5 + 1.5) = \mathrm{Po}(4)$; $\mathrm{P}(T = 4)$	M1
	$\boxed{0.1954}$	A1
		(2)
(b)	Po(8): $\mathrm{P}(\leqslant 5)$	M1
	$\boxed{0.1912}$	A1
		(2)
(c)	$\frac{\mathrm{e}^{-2.5}2.5^2}{2!} \times \frac{\mathrm{e}^{-1.5}1.5^2}{2!}$ (independence)	M1
	$\boxed{0.0644}$	A1
		(2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	P(no faults in t hours) = $e^{-0.2t} \geqslant 0.9$	M1
	$t \leqslant -\frac{\ln 0.9}{0.2} = 0.527$ hours	M1
	31.6 minutes	A1
		(3)
(b)	$e^{-\lambda} = 0.0498$, $\lambda = -\ln 0.0498$	M1
	$\lambda = 3$	A1
		(2)
(c)	$\frac{e^{-3}3^2}{2} =$ 0.2240	B1
		(1)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$X \sim$ Po(2.5): $1 - P(X \leqslant 4) = 1 - 0.8912$	M1
	$= 0.1088$ *	A1*
		(2)
(b)	$B \sim$ B(10, 0.1088)	B1
	$P(B \geqslant 2) = 1 - P(B = 0) - P(B = 1)$	M1
	$1 - 0.8912^{10} - 10(0.1088)(0.8912)^9$	M1
	0.298	A1
	Oct 2024 report: the binomial uses the Poisson probability as its p; it is not in the tables.	
		(4)
(c)	Po(10): $1 - P(\leqslant 11) = 1 - 0.6968$	M1
	0.3032	A1
		(2)
(d)	e.g. cars pass the checkpoint singly / independently / at a constant rate	B1
	Whether a minute is busy is independent of other minutes (with the same probability each minute)	B1
		(2)
		Total for Question 6: 10 marks

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7

10 marks

Part	Scheme	Marks
(a)	$1 - \mathrm{P}(X \leqslant 3) = 1 - 0.6472$ $\boxed{0.3528}$	**M1** **A1** **(2)**
(b)	$\dfrac{\mathrm{P}(X = 2)}{\mathrm{P}(X \geqslant 1)}$ $= \dfrac{0.2240}{1 - 0.0498}$ $\boxed{0.236}$	**M1** **M1** **A1** **(3)**
(c)	Po(6): $\mathrm{P}(\leqslant 6)$ $\boxed{0.6063}$	**M1** **A1** **(2)**
(d)	$\mathrm{P}(X = 3) = \dfrac{\mathrm{e}^{-3}3^3}{3!} = \dfrac{\mathrm{e}^{-3}3^2}{2!} = \mathrm{P}(X = 2)$ $\mathrm{P}(X = r+1) = \frac{3}{r+1}\mathrm{P}(X = r)$ increases for $r < 2$ and decreases for $r > 2$, so 2 and 3 are both modes	**M1** **A1** **(2)**
(e)	e.g. goals may not occur independently or at a constant rate (a team changes tactics after scoring)	**B1** **(1)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	Po(1): $e^{-1} = \boxed{0.3679}$	**B1** **(1)**
(b)	Po(5): $1 - P(\leqslant 2) = 1 - 0.1247$ $\boxed{0.8753}$	**M1** **A1** **(2)**
(c)	Rate 0.1 per metre: $e^{-0.1L} = 0.5$ $L = 10\ln 2$ $\boxed{6.93\,\text{m}}$	**M1** **M1** **A1** **(3)**
(d)	Each 5 m half: Po(0.5), independent; whole 10 m: Po(1) $\dfrac{P(2 \text{ in first half})\,P(0 \text{ in second half})}{P(2 \text{ in } 10\,\text{m})}$ $= \dfrac{e^{-0.5}\frac{0.5^2}{2} \times e^{-0.5}}{e^{-1}\frac{1^2}{2}}$ $\boxed{0.25}$	**B1** **M1** **M1** **A1** **(4)**
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{e^{-\lambda}\lambda^2}{2} = 3 \times \frac{e^{-\lambda}\lambda^4}{24}$	M1
	$\lambda^2 = 4$	A1
	$\lambda > 0$, so $\lambda = 2$ *	A1*
		(3)
(b)	$1 - P(X \leqslant 3) = 1 - 0.8571$	M1
	$\boxed{0.1429}$	A1
		(2)
(c)	$X + Y \sim Po(2+\mu)$	B1
	$e^{-(2+\mu)} = e^{-5}$	M1
	$\boxed{\mu = 3}$	A1
		(3)
(d)	Po(5): P($\leqslant 3$)	M1
	$\boxed{0.2650}$	A1
		(2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$P(X \geqslant 6) = 1 - P(X \leqslant 5) = 1 - 0.8576$ $\boxed{0.1424}$	**M1** **A1** **(2)**
(b)	Boats hired = $\min(X, 5)$: $\sum_{x=0}^{4} x\,P(X = x) + 5\,P(X \geqslant 5)$ $P(X = 1)$ to $P(X = 4)$: 0.1057, 0.1850, 0.2158, 0.1888; $P(X \geqslant 5) = 1 - 0.7254 = 0.2746$ $0.1057 + 0.3700 + 0.6474 + 0.7552 + 1.3730$ $\boxed{3.25}$ *The expectation is less than 3.5 because demand above 5 cannot be met.*	**M1** **M1** **M1** **A1** **(4)**
(c)	$P(X \leqslant 6) = 0.9347 < 0.95$, $P(X \leqslant 7) = 0.9733 > 0.95$ $\boxed{7 \text{ boats}}$	**M1** **A1** **(2)**
(d)	$\dfrac{P(X \geqslant 6)}{P(X \geqslant 4)} = \dfrac{0.1424}{1 - 0.5366}$ $\boxed{0.307}$	**M1** **A1** **(2)**
		Total for Question 10: 10 marks

3 Chapter 3: Approximations

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$X \sim \text{B}(150, 0.02)$	B1
	$\approx \text{Po}(3)$; $\text{P}(X \leqslant 2)$	M1
	$\boxed{0.4232}$	A1
		(3)
(b)	$n = 150$ is large and $p = 0.02$ is small (close to 0)	B1
	Summer 2023 scheme: "p is close to 0" or $p < 0.1$; $np < 10$ is condoned.	(1)
(c)	$\dfrac{\text{e}^{-3}3^4}{4!}$	M1
	$\boxed{0.1680}$	A1
		(2)
	Total for Question 1: 6 marks	

Question 2

6 marks

Part	Scheme	Marks
(a)	$X \sim \text{B}(150, 0.4) \approx \text{N}(60, 36)$	B1
	$\text{P}(Y > 69.5)$ (continuity correction)	M1
	$z = \frac{69.5-60}{6} = 1.58$	M1
	$1 - 0.9429 = \boxed{0.0571}$	A1
		(4)
(b)	$n = 150$ is large and $p = 0.4$ is close to 0.5	B1
		(1)
(c)	X is discrete (whole numbers) but the normal distribution is continuous	B1
		(1)
	Total for Question 2: 6 marks	

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3

6 marks

Part	Scheme	Marks
(a)	N(40, 40)	B1
	P($Y > 50.5$)	M1
	$z = \frac{10.5}{\sqrt{40}} = 1.66$	M1
	$1 - 0.9515 =$ 0.0485	A1
	Oct 2019 report: no continuity correction, or correcting the wrong way (49.5), are the common errors.	(4)
(b)	P($39.5 < Y < 40.5$): $z = \pm 0.08$	M1
	$2(0.5319 - 0.5) =$ 0.0638	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$p = 0.98$ is not close to 0.5 (the distribution is very skewed)	B1
		(1)
(b)	Failures $F \sim$ B(250, 0.02)	M1
	$\approx$ Po(5)	A1
	P($G > 245$) = P($F < 5$) = P($F \leqslant 4$)	M1
	0.4405	A1
	Oct 2019 report: it is essential to change to counting the failures before approximating.	(4)
(c)	P($F \leqslant 10$) = 0.9863	B1
		(1)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	Tables: $\boxed{0.1808}$	B1 (1)
(b)	N(15, 7.5); P($Y < 12.5$) $z = \frac{12.5-15}{\sqrt{7.5}} = -0.91$ $\boxed{0.1814}$	M1 M1 A1 (3)
(c)	$\frac{0.1814-0.1808}{0.1808} \times 100 = 0.33\%$ Very small error: a good approximation, as $p = 0.5$ gives a symmetric distribution	M1 A1 (2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	Po(1.4): $1 - \left(e^{-1.4} + 1.4e^{-1.4} + \frac{1.4^2}{2}e^{-1.4}\right)$ $\boxed{0.167}$	M1 A1 (2)
(b)	P(no customers) = $e^{-1.4} = 0.2466$ B(5, 0.2466): $5(0.2466)(0.7534)^4$ $\boxed{0.397}$	B1 M1 A1 (3)
(c)	Po(42) $\approx$ N(42, 42) P($Y > 49.5$) $z = \frac{7.5}{\sqrt{42}} = 1.16$ $1 - 0.8770 = \boxed{0.123}$ *Oct 2024 report: the 30-minute total is Poisson with mean 42; approximating a binomial is the wrong model.*	B1 M1 M1 A1 (4)
(d)	$\lambda = 42$ is large (so the Poisson distribution is approximately symmetric)	B1 (1)
		Total for Question 6: 10 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7

10 marks

Part	Scheme	Marks
(a)	$X \sim$ B(50, 0.25): mean 12.5	B1
	variance 9.375	B1
		(2)
(b)	N(12.5, 9.375); P($Y > 19.5$)	M1
	$z = \frac{19.5-12.5}{\sqrt{9.375}}$	M1
	$z = 2.29$	A1
	$1 - 0.9890 =$ 0.0110	A1
		(4)
(c)	P($X \geqslant k$) $\approx$ P($Y > k - 0.5$) < 0.01	M1
	$\frac{k-0.5-12.5}{\sqrt{9.375}} > 2.3263$	M1
	$k > 20.12$	A1
	$k = 21$	A1
	Check: exactly, P($X \geqslant 21$) = 0.0063 *and* P($X \geqslant 20$) = 0.0139.	
		(4)
	Total for Question 7: 10 marks	

Question 8

10 marks

Part	Scheme	Marks
(a)	B(400, 0.015) $\approx$ Po(6) (n large, p small)	M1
	P($X > 8$) = 1 − P($X \leqslant 8$)	M1
	$1 - 0.8472 = 0.1528$ *	A1*
		(3)
(b)	$R \sim$ B(60, 0.1528)	B1
	$\approx$ N(9.168, 7.767)	M1
	P($Y > 11.5$)	M1
	$z = \frac{11.5-9.168}{2.787} = 0.84$	M1
	$1 - 0.7995 =$ 0.2005	A1
		(5)
(c)	$p = 0.1528$ is not close to 0.5	B1
	but $np = 9.2$ and $n(1-p) = 50.8$ are both reasonably large, so the approximation is acceptable (exact value 0.198)	B1
		(2)
	Total for Question 8: 10 marks	

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9

10 marks

Part	Scheme	Marks
(a)	Po(25) $\approx$ N(25, 25); P($Y < 20.5$)	M1
	$z = \frac{20.5-25}{5} = -0.9$	M1
	$\boxed{0.1841}$	A1
		(3)
(b)	P($X > k$) $\approx$ P($Y > k + 0.5$) < 0.05	M1
	$\frac{k+0.5-25}{5} > 1.6449$	M1
	$k > 32.72$	A1
	$\boxed{k = 33}$	A1
		(4)
(c)	Po(125) $\approx$ N(125, 125); P($Y > 140.5$)	M1
	$z = \frac{15.5}{\sqrt{125}} = 1.39$	M1
	$1 - 0.9177 = \boxed{0.0823}$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$np = 6$, $np(1-p) = 5.88$	M1
	$\boxed{p = 0.02}$	A1
	$\boxed{n = 300}$	A1
		(3)
(b)	Po(6): P($X \leqslant 4$)	M1
	$\boxed{0.2851}$	A1
		(2)
(c)	N(6, 5.88); P($Y < 4.5$)	M1
	$z = \frac{-1.5}{\sqrt{5.88}} = -0.62$	M1
	$\boxed{0.2676}$	A1
		(3)
(d)	The Poisson approximation (0.2851 is closer to 0.2824)	B1
	n is large and p is small, so Po(np) is appropriate; p is far from 0.5, so the normal is not	B1
		(2)
		Total for Question 10: 10 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4 Chapter 4: Continuous Random Variables

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$k\int_0^3 (x^2+1)\,dx = k\left[\frac{x^3}{3}+x\right]_0^3 = 12k$	M1
	$12k = 1$, so $k = \frac{1}{12}$ *	A1*
		(2)
(b)	$\frac{1}{12}\int_1^2 (x^2+1)\,dx$	M1
	$\frac{1}{12}\left[\left(\frac{8}{3}+2\right)-\left(\frac{1}{3}+1\right)\right]$ (limits substituted)	M1
	$\boxed{\frac{5}{18}}$ (0.278)	A1
	Oct 2019 report: a calculator integral with no working loses the marks when algebraic integration is required.	
		(3)
(c)	$\boxed{0}$	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\int_0^x \frac{t}{8}\,dt = \frac{x^2}{16}$	M1
	$F(x) = \frac{x^2}{16}$ for $0 \leqslant x \leqslant 4$	A1
	$F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 4$	B1
	Oct 2024 report: omitting the 0 and 1 lines loses the B mark.	
		(3)
(b)	$\frac{m^2}{16} = 0.5$	M1
	$\boxed{2.83}$ $(2\sqrt{2})$	A1
		(2)
(c)	$1-\frac{9}{16} = \boxed{\frac{7}{16}}$	B1
		(1)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$\int_0^2 \frac{3}{8}x^3\,dx = \left[\frac{3}{32}x^4\right]_0^2$ 1.5	M1 A1 (2)
(b)	$E(X^2) = \int_0^2 \frac{3}{8}x^4\,dx = \left[\frac{3}{40}x^5\right]_0^2 = 2.4$ $\text{Var}(X) = 2.4 - 1.5^2$ $= 0.15$ * *Oct 2019 report:* $E(X^2)$ *is not* $[E(X)]^2$.	M1 M1 A1* (3)
(c)	E = 6, Var = 4×0.15 = 0.6	B1 (1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Differentiates: $f(x) = \frac{2x+2}{12}$ $f(x) = \frac{x+1}{6}$ for $1 \leqslant x \leqslant 3$, 0 otherwise *Oct 2019 report: integrating* $F(x)$ *is the common error.*	M1 A1 (2)
(b)	$F(4) - F(1.5) = 1 - \frac{2.25+3-3}{12}$ 0.8125 *Oct 2019 report:* $F(4) = F(3) = 1$ *must be recognised.*	M1 A1 (2)
(c)	$m^2 + 2m - 3 = 6$, $m^2 + 2m - 9 = 0$ $m = -1 + \sqrt{10} = 2.16$ (the other root is outside $1 \leqslant x \leqslant 3$)	M1 A1 (2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	Triangle with vertices (0, 0), (2, 0.5), (4, 0)	B1
	Area $\frac{1}{2} \times 4 \times 0.5 = 1$	B1
		(2)
(b)	$1 - 2 \times \left(\frac{1}{2} \times 1 \times \frac{1}{4}\right)$ (two small triangles removed; symmetry)	M1
	$\boxed{0.75}$	A1
		(2)
(c)	$P(X > q) = \frac{(4-q)^2}{8} = 0.25$	M1
	$\boxed{4 - \sqrt{2}}$	A1
		(2)
	Total for Question 5: 6 marks	

Question 6

10 marks

Part	Scheme	Marks
(a)	$f(x) \geqslant 0$ throughout	B1
	Area $\frac{1}{2} \times 6 \times \frac{1}{3} = 1$ (or $\frac{1}{3} + \frac{2}{3}$ by integration)	B1
		(2)
(b)	$\int_0^2 \frac{x^2}{6}\,dx + \int_2^6 \frac{6x - x^2}{12}\,dx$	M1
	$\frac{4}{9} + \frac{1}{12}\left[3x^2 - \frac{x^3}{3}\right]_2^6 = \frac{4}{9} + \frac{20}{9}$	M1
	$\boxed{\frac{8}{3}}$ (2.67)	A1
	Oct 2019 report: both integrals are needed, each with its own function.	
		(3)
(c)	$F(2) = \frac{1}{3}$	B1
	$F(x) = \frac{1}{3} + \int_2^x \frac{6-t}{12}\,dt = \frac{1}{3} + \frac{1}{12}\left(6x - \frac{x^2}{2} - 10\right)$	M1
	$= \frac{12x - x^2 - 12}{24}$ *	A1*
		(3)
(d)	$F(2) < 0.5$, so solve $12m - m^2 - 12 = 12$: $m^2 - 12m + 24 = 0$	M1
	$\boxed{6 - 2\sqrt{3}}$ (2.54)	A1
		(2)
	Total for Question 6: 10 marks	

Question 7

10 marks

Part	Scheme	Marks
(a)	$f'(x) = \frac{12x-3x^2}{108}$	**M1**
	$f'(x) = 0$: $3x(4-x) = 0$	**M1**
	Mode = 4 ($x = 0$ gives the minimum)	**A1**
	Oct 2019 report: solving $f(x) = 0$ instead of $f'(x) = 0$.	**(3)**
(b)	$\frac{1}{108}\int_0^6 \left(6x^3 - x^4\right) dx$	**M1**
	$\frac{1}{108}\left[\frac{3}{2}x^4 - \frac{x^5}{5}\right]_0^6 = \frac{388.8}{108}$	**M1**
	3.6	**A1**
		(3)
(c)	$F(x) = \frac{1}{108}\left(2x^3 - \frac{x^4}{4}\right)$; $F(3.6) = 0.475$, $F(3.7) = 0.504$	**M1**
	$0.475 < 0.5 < 0.504$, so the median lies between 3.6 and 3.7 *	**A1***
	Oct 2019 report: a conclusion is needed; "$F(3.6) < m < F(3.7)$" is not acceptable.	**(2)**
(d)	Mean (3.6) $<$ median (3.6 to 3.7) $<$ mode (4)	**M1**
	Negative skew	**A1**
		(2)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	Area $= \frac{1}{2}(2)k + 3k + \frac{1}{2}(1)k = 4.5k$ $4.5k = 1$, $k = \frac{2}{9}$ *	**M1** **A1*** **(2)**
(b)	$1 - \mathrm{P}(X < 1) - \mathrm{P}(X > 5.5) = 1 - \frac{k}{4} - \frac{k}{8}$ $\boxed{\frac{11}{12}}$	**M1** **A1** **(2)**
(c)	$\mathrm{P}(X \leqslant 2) = k = \frac{2}{9} < 0.5$, so the median is in the flat section $\frac{2}{9} + \frac{2}{9}(m-2) = \frac{1}{2}$ $\boxed{3.25}$	**M1** **M1** **A1** **(3)**
(d)	$\int_0^2 \frac{kx^2}{2}\,\mathrm{d}x + \int_2^5 kx\,\mathrm{d}x + \int_5^6 kx(6-x)\,\mathrm{d}x$ $\frac{8}{27} + \frac{63}{27} + \frac{16}{27}$ $\boxed{\frac{29}{9}}$ (3.22)	**M1** **M1** **A1** **(3)**
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$F(2) = 1$: $4a + 2b = 1$ *	**B1**
	$f(x) = 2ax + b$	**M1**
	$E(X) = \int_0^2 (2ax^2 + bx)\,dx = \frac{16a}{3} + 2b$	**M1**
	$\frac{16a}{3} + 2b = \frac{7}{6}$, so $16a + 6b = \frac{7}{2}$ *	**A1***
		(4)
(b)	Solves simultaneously	**M1**
	$\boxed{a = \frac{1}{8},\ b = \frac{1}{4}}$	**A1**
		(2)
(c)	$E(X^2) = \int_0^2 \frac{x^3 + x^2}{4}\,dx = \frac{5}{3}$	**M1**
	$\frac{5}{3} - \left(\frac{7}{6}\right)^2$	**M1**
	$\boxed{\frac{11}{36}}$ (0.306)	**A1**
		(3)
(d)	$1 - F\left(\frac{7}{6}\right) = \boxed{0.538}$ $\left(\frac{155}{288}\right)$	**B1**
		(1)
		Total for Question 9: 10 marks

Question 10

11 marks

Part	Scheme	Marks
(a)	$\frac{3}{32}\int_3^4 \left(4t - t^2\right) \mathrm{d}t$ $\frac{3}{32}\left[2t^2 - \frac{t^3}{3}\right]_3^4 = \frac{3}{32}\left(\frac{32}{3} - 9\right)$ $= \frac{5}{32}$ *	M1 M1 A1*
		(3)
(b)	$Y \sim \mathrm{B}\left(10, \frac{5}{32}\right)$ $1 - \left(\frac{27}{32}\right)^{10} - 10\left(\frac{5}{32}\right)\left(\frac{27}{32}\right)^{9}$ 0.478	B1 M1 A1
		(3)
(c)	$\mathrm{E}(T) =$ 2, as the pdf is symmetric about $t = 2$ $\mathrm{E}\left(T^2\right) = \frac{3}{32}\int_0^4 \left(4t^3 - t^4\right) \mathrm{d}t = 4.8$ $\mathrm{Var}(T) = 4.8 - 4 =$ 0.8 *Oct 2024 report: use the symmetry of the graph rather than a long integral for the mean.*	B1 M1 A1
		(3)
(d)	$\mathrm{E}(C) = 5(2) + 2 =$ 12 $\mathrm{Var}(C) = 25(0.8) = 20$, standard deviation 4.47	B1 B1
		(2)
		Total for Question 10: 11 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5 Chapter 5: Continuous Uniform Distribution

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$f(x) = \frac{1}{8}$ for $2 \leqslant x \leqslant 10$, 0 otherwise	B1
	Rectangle of height $\frac{1}{8}$ from 2 to 10	B1
		(2)
(b)	$E(X) = \boxed{6}$	B1
	$Var(X) = \frac{8^2}{12} = \boxed{\frac{16}{3}}$	B1
		(2)
(c)	$\frac{10-7.5}{8} = \boxed{0.3125}$	B1
		(1)
(d)	$\boxed{0}$	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\frac{a+b}{2} = 20$	B1
	$\frac{(b-a)^2}{12} = 48$, so $b - a = 24$ (square root, $b > a$)	M1
	Solves $a + b = 40$, $b - a = 24$	M1
	$\boxed{a = 8,\ b = 32}$	A1
	Oct 2024 report: there is no need to expand $(b-a)^2$ into a three-term quadratic.	
		(4)
(b)	$\frac{14-8}{24} = \boxed{0.25}$	B1
		(1)
(c)	$P(Y < 11) + P(Y > 29) = \frac{3+3}{24} = \boxed{0.25}$	B1
		(1)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$E \sim \mathrm{U}[-0.5, 0.5]$	B1 (1)
(b)	$\frac{0.2}{1} = 0.2$	B1 (1)
(c)	$\frac{1}{12}$ (0.0833)	B1 (1)
(d)	$N \sim \mathrm{B}(20, 0.2)$ $1 - \mathrm{P}(N \leqslant 5) = 1 - 0.8042$ 0.1958	B1 M1 A1 (3)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\mathrm{F}(x) = \frac{x+3}{8}$ for $-3 \leqslant x \leqslant 5$ Correct $\mathrm{F}(x) = 0$ for $x < -3$ and 1 for $x > 5$	M1 A1 B1 (3)
(b)	$\frac{q+3}{8} = 0.3$: $q = -0.6$	B1 (1)
(c)	$\mathrm{Var}(X) = \frac{64}{12}$, $\mathrm{E}(X) = 1$; $\mathrm{E}\left(X^2\right) = \frac{16}{3} + 1^2$ $\frac{19}{3}$ (6.33)	M1 A1 (2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$E(X) = \boxed{4}$ $Var(X) = \frac{36}{12} = \boxed{3}$	B1 B1 (2)
(b)	$E(Y) = 4(4) + 3 = \boxed{19}$ $Var(Y) = 16 \times 3 = \boxed{48}$ *Oct 2019 report: forgetting the +3 in $E(Y)$; using $4Var(X)$ instead of $4^2Var(X)$.*	B1 B1 (2)
(c)	$P(4X + 3 > 20) = P(X > 4.25)$ $\frac{7-4.25}{6} = \boxed{0.458}$ $\left(\frac{11}{24}\right)$	M1 A1 (2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$P(19.8 < L < 20.4) = \frac{20.4-19.8}{1}$ $= 0.6$ * *Oct 2019 report: some found the probability that the bolts could not be used.*	M1 A1* (2)
(b)	Unusable bolts $U \sim B(10, 0.4)$ $P(\text{usable} \geqslant 8) = P(U \leqslant 2)$ $\boxed{0.1673}$	M1 M1 A1 (3)
(c)	$B(200, 0.6) \approx N(120, 48)$ $P(Y > 125.5)$ $z = \frac{5.5}{\sqrt{48}} = 0.79$ $1 - 0.7852 = \boxed{0.2148}$ *The normal approximation is suitable here: n is large and $p = 0.6$ is close to 0.5*	B1 M1 M1 A1 (4)
(d)	e.g. the bolts' lengths are independent of each other	B1 (1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$X \sim \text{U}[0,30]$; the shorter piece is less than 10 cm $\text{P}(X < 10) + \text{P}(X > 20)$ $\boxed{\frac{2}{3}}$	**M1** **M1** **A1** **(3)**
(b)	Each length from 0 to 15 arises from two equally likely positions of the break, so all are equally likely Mean $\boxed{7.5\,\text{cm}}$ *Oct 2024 report: the uniform distribution must be on the correct interval;* $\text{U}[0,30]$ *for the shorter piece is wrong.*	**B1** **B1** **(2)**
(c)	$\text{Var} = \frac{15^2}{12} = 18.75$ $\boxed{4.33\,\text{cm}}$	**M1** **A1** **(2)**
(d)	The shorter piece is $X \sim \text{U}[0,15]$: $\text{P}(X > 6) = \frac{9}{15}$ $\boxed{0.6}$	**M1** **A1** **(2)**
(e)	The rod is equally likely to break at any point along its length	**B1** **(1)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$E(L) = \boxed{4}$, $Var(L) = \frac{16}{12} = \boxed{\frac{4}{3}}$	**B1**
	Both correct	**B1**
		(2)
(b)	$E(A) = E\left(L^2\right)$	**M1**
	$= Var(L) + [E(L)]^2 = \frac{4}{3} + 16$	**M1**
	$\boxed{\frac{52}{3}}$ (17.3)	**A1**
		(3)
(c)	$P\left(L > \sqrt{20}\right) = \frac{6-4.472}{4}$	**M1**
	$\boxed{0.382}$	**A1**
		(2)
(d)	Median of L is 4, and $A = L^2$ increases with L	**M1**
	Median of $A = \boxed{16}$	**A1**
	Mean (17.3) $>$ median (16): A is positively skewed	**B1**
		(3)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{4}{12} = \boxed{\frac{1}{3}}$	B1 (1)
(b)	$P(W \geqslant 9 \mid W > 5) = \frac{P(W \geqslant 9)}{P(W > 5)}$ $= \frac{3/12}{7/12}$ $\boxed{\frac{3}{7}}$	M1 M1 A1 (3)
(c)	$B\left(5, \frac{1}{3}\right)$ $\binom{5}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^3$ $\boxed{0.329}$ $\left(\frac{80}{243}\right)$	B1 M1 A1 (3)
(d)	$5 \times E(W) = 5 \times 6$ $\boxed{30 \text{ minutes}}$	M1 A1 (2)
(e)	e.g. buses may not run exactly every 12 minutes (traffic), or Jamal may arrive at a regular time, not at random	B1 (1)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$P(X > 1) = \dfrac{3a-1}{4a}$	M1
	$\dfrac{3a-1}{4a} = 0.6,\ 3a-1 = 2.4a$	M1
	$a = \frac{5}{3}$ *	A1*
		(3)
(b)	$E(X) = \frac{-a+3a}{2} = \boxed{\frac{5}{3}}$	B1
	$Var(X) = \frac{(4a)^2}{12} = \frac{100}{27} = \boxed{3.70}$	B1
		(2)
(c)	$P(-1 < X < 1) = \dfrac{2}{4a}$	M1
	$\boxed{0.3}$	A1
		(2)
(d)	$E(\lvert X\rvert) = \displaystyle\int_{-a}^{3a} \frac{\lvert x\rvert}{4a}\,dx = \frac{1}{4a}\left(\int_{-a}^{0}(-x)\,dx + \int_{0}^{3a} x\,dx\right)$	M1
	$= \frac{1}{4a}\left(\frac{a^2}{2} + \frac{9a^2}{2}\right) = \frac{5a}{4}$	M1
	$\boxed{\frac{25}{12}}$ (2.08)	A1
		(3)
		Total for Question 10: 10 marks

6 Chapter 6: Sampling and Sampling Distributions

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	The 1200 students in the school	B1 (1)
(b)	The head teacher's list of all the students	B1 (1)
(c)	Individual students	B1 (1)
(d)	Census: every student's view is included, so the result is accurate Sample: quicker / cheaper / easier to analyse	B1 B1 (2)
(e)	e.g. by chance it may contain mostly students from one year group, whose views differ	B1 (1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	(i) is a statistic: it uses only the sample values (ii) and (iv) are not statistics: they contain the unknown parameters μ and σ (iii) is a statistic: (i) and (iii) are statistics	B1 B1 B1 (3)
(b)	All possible values of the statistic together with their probabilities *Topic reports: the meaning of a sampling distribution is often poorly answered; "probabilities" must be mentioned.*	B1 (1)
(c)	$\frac{1}{3}(\mathrm{E}(X_1)+\mathrm{E}(X_2)+\mathrm{E}(X_3))$ $=\frac{1}{3}(3\mu)=\mu$ *	M1 A1* (2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)$ *B1 for at least 6 correct, or for the 6 distinct unordered samples.*	**B2** **(2)**
(b)	Means 1, 1.5, 2, 2.5, 3 Uses orders: e.g. $P(\bar{x} = 1.5) = 2(0.5)(0.3)$ 0.25, 0.3, 0.29, 0.12, 0.04 respectively Probabilities total 1	**B1** **M1** **A1** **A1** **(4)**
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	It is calculated from the sample values only, with no unknown parameters	**B1** **(1)**
(b)	$P(M = 2) = 0.7^3 =$ 0.343 $P(M = 5) =$ 0.657	**B1** **B1** **(2)**
(c)	$P(R = 0) = 0.7^3 + 0.3^3 =$ 0.37 $P(R = 3) =$ 0.63	**B1** **B1** **(2)**
(d)	$3 \times 0.63 =$ 1.89	**B1** **(1)**
		Total for Question 4: 6 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 5

6 marks

Part	Scheme	Marks
(a)	$Y \sim \text{B}(10, 0.3)$	B1 (1)
(b)	$1 - \text{P}(Y \leqslant 4) = 1 - 0.8497$ 0.1503	M1 A1 (2)
(c)	Mean 0.3 Variance $\frac{10(0.3)(0.7)}{100}$ = 0.021	B1 B1 (2)
(d)	$\frac{Y}{10}$ uses only the sample; $Y - 10p$ contains the unknown parameter p	B1 (1)
		Total for Question 5: 6 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6

10 marks

Part	Scheme	Marks
(a)	Bag A: P(1) = 0.75, P(2) = 0.25; bag B: P(2) = P(5) = 0.5 Samples (A,A,B): $(1,1,2)$, $(1,1,5)$, $(1,2,2)$, $(2,1,2)$, $(1,2,5)$, $(2,1,5)$, $(2,2,2)$, $(2,2,5)$ Probabilities: 0.28125, 0.28125, 0.09375 each for the four mixed-A cases, 0.03125, 0.03125 *Oct 2024 report: samples such as $(1,2,5)$ arise in two ways and must be counted twice; probabilities are not simply multiplied by 2, 3 or 6.*	**B1** **B1** **B1** **(3)**
(b)	Totals 4, 5, 6, 7, 8, 9 Combines the two orders for totals 5 and 8 0.28125, 0.1875, 0.03125, 0.28125, 0.1875, 0.03125	**B1** **M1** **A1** **(3)**
(c)	Median is 1 only when both coins from bag A are 1p P(M = 1) = 0.5625 P(M = 2) = 0.4375	**M1** **A1** **A1** **(3)**
(d)	The coin from bag B is at least 2, so the median is the smaller of the two bag A coins unless both are 2, and that smaller coin is 1 or 2 *Oct 2024 report: impossible samples such as $(1,1,1)$ must not be included.*	**B1** **(1)**
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$\mu = 0.4 + 0.4 = \boxed{0.8}$	**B1**
	$\sigma^2 = (0.4 + 0.8) - 0.64 = \boxed{0.56}$	**B1**
		(2)
(b)	Values of $\bar{X}$: 0, 0.5, 1, 1.5, 2	**B1**
	e.g. $P(\bar{X} = 1) = 0.4^2 + 2(0.4)(0.2)$	**M1**
	$\boxed{0.16, 0.32, 0.32, 0.16, 0.04}$	**A1**
	Total 1	**A1**
		(4)
(c)	$E(\bar{X}) = 0.16 + 0.32 + 0.24 + 0.08 = 0.8 = \mu$	**M1**
	Correct	**A1**
	$E(\bar{X}^2) = 0.08 + 0.32 + 0.36 + 0.16 = 0.92$, $\text{Var}(\bar{X}) = 0.92 - 0.64$	**M1**
	$= 0.28 = \frac{0.56}{2}$ *	**A1***
		(4)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	e.g. a census of all voters would be too costly and slow	**B1** **(1)**
(b)	The telephone directory Voters not in the directory (e.g. mobile only) cannot be chosen, and their views may differ	**B1** **B1** **(2)**
(c)	$Y \sim \text{B}(400, 0.35) \approx \text{N}(140, 91)$ $\hat{P} > 0.4$ means $Y > 160$: $\text{P}(W > 160.5)$ $z = \frac{20.5}{\sqrt{91}} = 2.15$ $1 - 0.9842 =$ $\boxed{0.0158}$	**B1** **M1** **M1** **A1** **(4)**
(d)	$\hat{P}$ is calculated from the sample only $\hat{P} - p$ involves the unknown parameter p, so its value cannot be calculated	**B1** **B1** **(2)**
(e)	$\boxed{p}$	**B1** **(1)**
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	It is calculated from the sample values only	**B1** **(1)**
(b)	27 equally likely samples $R = 0$: 3 samples, P $= \frac{1}{9}$ $R = 2$: contains both 1 and 3: $27 - 8 - 8 + 1 = 12$ P$(R = 2) = \frac{4}{9}$ P$(R = 1) = \frac{4}{9}$	**B1** **B1** **M1** **A1** **A1** **(5)**
(c)	$1 \times \frac{4}{9} + 2 \times \frac{4}{9}$ $\boxed{\frac{4}{3}}$	**M1** **A1** **(2)**
(d)	P(median 2) $= \frac{13}{27}$; median 2 and range 2: $(1, 2, 3)$ in 6 orders $\frac{6/27}{13/27} = \boxed{\frac{6}{13}}$	**M1** **A1** **(2)**
		Total for Question 9: 10 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10

10 marks

Part	Scheme	Marks
(a)	Possible values of $\bar{X}$: 1, $\frac{5}{3}$, $\frac{7}{3}$, 3	B1
	Uses the number of arrangements, e.g. $P\left(\bar{X}=\frac{5}{3}\right)=3p^2(1-p)$	M1
	$P(1)=p^3$, $P\left(\frac{5}{3}\right)=3p^2(1-p)$	A1
	$P\left(\frac{7}{3}\right)=3p(1-p)^2$, $P(3)=(1-p)^3$	A1
		(4)
(b)	$(1-p)^3=\frac{1}{8}$	M1
	$\boxed{p=\frac{1}{2}}$	A1
		(2)
(c)	Median 1 when at least two discs show 1: $3\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^3$	M1
	$\boxed{P(\text{median}=1)=P(\text{median}=3)=\frac{1}{2}}$	A1
		(2)
(d)	$p^3+\frac{5}{3}\left(3p^2(1-p)\right)+\frac{7}{3}\left(3p(1-p)^2\right)+3(1-p)^3=3-2p$ *	M1
	Population mean $=1\times p+3(1-p)=3-2p$, as expected since $E\left(\bar{X}\right)=\mu$	A1
		(2)
		Total for Question 10: 10 marks

7 Chapter 7: Hypothesis Testing

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
	$H_0: p = 0.3$, $H_1: p > 0.3$	B1
	Under H_0, $X \sim B(20, 0.3)$	M1
	$P(X \geqslant 10) = 1 - P(X \leqslant 9)$	M1
	$= 1 - 0.9520 = 0.0480$	A1
	$0.0480 < 0.05$: reject H_0	M1
	There is evidence that the spinner lands on red more often than 0.3	A1
	Jan 2014 report: $1 - P(X \leqslant 10)$ *is the common error.*	(6)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$X \sim B(25, 0.4)$: $P(X \leqslant 5) = 0.0294$	M1
	$P(X \leqslant 6) = 0.0736$	A1
	$X \leqslant 5$	A1
	Oct 2018 report: the critical region is a set of values, not a probability.	(3)
(b)	0.0294	B1
		(1)
(c)	6 is not in the critical region	M1
	Do not reject H_0: insufficient evidence that $p < 0.4$	A1
		(2)
		Total for Question 2: 6 marks

★★★★★ Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3

6 marks

Part	Scheme	Marks
(a)	$H_0: p = 0.25$, $H_1: p \neq 0.25$	B1 (1)
(b)	$X \sim B(20, 0.25)$; lower: $P(X \leqslant 1) = 0.0243$, $P(X \leqslant 2) = 0.0913$ Upper: $P(X \geqslant 9) = 1 - 0.9591 = 0.0409$, $P(X \geqslant 8) = 0.1018$ $X \leqslant 1$ or $X \geqslant 9$ *Oct 2024 report: using the full 10% in each tail is the common error.*	M1 M1 A1 (3)
(c)	$0.0243 + 0.0409 =$ 0.0652	B1 (1)
(d)	8 is not in the critical region: no evidence that the proportion is different from 25%	B1 (1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$H_0: \lambda = 1.5$, $H_1: \lambda < 1.5$ (per month)	B1 (1)
(b)	Under H_0, $X \sim Po(6)$ for 4 months $P(X \leqslant 2)$ $= 0.0620$ $0.0620 > 0.05$: do not reject H_0	B1 M1 A1 M1 (4)
(c)	There is insufficient evidence that the rate of accidents has decreased *Oct 2024 report: a conclusion needs the context; do not refer to a claim that was not made.*	B1 (1)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$X \sim \text{B}(20, 0.05)$; the most extreme value is $X = 0$	M1
	$\text{P}(X = 0) = 0.95^{20} = 0.3585$	A1
	$0.3585 > 0.05$, so even $X = 0$ would not lead to rejecting H_0: there is no critical region *	A1*
	Summer 2023 scheme: "the sample size is too small" on its own is not enough.	(3)
(b)	$0.95^n \leqslant 0.05$	M1
	$n \geqslant \dfrac{\ln 0.05}{\ln 0.95} = 58.4$	A1
	$\boxed{n = 59}$	A1
		(3)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\text{H}_0 : p = 0.25$	B1
	$\text{H}_1 : p < 0.25$	B1
		(2)
(b)	$X \sim \text{B}(30, 0.25)$: $\text{P}(X \leqslant 3)$	M1
	$= 0.0374$	A1
	$0.0374 < 0.05$: reject H_0	M1
	Evidence that fewer than 25% of workers cycle to work	A1
		(4)
(c)	$\text{P}(X \leqslant 3) = 0.0374$, $\text{P}(X \leqslant 4) = 0.0979$	M1
	$\boxed{X \leqslant 3}$	A1
		(2)
(d)	$\boxed{0.0374}$	B1
		(1)
(e)	The council suspects the proportion is lower, so the test must look for evidence in the lower tail	B1
	Oct 2019 report: choosing the upper tail gives a probability close to 1 (here $\text{P}(X \geqslant 3) = 0.989$), a sign of the wrong tail.	(1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	e.g. calls arrive singly / independently / at a constant rate	**B1** **(1)**
(b)	$H_0 : \lambda = 7$, $H_1 : \lambda \neq 7$	**B1** **(1)**
(c)	$X \sim$ Po(7); lower: $P(X \leqslant 2) = 0.0296$, $P(X \leqslant 3) = 0.0818$ Lower tail $X \leqslant 2$ Upper: $P(X \geqslant 13) = 1 - 0.9730 = 0.0270$, $P(X \geqslant 12) = 1 - 0.9467 = 0.0533$ $\boxed{X \leqslant 2 \text{ or } X \geqslant 13}$ *0.0533 is nearer to 0.05, but it exceeds 5%, so 12 is not included.*	**M1** **A1** **M1** **A1** **(4)**
(d)	$0.0296 + 0.0270 = \boxed{0.0566}$	**B1** **(1)**
(e)	12 is not in the critical region, so do not reject H_0 There is insufficient evidence that the *rate of calls* has changed	**M1** **A1** **(2)**
(f)	e.g. the 10-minute period is chosen at random, or calls still occur independently	**B1** **(1)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$H_0: \lambda = 30$, $H_1: \lambda < 30$ *Oct 2019 report:* H_0 *is an equality, not* $\lambda > 30$	**B1** **(1)**
(b)	$\lambda = 30$ is large	**B1** **(1)**
(c)	$Po(30) \approx N(30, 30)$ $P(X \leqslant 20) \approx P(Y < 20.5)$ $z = \frac{20.5-30}{\sqrt{30}} = -1.73$ $0.0418 < 0.05$ Reject H_0: evidence that the mean number of emails has decreased	**B1** **M1** **M1** **A1** **A1** **(5)**
(d)	$\frac{c+0.5-30}{\sqrt{30}} < -1.6449$ $c < 20.49$ $X \leqslant 20$	**M1** **A1** **A1** **(3)**
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$H_0: p = 0.4$, $H_1: p < 0.4$	**B1**
		(1)
(b)	$X \sim B(150, 0.4) \approx N(60, 36)$	**B1**
	$P(X \leqslant 48) \approx P(Y < 48.5)$	**M1**
	$z = \frac{48.5-60}{6} = -1.92$	**M1**
	$0.0274 < 0.05$	**A1**
	Reject H_0: evidence that support is lower than 40%	**A1**
		(5)
(c)	$0.0274 > 0.01$	**M1**
	At 1% H_0 would not be rejected: the conclusion would change	**A1**
		(2)
(d)	$\frac{c+0.5-60}{6} < -1.6449$, $c < 49.6$	**M1**
	$X \leqslant 49$	**A1**
		(2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$X \sim \text{B}(50, 0.1)$: $\text{P}(X \leqslant 1) = 0.0338$ $\text{P}(X \leqslant 2) = 0.1117$ $\boxed{X \leqslant 1}$	M1 A1 A1 (3)
(b)	$\boxed{0.0338}$	B1 (1)
(c)	$X \sim \text{B}(50, 0.05)$: $\text{P}(X \leqslant 1)$ $\boxed{0.2794}$	M1 A1 (2)
(d)	The test detects a genuine halving of the defective rate only about 28% of the time, so it is not very powerful Use a larger sample (or a higher significance level)	B1 B1 (2)
(e)	$1 - (1 - 0.0338)^5$ $\boxed{0.158}$	M1 A1 (2)
		Total for Question 10: 10 marks

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk