



eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	2	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

[Full mark scheme](#)

[Free courses](#)

[More worksheets](#)

Name

Class

Date



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Page 1 of 27

6

CHAPTER 6 • KEY SKILLS

Integration

Specification P4: Integration • 10 questions • 86 marks • about 103 minutes

1 Integration by parts

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx. \text{ Let } u = x \text{ (or } x^2), \text{ but always } u = \ln x.$$

2 Substitution

Find $\frac{du}{dx}$, replace dx (not just rename it du), change the limits, and leave no x in the integral.

3 Partial fractions

Split first.

$$\int \frac{k}{ax+b} dx = \frac{k}{a} \ln|ax+b| + c: \text{ do not lose the } \frac{1}{a}.$$

4 Trigonometric integrals

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x);$$

$$\int \sec^2 x dx = \tan x;$$

$$\int \tan x dx = \ln|\sec x|$$

5 Parametric areas

$$\text{Area} = \int y \frac{dx}{dt} dt, \text{ with the limits as values of } t.$$

6 Volumes of revolution

$$V = \pi \int y^2 dx: \text{ square the whole of } y.$$

$$\text{Parametric: } \pi \int y^2 \frac{dx}{dt} dt$$

7 Separable equations

Separate, integrate both sides and add $+c$ at once. Use the condition to find c , then rearrange.

8 Modelling

As $t \rightarrow \infty$, $e^{-kt} \rightarrow 0$ gives the long-term value. Find times exactly with $\ln$.

9 Exact answers

Combine logarithms: $\ln 5 - \ln \frac{1}{3} = \ln 15$.
Give the form asked for, e.g. $a + b \ln 2$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	6	8	5	6	11	10	12	12	11
1 Integration by parts	•						•		•	•
2 Substitution		•				•				
3 Partial fractions			•					•		
4 Trigonometric integrals	•				•	•				•
5 Parametric areas						•				
6 Volumes of revolution				•		•			•	
7 Separable equations					•			•		•
8 Modelling								•	•	
9 Exact answers	•	•	•	•			•	•		•
Interleaved: trigonometric identities and inverse functions (P3)					•	•				•
Interleaved: partial fractions and parametric curves (Chapters 2, 3)			•			•		•		
Interleaved: e, ln and numerical methods (P3)							•	•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Quote the formula first ($V = \pi \int y^2 dx$, the parts formula). Include $+c$ as soon as you integrate a differential equation. With a substitution, change dx and the limits. Write the answer in the exact form asked for.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Use integration by parts to find the exact value of

$$\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx$$

giving your answer in the form $a\pi + b \ln 2$, where a and b are rational constants.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 1 is 5 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Use the substitution $u = \sqrt{x}$ to show that

$$\int_4^9 \frac{1}{\sqrt{x}-1} dx = a + b \ln 2$$

where a and b are integers to be found.

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{5}{(2x-1)(x+2)} \quad x > \frac{1}{2}$$

(3)

$$\int_k^{2k} f(x) \, dx = \ln \frac{14}{9} \quad k > \frac{1}{2}$$

(5)

Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 3 is 8 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

4. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

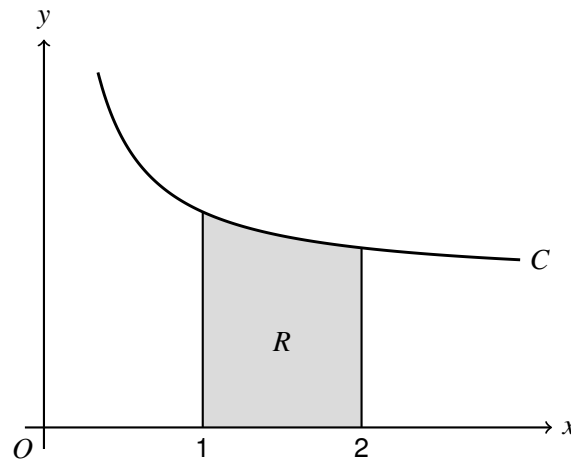


Figure 1

Figure 1 shows part of the curve C with equation $y = 2 + \frac{1}{x}$, $x > 0$

The region R , shown shaded, is bounded by C , the lines $x = 1$ and $x = 2$ and the x -axis. The region R is rotated through 2π radians about the x -axis.

Use algebraic integration to find the exact volume of the solid generated, giving your answer in the form $\pi(p + q \ln 2)$, where p and q are rational.

(5)



Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 4 is 5 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

5. Find the particular solution of the differential equation

$$\frac{dy}{dx} = (2x + 1) \cos^2 y \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

for which $y = \frac{\pi}{4}$ when $x = 0$, giving your answer in the form $y = f(x)$

(6)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 5 is 6 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

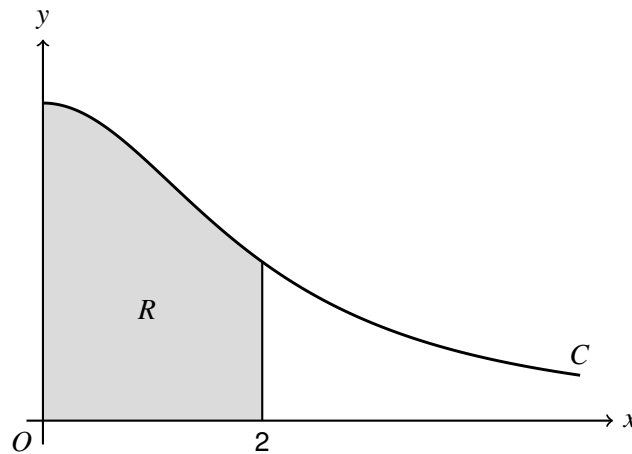


Figure 2

Figure 2 shows part of the curve C with parametric equations

$$x = 2 \tan t \quad y = 4 \cos^2 t \quad 0 \leq t < \frac{\pi}{2}$$

The region R , shown shaded, is bounded by C , the coordinate axes and the line $x = 2$

- (a) Show that the area of R is given by $\int_0^{\frac{\pi}{4}} k \, dt$, where k is a constant to be found, and hence find the exact area of R .

(4)

- (b) Show that a Cartesian equation of C is $y = \frac{16}{4+x^2}$

(2)

The region R is rotated through 2π radians about the x -axis.

- (c) Use algebraic integration to find the exact volume of the solid generated, giving your answer in the form $\pi(a\pi + b)$, where a and b are integers.

(5)



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 6 is 11 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

7. (a) Find $\int e^{-x} \sin x dx$

(5)

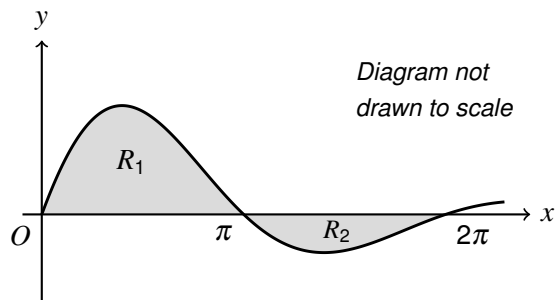


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = e^{-x} \sin x$, $x \geq 0$

The region R_1 is bounded by the curve and the x -axis for $0 \leq x \leq \pi$. The region R_2 is bounded by the curve and the x -axis for $\pi \leq x \leq 2\pi$

(b) Find the exact area of R_1 **(2)**

(c) Show that the area of R_1 is e^π times the area of R_2 (3)

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 10 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

8. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

The number of fish in a lake, P thousands, t years after the start of a study, is modelled by the differential equation

$$\frac{dP}{dt} = \frac{P(10 - P)}{20} \quad 0 < P < 10$$

At the start of the study there were 2000 fish in the lake.

- (a) Express $\frac{1}{P(10 - P)}$ in partial fractions. (3)
- (b) Solve the differential equation to show that $P = \frac{10}{1 + 4e^{-\frac{1}{2}t}}$ (6)
- (c) Find, according to the model, the number of fish in the lake in the long term. (1)
- (d) Find the exact time, in years, at which there are 8000 fish in the lake, giving your answer in the form $a \ln 2$ (2)



Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 12 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

9.

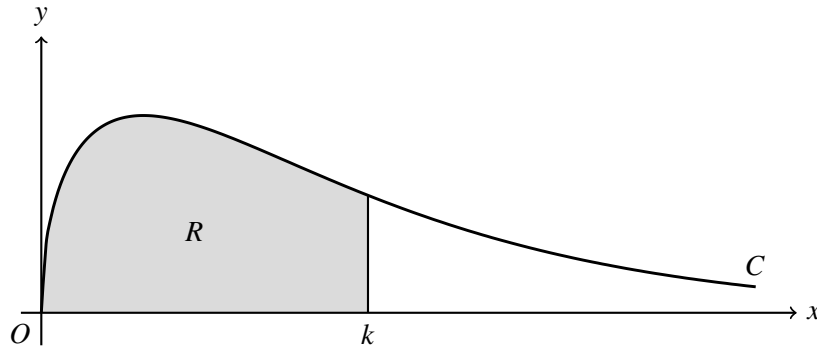


Figure 4

Figure 4 shows part of the curve C with equation $y = \sqrt{x}e^{-\frac{1}{2}x}$, $x \geq 0$

The region R , shown shaded, is bounded by C , the x -axis and the line $x = k$, where $k > 0$. The region R is rotated through 2π radians about the x -axis to form a solid of volume V .

(a) Show that $V = \pi(1 - (k+1)e^{-k})$ (5)

(b) State the limit of V as $k \rightarrow \infty$ (1)

The value of k for which V is 90% of this limit is β .

(c) Show that β satisfies the equation $\beta = \ln(10\beta + 10)$ (2)

(d) Using the iteration formula $\beta_{n+1} = \ln(10\beta_n + 10)$ with $\beta_1 = 4$, find the values of β_2 and β_3 , giving your answers to 4 decimal places. (2)

(e) Show that $\beta = 3.890$ correct to 3 decimal places. (2)



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 12 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

10.

(a) Find $\int x \cos 2x \, dx$

(3)

(b) Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{x \cos^2 x}{y} \quad y > 0$$

to show that $y^2 = \frac{1}{2}x^2 + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + \frac{3}{4}$

(6)

(c) Find the exact value of y when $x = \frac{\pi}{2}$

(2)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 6 IS 86 MARKS

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk