



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Trigonometric Functions

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Trigonometric Functions worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Trigonometric Functions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $\frac{1}{\sin 45^\circ}, \frac{1}{\cos 150^\circ}, \frac{1}{\tan 300^\circ}$: $\sqrt{2}, -\frac{2\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}$

A2 $\frac{1}{\sin x} - \sin x = \frac{1 - \sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \cos x \times \frac{\cos x}{\sin x} = \cos x \cot x$ **as required**

A3 $\tan x = -2$: $180^\circ - 63.4^\circ$ and $360^\circ - 63.4^\circ$: $116.6^\circ, 296.6^\circ$

STRETCH

A4 $\operatorname{cosec}^2 x - 1 - \operatorname{cosec} x - 1 = 0$, so $(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 1) = 0$: $\sin x = \frac{1}{2}$ or -1 : $30^\circ, 150^\circ, 270^\circ$

A5 $\frac{\pi}{6}, -\frac{\pi}{4}, \frac{\pi}{3}$

A6 Opposite 2, adjacent 1, hypotenuse $\sqrt{5}$: $\frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

A7 $\sin 2x = 0$ at $x = \frac{\pi}{2}$: $x = \frac{\pi}{2}$; turning points $\left(\frac{\pi}{4}, 1\right)$, **minimum** and $\left(\frac{3\pi}{4}, -1\right)$, **maximum**



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\sqrt{2}$ (b) $-\frac{2\sqrt{3}}{3}$ (c) $\frac{\sqrt{3}}{3}$ (d) -1

Each reciprocal has the same sign as the function it comes from.

B2 (a) $\frac{\sin x}{\cos x} \times \frac{1}{\sin x} = \sec x$ (b) $\operatorname{cosec} x$ (c) 1 (d) $\tan^2 x$

Write everything in sin and cos, then cancel.

B3 (a) $30^\circ, 150^\circ$ (b) $120^\circ, 240^\circ$ (c) $30^\circ, 210^\circ$ (d) $\sin x = 2$: **no solutions**

$|\operatorname{cosec} x| \geq 1$ and $|\sec x| \geq 1$ always.

B4 (a) $\frac{5}{4}$ (b) $\frac{5}{3}$ (c) $\frac{4}{3}$ (d) third quadrant, $\cos \theta < 0$: $-\frac{5}{4}$

The 3, 4, 5 triangle gives the sizes; the quadrant gives the signs.

B5 (a) $x = \frac{\pi}{2}, x = \frac{3\pi}{2}$ (b) $y \leq -1$ or $y \geq 1$ (c) π

Asymptotes of sec are where $\cos x = 0$

B6 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\sin \frac{5\pi}{6} = \frac{1}{2}$: $\frac{\pi}{6}$

An inverse function always returns a value in its own range.

STRETCH

B7 (a) $\sec^2 x - 1 = 2 \sec x - 1$, $\sec x(\sec x - 2) = 0$; $\sec x = 0$ is impossible: $60^\circ, 300^\circ$

(b) $\tan x = \pm \frac{1}{\sqrt{3}}$: $30^\circ, 150^\circ, 210^\circ, 330^\circ$

(c) $2 \operatorname{cosec}^2 x + 3 \operatorname{cosec} x - 2 = 0$, $(2 \operatorname{cosec} x - 1)(\operatorname{cosec} x + 2) = 0$; $\operatorname{cosec} x = \frac{1}{2}$ is impossible: $210^\circ, 330^\circ$

(d) $\cos x = \pm \frac{1}{2}$: $60^\circ, 120^\circ, 240^\circ, 300^\circ$

Replace the squared term with an identity so that only one function remains.



B8 (a) $\frac{1 - \cos^2 x}{\sin^2 x} = 1$ (b) $\sec^2 x - \tan^2 x = 1$ (c) $\frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$ (d) $\frac{\operatorname{cosec}^2 x}{\sec^2 x} = \frac{\cos^2 x}{\sin^2 x}$

each identity holds

Start from the more complicated side and use one identity at a time.

B9 (a) domain $-1 \leq x \leq 1$, range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ (b) domain $-1 \leq x \leq 1$, range $0 \leq y \leq \pi$

(c) domain $x \in \mathbb{R}$, range $-\frac{\pi}{2} < y < \frac{\pi}{2}$ (d) $2x - 1 = \tan \frac{\pi}{4} = 1: x = 1$

Each range is chosen so that the inverse is a function.

B10 (a) $\frac{4}{5}$ (b) opposite $\sqrt{5}$, adjacent 2: $\frac{\sqrt{5}}{2}$ (c) $\sqrt{1-x^2}$ (d) $\sqrt{1+x^2}$

A triangle with the right sides turns any composition into a ratio of sides.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $\sec$ is $\frac{1}{\cos}$, not $\frac{1}{\sin}$: $\sec 30^\circ = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

C2 $\frac{2\pi}{3}$ is outside the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ of $\arcsin$. $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$, so the value is $\frac{\pi}{3}$

C3 $\sec x = 0.5$ means $\cos x = 2$, not 0.5. Since $|\cos x| \leq 1$, there are no solutions

C4 $1 + \tan^2 x \equiv \sec^2 x$. Dividing $\sin^2 x + \cos^2 x \equiv 1$ by $\sin^2 x$ gives $1 + \cot^2 x \equiv \operatorname{cosec}^2 x$

STRETCH

C5 $\frac{4\pi}{3}$ is outside $-\pi \leq x \leq \pi$. $\cos x = -\frac{1}{2}$ in the interval: $x = \pm \frac{2\pi}{3}$

C6 The interval is in radians, so the answer must be too; also $2y = \frac{\pi}{6}$ needs dividing by 2: $y = \frac{\pi}{12}$



C7 The 2 must be squared as well: $(2 \sin x \cos x)^2 = 4 \sin^2 x \cos^2 x$: $\frac{1}{4 \sin^2 x \cos^2 x}$

C8 (a) In a right-angled triangle with sides x and 1, the two acute angles are $\arctan x$ and $\arctan \frac{1}{x}$, and they add to $\frac{\pi}{2}$ **as required**

(b) $\arctan x = 2 \left(\frac{\pi}{2} - \arctan x \right)$, so $3 \arctan x = \pi$: $x = \tan \frac{\pi}{3} = \sqrt{3}$

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $2(\sec^2 x - 1) - 3 \sec x = 0$, so $2 \sec^2 x - 3 \sec x - 2 = 0$, $(2 \sec x + 1)(\sec x - 2) = 0$

$\sec x = -\frac{1}{2}$ is impossible ($|\sec x| \geq 1$); $\cos x = \frac{1}{2}$: $60^\circ, 300^\circ$

D2 (a) $\frac{4 \cos x}{\sin x} = \frac{3}{\cos x}$, so $4 \cos^2 x = 3 \sin x$, and $4(1 - \sin^2 x) = 3 \sin x$: $4 \sin^2 x + 3 \sin x - 4 = 0$ **as required**

(b) $\sin x = \frac{-3 + \sqrt{73}}{8} \approx 0.6930$ (the other root is less than -1): $x = 0.766, x = 2.38$

D3 (a) $x = 0$: $a + b = 5$; $x = \pi$: $-a + b = 1$: $a = 2, b = 3$

(b) $y \leq 1$ or $y \geq 5$

D4 $-\frac{\pi}{2} \leq \theta \leq 0$, so $\cos \theta > 0$, $\tan \theta < 0$. Triangle 3, 4, 5: $\cos \theta = \frac{4}{5}$, $\tan \theta = -\frac{3}{4}$, $\sec \theta = \frac{5}{4}$

STRETCH

D5 (a) $\frac{(\operatorname{cosec} x + 1) + (\operatorname{cosec} x - 1)}{\operatorname{cosec}^2 x - 1} = \frac{2 \operatorname{cosec} x}{\cot^2 x} = \frac{2}{\sin x} \times \frac{\sin^2 x}{\cos^2 x} = \frac{2 \sin x}{\cos^2 x} = 2 \tan x \sec x$ **as required**

(b) $\frac{2 \sin x}{\cos^2 x} = 4 \sin x$, so $\sin x \left(\frac{2}{\cos^2 x} - 4 \right) = 0$. $\sin x = 0$ makes $\operatorname{cosec} x$ undefined, so reject it; $\cos^2 x = \frac{1}{2}$:

$45^\circ, 135^\circ, 225^\circ, 315^\circ$



- D6** (a) $\frac{1}{\cos x} = 2 + \cos x$, so $1 = 2 \cos x + \cos^2 x$: $\cos^2 x + 2 \cos x - 1 = 0$ as required
- (b) $\cos x = -1 \pm \sqrt{2}$; $-1 - \sqrt{2} < -1$ is impossible, so $\cos x = \sqrt{2} - 1$: $x = 1.14, x = 5.14$

- D7** $-30^\circ \leq 2x - 30^\circ < 690^\circ$ and $\cos(2x - 30^\circ) = \frac{1}{2}$: $2x - 30^\circ = 60^\circ, 300^\circ, 420^\circ, 660^\circ$
- $x = 45^\circ, 165^\circ, 225^\circ, 345^\circ$

- D8** (a) $(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x) = 1 \times (\sec^2 x + \tan^2 x)$ as required
- (b) $1 + 2 \tan^2 x = 3 \tan x$, $(2 \tan x - 1)(\tan x - 1) = 0$: $x = \frac{\pi}{4}$, $x = 0.464$

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

- E1** (a) $2(\operatorname{cosec}^2 x - 1) + 5 \operatorname{cosec} x + 4 = 0$ *M1: uses $1 + \cot^2 x = \operatorname{cosec}^2 x$*
- $2 \operatorname{cosec}^2 x + 5 \operatorname{cosec} x + 2 = 0$ as required A1*
- (b) $(2 \operatorname{cosec} x + 1)(\operatorname{cosec} x + 2) = 0$; $\operatorname{cosec} x = -\frac{1}{2}$ is impossible M1
- $\sin x = -\frac{1}{2}$: 210° A1
- 330° A1
-
- E2** (a) $\frac{\pi}{3}$, $-\frac{\pi}{6}$ B1 B1
- (b) $0 \leq \theta \leq \pi$, so $\sin \theta \geq 0$; triangle 1, $\sqrt{15}$, 4 M1
- $\sin \theta = \frac{\sqrt{15}}{4}$, $\cot \theta = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$ A1 A1
- (c) $\arccos$ returns values in $[0, \pi]$; $\cos \frac{5\pi}{3} = \frac{1}{2}$: $\frac{\pi}{3}$ B1



STRETCH

- E3** (a) $\cos 2x = 0$: $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$ B1 B1
- (b) $M\left(\frac{\pi}{2}, 0\right)$ B1
- (c) $\sec 2x = 2$, $\cos 2x = \frac{1}{2}$, with $0 \leq 2x \leq 2\pi$ M1
- $2x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ dM1
- $x = \frac{\pi}{6}$, $x = \frac{5\pi}{6}$ A1
- (d) Two solutions for $k \geq 2$ (the two outer branches, including $x = 0$ and π when $k = 2$) and for $k < 0$ M1
- $k < 0$ or $k \geq 2$ A1

