eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 1 (WFM01)

Further Pure 1 Practice Paper 3

Edexcel IAL Further Pure Mathematics 1

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM01/01) • Calculator allowed

A full 75-mark paper set above the hardest recent series: long questions, few printed results and a trap in every question

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 7 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	A quartic with unknown coefficients	10	
2	Roots of a quadratic with a parameter	10	
3	A series from n to $2n$	11	
4	Numerical methods near an asymptote	11	
5	A transformation with a given area	11	
6	A normal to a parabola and a locus	11	
7	Induction for a two-step recurrence	11	
	Total	**75**	

Rough grade guide (out of 75): A **53** • B **46** • C **39**

Based on recent WFM01 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(z) = z^4 + az^2 + bz + 10$$

where a and b are real constants. Given that $1 + 2i$ is a root of the equation $f(z) = 0$

(a) find the value of a and the value of b

(5)

(b) find the other three roots of the equation $f(z) = 0$

(3)

The four roots are represented by the points A, B, C and D on an Argand diagram.

(c) Find the area of the quadrilateral formed by A, B, C and D

(2)

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Question 1 continued

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(Total for Question 1 is 10 marks)

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2. The equation

$$x^2 + kx + k + 6 = 0$$

where k is a constant, has **real** roots α and β

(a) Find an expression for $\alpha^2 + \beta^2$ in terms of k

(2)

Given that $\alpha^2 + \beta^2 = 12$

(b) find the value of k, giving a reason for rejecting any other value.

(5)

(c) Using this value of k, find a quadratic equation, with integer coefficients, whose roots are α^3 and β^3

(3)

(Total for Question 2 is 10 marks)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Use the standard results for $\sum_{r=1}^{m} r$ and $\sum_{r=1}^{m} r^2$ to find $\sum_{r=1}^{m} r(r+1)$, giving your answer in fully factorised form.

(3)

(b) Hence show that, for all positive integers n,

$$\sum_{r=n}^{2n} r(r+1) = \tfrac{1}{3}n(n+1)(7n+5)$$

(4)

(c) Find the value of n for which

$$\sum_{r=n}^{2n} r(r+1) = \sum_{r=1}^{n} (28r - 15)$$

giving a reason for rejecting any other solutions.

(4)

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Question 3 continued

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(Total for Question 3 is 11 marks)

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4.

$$f(x) = x^2 - 4 + \frac{1}{x-1}, \qquad x \neq 1$$

(a) Show that f(0.5) and f(1.5) have opposite signs, and explain why this does **not** show that the equation $f(x) = 0$ has a root in the interval [0.5, 1.5]

(3)

(b) Show that the equation $f(x) = 0$ has a root α in the interval $[-2.1, -2]$

(2)

(c) Taking -2 as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(3)

A student instead applies the Newton–Raphson process once to $f(x)$ with a first approximation of 1.6

(d) Find the second approximation that the student obtains, giving your answer to 3 decimal places, and explain why 1.6 is not a suitable first approximation.

(3)

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Question 4 continued

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(Total for Question 4 is 11 marks)

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5.

$$\mathbf{P} = \begin{pmatrix} k & k+2 \\ -1 & 3 \end{pmatrix}$$

where k is a constant.

(a) Find $\det \mathbf{P}$ in terms of k, and hence find the value of k for which $\mathbf{P}$ is singular.

(2)

The triangle T has vertices $O(0, 0)$, $A(3, 0)$ and $B(0, 2)$. The image of T under the transformation represented by $\mathbf{P}$ has area 30

(b) Find the possible values of k

(4)

(c) For the negative value of k, find the coordinates of the images of A and B

(2)

The transformation represented by $\mathbf{P}$, with this negative value of k, is followed by a rotation through 90° anticlockwise about $(0, 0)$. The combined transformation is represented by the matrix $\mathbf{R}$

(d) Find the coordinates of the point that is mapped by $\mathbf{R}$ onto the point $(5, 5)$

(3)

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Question 5 continued

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(Total for Question 5 is 11 marks)

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6.

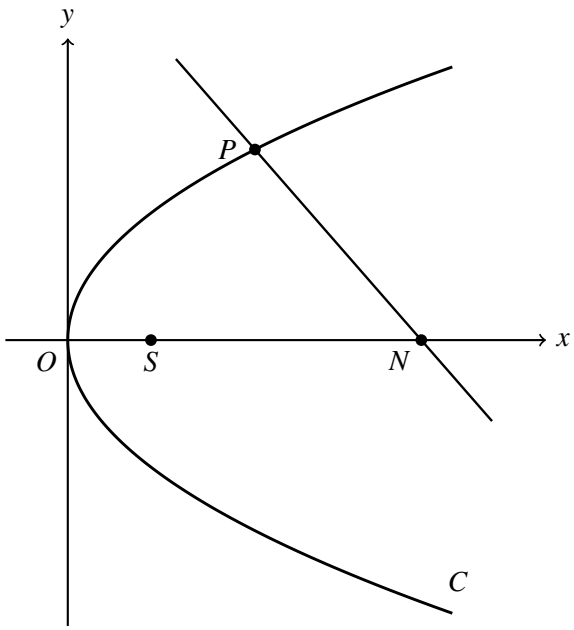

Figure 1

The parabola C has equation $y^2 = 8x$ and focus S. The point $P\left(2p^2, 4p\right)$, $p \neq 0$, lies on C. The normal to C at P meets the x-axis at the point N, as shown in Figure 1.

(a) Show that an equation of the normal to C at P is

$$y + px = 2p^3 + 4p$$

(4)

(b) Find, in terms of p, the area of the triangle SPN when $p > 0$

(3)

(c) Given that the area of the triangle SPN is 40, find the possible coordinates of P

(2)

The point M is the midpoint of PN

(d) Show that, as p varies, M lies on a parabola, and find a Cartesian equation of this parabola.

(2)

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Question 6 continued

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(Total for Question 6 is 11 marks)

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7. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 5, \qquad u_2 = 13, \qquad u_{n+2} = 5u_{n+1} - 6u_n$$

(a) Prove by induction that, for all positive integers n,

$$u_n = 2^n + 3^n$$

(6)

(b) Using the result in part (a), prove by induction that u_{2m-1} is divisible by 5 for all positive integers m

(5)

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Question 7 continued

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TOTAL FOR PAPER IS 75 MARKS

END

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