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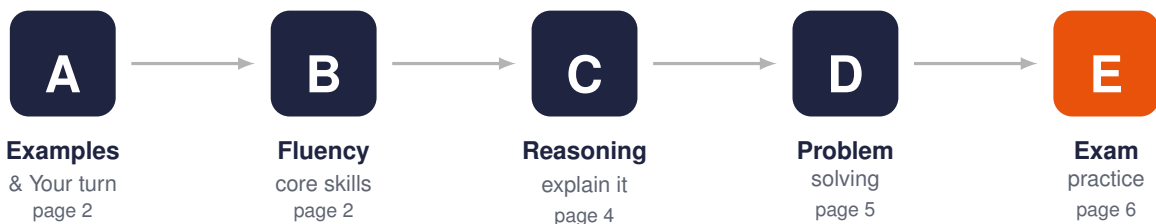
A-LEVEL MATHEMATICS • PURE 4

Coordinate Geometry

Worked Solutions

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Coordinate Geometry worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 4 Coordinate Geometry worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Coordinate Geometry revision engine

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $t = 3 - x$, so $y = (3 - x)^2 + 2(3 - x)$: $y = x^2 - 8x + 15$

A2 $\cos t = \frac{x-2}{3}$, $\sin t = \frac{y+1}{3}$: $(x-2)^2 + (y+1)^2 = 9$, a circle with centre $(2, -1)$ and radius 3

A3 $t^2 = 2(t+1) + 1$, so $t^2 - 2t - 3 = 0$, $t = 3$ or $t = -1$: $(4, 9)$ and $(0, 1)$

STRETCH

A4 x is linear: $t = -2$ gives -8 , $t = 1$ gives 1 ; y has its minimum 1 at $t = 0$, and $t = -2$ gives 5

$$-8 \leq x \leq 1, 1 \leq y \leq 5$$

A5 $\cos 2t + 1 = 2\cos^2 t = 2\left(\frac{x}{2}\right)^2$: $y = \frac{x^2}{2}$, $-2 \leq x \leq 2$

A6 $\cot t = x$, $\operatorname{cosec} t = \frac{y-1}{2}$, and $\operatorname{cosec}^2 t - \cot^2 t = 1$: $\frac{(y-1)^2}{4} - x^2 = 1$

A7 $y = 0$: $t = 3$ or -1 , so $(7, 0)$, $(-1, 0)$; $x = 0$: $t = -\frac{1}{2}$, so $\left(0, -\frac{7}{4}\right)$

$y = (t-1)^2 - 4$ is least at $t = 1$: $(7, 0)$, $(-1, 0)$, $\left(0, -\frac{7}{4}\right)$; **lowest point** $(3, -4)$

B**Fluency solutions**One mark for each correct part. The notes in *italics* explain why each sequence was chosen.**CORE**

B1 (a) $(5, 4)$ (b) $y = 7$ gives $t = 3$, and $3^2 + 1 = 10$: $t = 3$

(c) $t = -2$: $x = 5$, $y = -8$: **on the curve** (d) $x = 2$ needs $t = \pm 1$, giving $y = 1$ or -5 : **not on the curve**

A point is on the curve only if one value of t gives both coordinates.



B2 (a) $y = 2x - 7$ (b) $y = \frac{x^2}{4} - 1$ (c) $y = \frac{1}{x} + 2$ (d) $y = x^2 - 4, x \geq 0$

Make t the subject of the simpler equation; in (d) $\sqrt{t}$ cannot be negative, so the domain is limited.

B3 (a) $x^2 + y^2 = 16$ (b) $\frac{x^2}{4} + \frac{y^2}{25} = 1$ (c) $(x-3)^2 + \frac{y^2}{4} = 1$

(d) $x^2 + y^2 = 2\sin^2 t + 2\cos^2 t$: $x^2 + y^2 = 2$

Isolate $\cos t$ and $\sin t$ first; in (d) the cross terms cancel when you add the squares.

B4 (a) $4t = 2t^2$: $(0, 0), (4, 8)$ (b) $\frac{1}{t} = \frac{t}{4}, t = \pm 2$: $\left(4, \frac{1}{2}\right), \left(-4, -\frac{1}{2}\right)$

(c) $y = 0: t = 0; x = 0: t = -1$: $(1, 0), (0, 1)$ (d) $\tan t = 1$: $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right), \left(-\frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

Substitute the parametric forms into the line, solve for t , then find both coordinates.

B5 (a) $(2, 0)$ and $(-2, 0)$ (b) $\sin t \geq 0$ here: $0 \leq y \leq 1$

(c) $\frac{x^2}{4} + y^2 = 1$, the upper half ($y \geq 0$)

The interval for t decides which part of the Cartesian curve is drawn.

B6 (a) $x = (t-1)^2 - 1$: $-1 \leq x \leq 3$ (b) $1 \leq y \leq 4$ (c) $t = y - 1$: $x = y^2 - 4y + 3$

(d) x is least at $t = 1$, inside the interval; the ends give only 0 and 3

Look for a turning point inside the interval before using the end values.

STRETCH

B7 (a) $y = 2x^2 - 1$ (b) $x^2 = 1 + 2\sin t \cos t$: $y = x^2 - 1$

(c) $x = 2 - 2\sin^2 t$: $x = 2 - 2y^2$ (d) $\cos 3t = 4\cos^3 t - 3\cos t$: $y = 4x^3 - 3x$

Choose the form of the double-angle formula that matches the other equation.

B8 (a) $x^2 - y^2 = 1$ (b) $\cos^2 t = \frac{1}{1 + \tan^2 t}$: $y = \frac{4}{4 + x^2}$

(c) $x^2 - (y-1)^2 = 1$ (d) $(\sec t + \tan t)(\sec t - \tan t) = \sec^2 t - \tan^2 t$: $xy = 1$

Each pair is linked by $\sec^2 = 1 + \tan^2$ or $\operatorname{cosec}^2 = 1 + \cot^2$



- B9** (a) $t^3 = 2t^2$, $t = 0$ or 2 : $(0, 0), (4, 8)$ (b) $x^2 + (x+2)^2 = 4$: $(0, 2), t = \frac{\pi}{2}; (-2, 0), t = \pi$
- (c) $t^2 - t - 1 = 0$, $t = \frac{1 \pm \sqrt{5}}{2}$, and $t + \frac{1}{t} = \pm\sqrt{5}$: $(\sqrt{5}, 1), (-\sqrt{5}, 1)$
- (d) $mt^2 - 2t + 1 = 0$ has one root when $4 - 4m = 0$: $m = 1$

Every part ends in an equation in t ; in (d) the discriminant counts the meeting points.

- B10** (a) $t = \frac{x}{20}$: $y = \frac{3x}{4} - \frac{x^2}{80}$ (b) $y = 0$ at $t = 3$: **60 m**
- (c) $y = \frac{45}{4} - 5\left(t - \frac{3}{2}\right)^2$: **11.25 m** (d) $0 \leq x \leq 60$

The parameter is time, so its interval gives the domain of the path.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** It still contains t . A Cartesian equation links x and y only. $t = x + 1$: $y = x^2 + 4x + 3$
- C2** $y = 4 - (t - 2)^2$ has its maximum 4 at $t = 2$, inside the interval: $0 \leq y \leq 4$
- C3** The signs of the centre are wrong, and the radius is 2, not 4 (it is not squared here).
centre (3, 1), radius 2: $(x - 3)^2 + (y - 1)^2 = 4$
- C4** $t^2 = 4$ also has the root $t = -2$, which gives a second point. **(4, 4) and (4, -4)**

STRETCH

- C5** $x = 2 \sin t$ only takes values from -2 to 2 , and $y = \cos 2t$ from -1 to 1
domain $-2 \leq x \leq 2$, range $-1 \leq y \leq 1$: only the arc of the parabola between $(-2, -1)$ and $(2, -1)$
- C6** Both give $y = x^2$. But $x = t^2 \geq 0$, so the first is only the right half. **$x \geq 0$ for the first; all x for the second**



C7 $x^2 = t^2 + 2 + \frac{1}{t^2}$ and $y^2 = t^2 - 2 + \frac{1}{t^2}$ (the middle terms were missed)

Subtracting: $x^2 - y^2 = 4$

C8 (a) $x^2 + y^2 = \frac{4t^2 + (1 - t^2)^2}{(1 + t^2)^2} = \frac{1 + 2t^2 + t^4}{(1 + t^2)^2} = \frac{(1 + t^2)^2}{(1 + t^2)^2}$: $= 1$

(b) $y = -1$ needs $1 - t^2 = -1 - t^2$, which has no solution.

So $(0, -1)$ is never reached

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $\cos^2 t = \frac{x-1}{2}$, $\sin^2 t = \frac{y}{3}$, and they add to 1: $3(x-1) + 2y = 6$: $3x + 2y = 9$

(b) $A(3, 0)$ at $t = 0$, $B(1, 3)$ at $t = \frac{\pi}{2}$: $AB = \sqrt{2^2 + 3^2}$: $\sqrt{13}$

D2 (a) $(x-1)^2 + (y-2)^2 = 4$: centre $(1, 2)$, radius 2

(b) $x = 0$: $\cos t = -\frac{1}{2}$, so $\sin t = \pm \frac{\sqrt{3}}{2}$: $P(0, 2 + \sqrt{3})$, $Q(0, 2 - \sqrt{3})$

D3 $2t + \frac{6}{t} = 8$, so $t^2 - 4t + 3 = 0$, $t = 1$ or 3 : $A(2, 6)$, $B(6, 2)$

$AB = \sqrt{4^2 + 4^2}$: $4\sqrt{2}$

D4 (a) $t^2 + 2t = 3$, so $(t+3)(t-1) = 0$: $a = -3$, $b = 1$

(b) $y = 3 - t$ decreases from 6 to 2: $2 \leq y \leq 6$

STRETCH

D5 (a) $1 + \tan^2 t = \sec^2 t$, so $1 + x = \left(\frac{y}{2}\right)^2$: $y^2 = 4(x+1)$

(b) $\tan^2 t \geq 0$ and $\sec t \geq 1$ for $0 \leq t < \frac{\pi}{2}$: $x \geq 0$, $y \geq 2$



D6 (a) $x = 1 - (1 - 2\sin^2 t) = 2\sin^2 t = 2\left(\frac{y}{2}\right)^2$: $x = \frac{y^2}{2}$

(b) $y = \frac{y^2}{2}$, so $y = 0$ or 2 : $P(2, 2)$, $\sin t = 1$: $t = \frac{\pi}{2}$

D7 (a) $1 - x = \frac{1}{1+t}$, so $t = \frac{x}{1-x}$ and $y = tx$: $y = \frac{x^2}{1-x}$, with $x < 1$ (as $\frac{1}{1+t} > 0$)

(b) $\frac{x^2}{1-x} = 2x$, so $x = 0$ or $x = 2(1-x)$: $R\left(\frac{2}{3}, \frac{4}{3}\right)$, $t = \frac{2/3}{1/3}$: $t = 2$

D8 (a) $\frac{x^2}{16} + \frac{y^2}{9} = 1$

(b) $12\sin t = 12\cos t + 12$, so $\sin t - \cos t = 1$, i.e. $\sqrt{2}\sin\left(t - \frac{\pi}{4}\right) = 1$

$t - \frac{\pi}{4} = \frac{\pi}{4}$ or $\frac{3\pi}{4}$: $t = \frac{\pi}{2}$ at $B(0, 3)$; $t = \pi$ at $A(-4, 0)$

(c) $\frac{1}{2} \times 4 \times 3$: **6**

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $\cos t = \frac{x}{5}$, $\sin t = \frac{y}{3}$, and $\cos^2 t + \sin^2 t = 1$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

(b) $5\cos t = 4$, so $\cos t = \frac{4}{5}$ and $\sin t = \pm \frac{3}{5}$

$P\left(4, \frac{9}{5}\right), Q\left(4, -\frac{9}{5}\right)$

$PQ = \frac{18}{5}$

M1

A1

M1

A1

A1



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- E2** (a) $t = \frac{x-3}{2}$ M1
- $$y = \frac{(x-3)^2}{4} - 2(x-3)$$
- dM1
- $$y = \frac{1}{4}x^2 - \frac{7}{2}x + \frac{33}{4}$$
- A1
- (b) $1 \leq x \leq 13$ B1
- (c) $y = (t-2)^2 - 4$: least value -4 at $t = 2$; at $t = -1$ and $t = 5$, $y = 5$ M1
- $$-4 \leq y \leq 5$$
- A1

STRETCH

- E3** (a) $t = \frac{y}{6}, x = 3\left(\frac{y}{6}\right)^2$: $y^2 = 12x$ B1
- (b) $6t = 3t^2 - 12$ M1
- $$t^2 - 2t - 4 = 0$$
- A1*
- (c) $t = 1 \pm \sqrt{5}$ M1
- $$A(18 + 6\sqrt{5}, 6 + 6\sqrt{5}), B(18 - 6\sqrt{5}, 6 - 6\sqrt{5})$$
- A1
- (d) Gradient of O to $(3t^2, 6t)$ is $\frac{6t}{3t^2} = \frac{2}{t}$ M1
- product of gradients $= \frac{4}{t_1 t_2}$, and $t_1 t_2 = -4$ from (b) dM1
- $$\text{product} = -1, \text{ so } OA \text{ is perpendicular to } OB$$
- A1

