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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	3	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	10	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	16	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	23	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	30	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	38	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	46	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	52	

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How to use this mark scheme

The same marking rules Pearson examiners use for WFM01.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

"Solutions relying entirely on calculator technology are not acceptable." When a WFM01 question says this, a correct final answer with no algebraic working scores **no marks**. Exact answers (surds, multiples of π) are required when asked for; otherwise give 3 significant figures, with arguments in radians.


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1 Chapter 1: Complex Numbers

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$3+12i-2i-8i^2$	M1
	$\boxed{11+10i}$	A1
		(2)
(b)	$\frac{(3-2i)(1-4i)}{(1+4i)(1-4i)}$	M1
	$=\frac{3-12i-2i-8}{17}$	A1
	$\boxed{-\frac{5}{17}-\frac{14}{17}i}$	A1
	Summer 2019 report: the final answer must be a complex number in the form asked for.	(3)
(c)	$3-2i+1-4i=\boxed{4-6i}$	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\sqrt{9+27}=\boxed{6}$	B1
		(1)
(b)	Second quadrant: $\pi-\tan^{-1}\sqrt{3}$	M1
	$\boxed{\frac{2\pi}{3}}$	A1
	Jan 2021 report: $\tan^{-1}$ *alone gives the wrong quadrant; subtract from* π.	(2)
(c)	$\boxed{6\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)}$	B1
		(1)
(d)	$\lvert zw\rvert=6\sqrt{2}$	B1
	$\arg(zw)=\frac{2\pi}{3}-\frac{\pi}{4}=\boxed{\frac{5\pi}{12}}$	B1
		(2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$(z+3)^2+4=0$ or $z=\dfrac{-6\pm\sqrt{36-52}}{2}$	M1
	$z+3=\pm 2\mathrm{i}$	A1
	$\boxed{z=-3\pm 2\mathrm{i}}$	A1
		(3)
(b)	Both points in the second and third quadrants, symmetric about the real axis	B1
		(1)
(c)	$\boxed{\sqrt{13}}$	B1
		(1)
(d)	The roots are a conjugate pair $p\pm q\mathrm{i}$, so both have modulus $\sqrt{p^2+q^2}$	B1
		(1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$z=x+\mathrm{i}y$: $2x+2\mathrm{i}y+\mathrm{i}(x-\mathrm{i}y)=5+4\mathrm{i}$	M1
	Real: $2x+y=5$; imaginary: $x+2y=4$	M1
	Both correct	A1
	$\boxed{z=2+\mathrm{i}}$	A1
		(4)
(b)	$\lvert -1+\mathrm{i}\rvert$	M1
	$\boxed{\sqrt{2}}$	A1
		(2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$a^2 - b^2 + 2ab\text{i} = 5 - 12\text{i}$	M1
	$a^2 - b^2 = 5$ and $ab = -6$	A1
	Eliminates: $a^4 - 5a^2 - 36 = 0$	M1
	$a^2 = 9$ (rejecting -4)	A1
	$\boxed{a = 3,\ b = -2 \text{ or } a = -3,\ b = 2}$	A1
		(5)
(b)	$\boxed{z = \pm(3 - 2\text{i})}$	B1
		(1)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\boxed{2 - 3\text{i}}$	B1
		(1)
(b)	$(z - 2 - 3\text{i})(z - 2 + 3\text{i})$ or sum 4, product 13	M1
	$\boxed{z^2 - 4z + 13}$	A1
	Jan 2021 report: multiplying the two root factors is the efficient method.	
		(2)
(c)	$f(z) = (z^2 - 4z + 13)(z - 1)$	M1
	$\boxed{z = 1,\ 2 \pm 3\text{i}}$	A1
		(2)
(d)	$2 \pm 3\text{i}$ plotted symmetrically about the real axis	B1
	1 on the real axis, at a realistic distance	B1
	Summer 2024 report: real roots must be plotted on the real axis, not the imaginary axis.	
		(2)
(e)	Base from $(2, 3)$ to $(2, -3)$ has length 6	B1
	Height $2 - 1 = 1$	M1
	Area $\boxed{3}$	A1
		(3)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$(1+2i)^2 = -3+4i$, $(1+2i)^4 = -7-24i$	**B1**
	Substitutes: $-7-24i+p(-3+4i)+q(1+2i)+50=0$	**M1**
	Real: $43-3p+q=0$; imaginary: $-24+4p+2q=0$	**M1**
	$\boxed{p=11,\ q=-10}$	**A1**
		(4)
(b)	$1-2i$ is a root; factor z^2-2z+5	**B1**
	$z^4+11z^2-10z+50=(z^2-2z+5)(z^2+2z+10)$	**M1**
	Solves $z^2+2z+10=0$	**M1**
	$\boxed{1-2i,\ -1\pm 3i}$	**A1**
		(4)
(c)	Two conjugate pairs, each symmetric about the real axis	**B1**
	Correct relative positions	**B1**
		(2)
		Total for Question 7: 10 marks

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Question 8

10 marks

Part	Scheme	Marks
(a)	$\frac{(k+6\mathrm{i})(2+\mathrm{i})}{(2-\mathrm{i})(2+\mathrm{i})}$	M1
	$= \frac{2k-6+(k+12)\mathrm{i}}{5}$	A1
	$\boxed{a = \frac{2k-6}{5},\ b = \frac{k+12}{5}}$	A1
		(3)
(b)	$\arg w = \frac{\pi}{4}$: real and imaginary parts equal and positive	M1
	$2k-6 = k+12$	M1
	$\boxed{k = 18}$	A1
	Summer 2019 report: think geometrically about what the argument condition means.	(3)
(c)	$w = 6+6\mathrm{i}$: $\boxed{6\sqrt{2}}$	B1
		(1)
(d)	$\lvert 6+(6+c)\mathrm{i}\rvert = 10$: $36+(6+c)^2 = 100$	M1
	$(6+c)^2 = 64$	A1
	$\boxed{c = 2 \text{ or } c = -14}$	A1
		(3)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\|z\| = \boxed{2}$, $\arg z = \boxed{\frac{\pi}{3}}$	B1
	Both correct	B1
		(2)
(b)	$z^2 = 1 + 2\sqrt{3}\mathrm{i} - 3 = \boxed{-2 + 2\sqrt{3}\mathrm{i}}$	B1
	$\left\|z^2\right\| = \sqrt{4+12} = 4 = \|z\|^2$	M1
	$\arg\left(z^2\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3} = 2\arg z$ *	A1*
		(3)
(c)	$z^3 = z^2 \times z = (-2 + 2\sqrt{3}\mathrm{i})(1 + \sqrt{3}\mathrm{i})$	M1
	$= -2 - 2\sqrt{3}\mathrm{i} + 2\sqrt{3}\mathrm{i} - 6 = -8$ *	A1*
		(2)
(d)	$z = -2$ is a root, so $z^3 + 8 = (z+2)(z^2 - 2z + 4)$	M1
	$z^2 - 2z + 4 = 0$ gives $z = 1 \pm \sqrt{3}\mathrm{i}$	M1
	$\boxed{-2,\ 1 \pm \sqrt{3}\mathrm{i}}$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	Discriminant $4k^2 - 4(k^2+16) = -64$ $-64 < 0$ for all k, so the roots are not real * *Jan 2024 report: completing the square, $(z-k)^2+16=0$, is equally valid; take care with the constant.*	**M1** **A1*** **(2)**
(b)	$(z-k)^2 = -16$ $\boxed{z_1 = k+4\text{i},\ z_2 = k-4\text{i}}$	**M1** **A1** **(2)**
(c)	$\arg z_1 = \frac{\pi}{4}$: $k = 4$ (and $k > 0$) $\boxed{k = 4}$	**M1** **A1** **(2)**
(d)	$PQ = 8$; equilateral so $OP = 8$ $k^2 + 16 = 64$ (or angle $POQ = \frac{\pi}{3}$, so $\tan\frac{\pi}{6} = \frac{4}{k}$) $\boxed{k = 4\sqrt{3}}$	**M1** **M1** **A1** **(3)**
(e)	$\frac{1}{2} \times 8 \times 4\sqrt{3} = \boxed{16\sqrt{3}}$	**B1** **(1)**
		Total for Question 10: 10 marks

2 Chapter 2: Roots of Quadratic Equations

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$\boxed{\alpha+\beta=\frac{5}{2},\ \alpha\beta=2}$	B1 (1)
(b)	$\left(\frac{5}{2}\right)^2-2(2)$ $\boxed{\frac{9}{4}}$	M1 A1 (2)
(c)	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ $\frac{125}{8}-3(2)\left(\frac{5}{2}\right)$ $\boxed{\frac{5}{8}}$ *Oct 2021 report: the sum of cubes identity causes more errors than the sum of squares.*	M1 M1 A1 (3)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=-3,\ \alpha\beta=-7$: $\dfrac{-3}{-7}$ $\boxed{\frac{3}{7}}$	M1 A1 (2)
(b)	$\dfrac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=\dfrac{9+14}{49}$ $\boxed{\frac{23}{49}}$	M1 A1 (2)
(c)	$(\alpha-\beta)^2=9+28=37$ $\boxed{\sqrt{37}}$ (positive, since $\alpha>\beta$)	M1 A1 (2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$2(\alpha+\beta)+2=2(4)+2$	M1
	$\boxed{10}$	A1
		(2)
(b)	$4\alpha\beta+2(\alpha+\beta)+1=8+8+1$	M1
	$\boxed{17}$	A1
		(2)
(c)	$x^2-10x+17$	M1
	$\boxed{x^2-10x+17=0}$ $(p=-10,\ q=17)$	A1
	Oct 2021 report: a few omitted the "= 0".	
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=-\frac{2}{3},\ \alpha\beta=\frac{5}{3}$	B1
	$\frac{\alpha^2+\beta^2}{\alpha\beta}$ with $\alpha^2+\beta^2=\frac{4}{9}-\frac{10}{3}=-\frac{26}{9}$	M1
	$\frac{-26/9}{5/3}=-\frac{26}{15}$ *	A1*
		(3)
(b)	Product $\frac{\alpha}{\beta}\times\frac{\beta}{\alpha}=1$	B1
	$x^2+\frac{26}{15}x+1=0$	M1
	$\boxed{15x^2+26x+15=0}$	A1
		(3)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	Roots α and 2α: product $2\alpha^2 = 18$	M1
	$\alpha = \pm 3$	A1
	$k = \alpha + 2\alpha = 3\alpha$	M1
	$k = 9$ or $k = -9$	A1
		(4)
(b)	3 and 6	B1
		(1)
(c)	$k^2 = 72$: $k = \pm 6\sqrt{2}$	B1
		(1)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\alpha + \beta = -2$, $\alpha\beta = 6$	B1
		(1)
(b)	$(-2)^2 - 2(6)$	M1
	$= -8$ *	A1*
		(2)
(c)	$(-2)^3 - 3(6)(-2)$	M1
	28	A1
		(2)
(d)	Sum 28; product $(\alpha\beta)^3 = 216$	B1
	$x^2 - 28x + 216$	M1
	$x^2 - 28x + 216 = 0$	A1
		(3)
(e)	The discriminant $4 - 24 < 0$, so the roots are complex, and squares of non-real numbers can have negative real parts	B1
	$-1 \pm \sqrt{5}i$	B1
		(2)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=\frac{3}{2},\ \alpha\beta=2$	B1 (1)
(b)	$(\alpha+\beta)+\dfrac{\alpha+\beta}{\alpha\beta}$ $\frac{3}{2}+\frac{3}{4}$ $=\frac{9}{4}$ *	M1 M1 A1* (3)
(c)	$\alpha\beta+1+1+\dfrac{1}{\alpha\beta}$ $2+2+\frac{1}{2}$ $\boxed{\frac{9}{2}}$ *Oct 2021 report: many formed the sum correctly but not the product.*	M1 M1 A1 (3)
(d)	$x^2-\frac{9}{4}x+\frac{9}{2}=0$ Multiplies by 4 $\boxed{4x^2-9x+18=0}$	M1 M1 A1 (3)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=5$, $\alpha\beta=c$: $25-2c=13$ Correct equation $\boxed{c=6}$ *Summer 2024 report: the roots are α and β, not the coefficients.*	**M1** **A1** **A1** **(3)**
(b)	Sum 13; product $(\alpha\beta)^2=36$ Both correct $\boxed{x^2-13x+36=0}$	**M1** **A1** **A1** **(3)**
(c)	$(\alpha-\beta)^2=25-24=1$, so $\alpha-\beta=1$ $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$ $=1\times(13+6)$ $\boxed{19}$	**M1** **M1** **M1** **A1** **(4)**
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	Real coefficients, so $\beta=\alpha^*$ and $\alpha\alpha^*=\lvert\alpha\rvert^2$ $k=\alpha\beta=25$ $\boxed{k=25}$	**B1** **M1** **A1** **(3)**
(b)	$(x-2)^2=-21$ $\boxed{2\pm\sqrt{21}\,\mathrm{i}}$	**M1** **A1** **(2)**
(c)	Sum $\alpha^2+\beta^2=16-50=-34$; product $(\alpha\beta)^2=625$ Both correct $\boxed{x^2+34x+625=0}$	**M1** **A1** **A1** **(3)**
(d)	$\lvert\alpha^2\rvert=\lvert\alpha\rvert^2=25$ $\lvert\alpha^2\rvert^2=625=\alpha^2\beta^2$ *	**M1** **A1*** **(2)**
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$\alpha+\beta=k+2,\ \alpha\beta=2k+1$ $(k+2)^2-2(2k+1)$ $=k^2+4k+4-4k-2=k^2+2$ *	B1 M1 A1* (3)
(b)	$\boxed{2}$ (when $k=0$)	B1 (1)
(c)	$(\alpha-\beta)^2=(k+2)^2-4(2k+1)=k^2-4k$ $k^2-4k=4$ $\boxed{k=2\pm2\sqrt{2}}$	M1 M1 A1 (3)
(d)	$k^2-4k=0$ $\boxed{k=0 \text{ or } k=4}$	M1 A1 (2)
(e)	Sum $6+9=15$, product 54: $\boxed{x^2-15x+54=0}$	B1 (1)
		Total for Question 10: 10 marks

3 Chapter 3: Numerical Solutions of Equations

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$f(2) = -1, f(3) = 16$ Sign change and f is continuous, so a root lies in $[2,3]$ * *Jan 2020 report: a reason (sign change) and a conclusion are both needed.*	M1 A1* (2)
(b)	$f(2.5) = 5.625 > 0$ Root in [2, 2.5] $f(2.25) = 1.89 > 0$ [2, 2.25]	M1 A1 M1 A1 (4)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$f(3) = -1, f(4) = 3$ Sign change, f continuous, so a root in $[3,4]$ *	M1 A1* (2)
(b)	$\frac{\alpha - 3}{1} = \frac{4 - \alpha}{3}$ (similar triangles) $3\alpha - 9 = 4 - \alpha$ $\alpha \approx 3.25$	M1 A1 A1 (3)
(c)	$\alpha = 3.30$: an under-estimate	B1 (1)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$3x^2+2$	B1 (1)
(b)	$f(1.5) = -0.625$, $f'(1.5) = 8.75$ $x_1 = 1.5 - \frac{-0.625}{8.75}$ 1.571	M1 M1 A1 (3)
(c)	$x_2 = 1.5714 - \frac{0.0231}{9.408}$ 1.569	M1 A1 (2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\frac{3}{2}x^{\frac{1}{2}}$ $f'(x) = \frac{3}{2}x^{\frac{1}{2}} - \frac{2}{x^2}$ *Jan 2020 report: errors in differentiating fractional and negative powers.*	B1 B1 (2)
(b)	$f(3) = -0.1372$, $f'(3) = 2.3759$ $3 - \frac{-0.1372}{2.3759}$ 3.058	M1 M1 A1 (3)
(c)	$f(3) < 0$, $f(3.1) = 0.103 > 0$; f is continuous for $x > 0$, so a root lies between them	B1 (1)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	$1+1-8=-6<0$ and $8+2-8=2>0$ Sign change, continuous, so $\alpha \in [1,2]$ *	M1 A1* (2)
(b)	Evaluates at 1.825 and 1.835: -0.0966 and 0.0139 Sign change, continuous So $1.825 < \alpha < 1.835$ and $\alpha = 1.83$ to 2 d.p. *Jan 2020 report: "the root lies between 1.825 and 1.835" alone is not the required conclusion; f(1.82) and f(1.84) do not show 2 d.p. accuracy.*	M1 A1 A1 (3)
(c)	One value only shows how close f(1.83) is to 0; it does not show the root lies within ± 0.005 of 1.83	B1 (1)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	f(3.5) = -4.125, f(4) = 2 Sign change, continuous, so a root in $[3.5,4]$ *	M1 A1* (2)
(b)	$f'(x) = 3x^2 - 8x$ $f'(3.5) = 8.75$ $3.5 - \dfrac{-4.125}{8.75}$ 3.971 *Jan 2020 report: $x_0 + \frac{f}{f'}$ is the wrong formula.*	B1 M1 M1 A1 (4)
(c)	f(3.865) = -0.0167, f(3.875) = 0.123 Sign change, continuous So $\alpha = 3.87$ to 2 d.p. *	M1 A1 A1* (3)
(d)	$f'\left(\frac{8}{3}\right) = 0$, so the formula divides by zero (the tangent is horizontal)	B1 (1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$f(2) = -1, f(3) = 2$ Sign change, continuous, so a root in $[2,3]$ *	**M1** **A1*** **(2)**
(b)	$f(2.5) = 0.157 > 0$, root in [2, 2.5] $f(2.25) = -0.493 < 0$ $\boxed{[2.25, 2.5]}$	**M1** **M1** **A1** **(3)**
(c)	$\frac{\alpha - 2}{1} = \frac{3 - \alpha}{2}$ $3\alpha = 7$ $\boxed{2.333}$	**M1** **A1** **A1** **(3)**
(d)	Bisection: the interval contains 2.445 but only locates it within 0.25 Interpolation: 2.333 is out by about 0.11, because the curve is not close to a straight line on $[2,3]$	**B1** **B1** **(2)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$g(1) = 1 - 3 = -2$ $g(3) = 3 + 3 = 6$: opposite signs *	**M1** **A1*** **(2)**
(b)	$x(x-2) + 3 = 0$ $x^2 - 2x + 3 = 0$ Discriminant $4 - 12 < 0$: no real roots *	**M1** **A1** **A1*** **(3)**
(c)	g is not continuous on $[1, 3]$ (it has an asymptote at $x = 2$), so a sign change does not imply a root	**B1** **(1)**
(d)	$h'(x) = 3x^2 - 3$ $h'(1) = 0$, so the formula involves division by zero	**M1** **A1** **(2)**
(e)	$0.5 - \frac{-0.375}{-2.25}$ $\boxed{0.333}$	**M1** **A1** **(2)**
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	$f'(x) = 2x$	**B1**
	$x_{n+1} = x_n - \dfrac{x_n^2 - 10}{2x_n}$	**M1**
	$= \dfrac{x_n^2 + 10}{2x_n} = \dfrac{1}{2}\left(x_n + \dfrac{10}{x_n}\right)$ *	**A1***
		(3)
(b)	$x_1 = \frac{1}{2}\left(3 + \frac{10}{3}\right)$	**M1**
	$\boxed{x_1 = 3.16667}$	**A1**
	$\boxed{x_2 = 3.16228}$	**A1**
		(3)
(c)	The tangent at $(x_n, f(x_n))$ has gradient $f'(x_n)$	**B1**
	It meets the x-axis where $0 - f(x_n) = f'(x_n)(x - x_n)$, which gives x_{n+1}	**B1**
		(2)
(d)	$\sqrt{10} = 3.1622777$; error $\boxed{3.0 \times 10^{-6}}$	**B1**
	Very fast: the error roughly squares at each step	**B1**
		(2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$8-2k+1=1$: $\boxed{k=4}$	B1 (1)
(b)	$f(-3)=-14$, $f(-2)=1$, $f(0)=1$, $f(1)=-2$, $f(2)=1$ Three sign changes; f is continuous $\boxed{[-3,-2],[0,1],[1,2]}$	M1 A1 A1 (3)
(c)	$f(1.8)=-0.368$, $f'(1.8)=5.72$ $\boxed{x_1=1.8643}$ Second iteration with f(1.8643) and f′(1.8643) $\boxed{x_2=1.8608}$	M1 A1 M1 A1 (4)
(d)	$\frac{\alpha-0}{1}=\frac{1-\alpha}{2}$ $\boxed{\frac{1}{3}}$ (0.333)	M1 A1 (2)
		Total for Question 10: 10 marks

4 Chapter 4: Coordinate Systems

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$S(3, 0)$	B1
	$x = -3$	B1
		(2)
(b)	$x = \frac{36}{12} = 3$; $PS = 3 + 3$	M1
	6	A1
		(2)
(c)	$x = 3t^2$, $y = 6t$	B1
		(1)
(d)	$x + 3 = 15$, $x = 12$: (12, 12) and (12, −12)	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$t = \frac{y}{10}$, so $x = 5\left(\frac{y}{10}\right)^2$	M1
	C_1: $y^2 = 20x$	A1
	C_2: $xy = 50$	B1
		(3)
(b)	$x = \frac{50}{y}$: $y^2 = \frac{1000}{y}$	M1
	$y^3 = 1000$, $y = 10$	A1
	(5, 10)	A1
	Jan 2021 report: answers with no algebra score no marks.	
		(3)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = \frac{2a}{2at} = \frac{1}{t}$	M1
	Correct	A1
	$y - 2at = \frac{1}{t}\left(x - at^2\right)$	M1
	$ty - 2at^2 = x - at^2$, so $ty = x + at^2$ *	A1*
	Jan 2021 report: t is not the gradient; differentiate.	(4)
(b)	$a = 3$, $t = 2$: $2y = x + 12$	M1
	$\boxed{2y = x + 12}$	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$y = 9x^{-1}$, $\frac{dy}{dx} = -\frac{9}{x^2} = -\frac{1}{t^2}$ at P	M1
	Correct	A1
	$y - \frac{3}{t} = -\frac{1}{t^2}(x - 3t)$	M1
	$t^2y - 3t = -x + 3t$, so $x + t^2y = 6t$ *	A1*
		(4)
(b)	$x + 9y = 18$: $A(18, 0)$, $B(0, 2)$	M1
	Area $\boxed{18}$	A1
		(2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$S(2, 0)$, directrix $x = -2$	B1
	PS equals the distance from P to the directrix, $2t^2 + 2$	M1
	$PS = 2\left(t^2 + 1\right)$ *	A1*
		(3)
(b)	$2(t^2 + 1) = 10$, $t^2 = 4$	M1
	$t = \pm 2$	A1
	$\boxed{(8, 8) \text{ and } (8, -8)}$	A1
		(3)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = -\frac{16}{x^2} = -\frac{1}{t^2}$	M1
	Normal gradient t^2	M1
	$y - \frac{4}{t} = t^2(x - 4t)$	M1
	Multiplies by t: $ty - 4 = t^3x - 4t^4$	A1
	$t^3x - ty = 4t^4 - 4$ *	A1*
		(5)
(b)	$t = 2$: $8x - 2y = 60$, so $y = 4x - 30$	B1
	$x(4x - 30) = 16$: $2x^2 - 15x - 8 = 0$	M1
	$\boxed{B\left(-\frac{1}{2}, -32\right)}$	A1
	Jan 2020 report: $x = -\frac{1}{2}$ is not the parameter t; B is not $(-2, -8)$.	
		(3)
(c)	$AB = \sqrt{8.5^2 + 34^2}$	M1
	$\boxed{\frac{17}{2}\sqrt{17}}$	A1
		(2)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$x = \frac{32}{y}$: $y^2 = \frac{512}{y}$	M1
	$y^3 = 512$ has one real solution, $y = 8$	A1
	$\boxed{P(4, 8)}$	A1
		(3)
(b)	$2y\frac{dy}{dx} = 16$, gradient $\frac{16}{16} = 1$ at P	M1
	Normal gradient -1: $y - 8 = -(x - 4)$	M1
	$\boxed{x + y = 12}$	A1
	Jan 2021 report: the answer must be in the form asked for; use the parabola, not the hyperbola.	
		(3)
(c)	$y^2 = 16(12 - y)$: $y^2 + 16y - 192 = 0$	M1
	$(y + 24)(y - 8) = 0$	M1
	$\boxed{Q(36, -24)}$	A1
	Jan 2021 report: show the quadratic and its solution, not a calculator answer alone.	
		(3)
(d)	$\sqrt{32^2 + 32^2} = \boxed{32\sqrt{2}}$	B1
		(1)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$S(2, 0)$; gradient $PS = \dfrac{4p}{2p^2-2}$, gradient $QS = \dfrac{4q}{2q^2-2}$	**M1**
	P, S, Q collinear: $\dfrac{2p}{p^2-1} = \dfrac{2q}{q^2-1}$	**M1**
	$pq^2 - p = p^2q - q$, so $(q-p)(pq+1) = 0$	**A1**
	$p \neq q$, so $pq = -1$ *	**A1***
		(4)
(b)	Tangent gradients $\frac{1}{p}$ and $\frac{1}{q}$	**M1**
	Product $\frac{1}{pq} = -1$: perpendicular *	**A1***
		(2)
(c)	Tangents $py = x + 2p^2$ and $qy = x + 2q^2$	**B1**
	Subtracts: $(p-q)y = 2(p^2-q^2)$, $y = 2(p+q)$	**M1**
	$x = py - 2p^2 = 2pq$	**M1**
	$x = -2$: on the directrix $x = -2$ *	**A1***
		(4)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{\frac{4}{q}-\frac{4}{p}}{4q-4p}$	M1
	$=\frac{\frac{p-q}{pq}}{q-p}$	M1
	$=-\frac{1}{pq}$ *	A1*
		(3)
(b)	$x+p^2y=8p$ and $x+q^2y=8q$	B1
	Subtracts: $(p^2-q^2)y=8(p-q)$	M1
	$y=\frac{8}{p+q}$	A1
	$R\left(\frac{8pq}{p+q},\frac{8}{p+q}\right)$	A1
		(4)
(c)	$P(4,4)$, $Q\left(12,\frac{4}{3}\right)$, $R(6,2)$	B1
	$\frac{1}{2}\left\|4\left(\frac{4}{3}-2\right)+12(2-4)+6\left(4-\frac{4}{3}\right)\right\|$	M1
	$\frac{16}{3}$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	Tangent gradient $\frac{1}{t}$, so normal gradient $-t$	**M1**
	$y-2at=-t\left(x-at^2\right)$	**M1**
	$y+tx=2at+at^3$ *	**A1***
		(3)
(b)	Substitutes $\left(as^2,2as\right)$: $2as+tas^2=2at+at^3$	**M1**
	$t\left(s^2-t^2\right)+2(s-t)=0$	**M1**
	$(s-t)(t(s+t)+2)=0$	**A1**
	$s\neq t$, so $s=-t-\frac{2}{t}$ *	**A1***
		(4)
(c)	$s=-3$: $\boxed{Q(9,-6)}$	**B1**
	$P(1,2)$: area $\frac{1}{2}\lvert1(-6)-9(2)\rvert$	**M1**
	$\boxed{12}$	**A1**
		(3)
		Total for Question 10: 10 marks

5 Chapter 5: Matrices

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	2×2 result; row by column	M1
	Two entries correct	A1
	$\begin{pmatrix} 2 & -1 \\ 10 & 4 \end{pmatrix}$	A1
		(3)
(b)	3×3	B1
		(1)
(c)	$\mathbf{A}$ is 2×3: the number of columns (3) is not equal to the number of rows (2)	B1
		(1)
(d)	$8 - (-10) = 18$	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$k(k-5) - 6 = 0$	M1
	$k^2 - 5k - 6 = (k-6)(k+1)$ *	A1*
	$k = 6$ or $k = -1$	A1
	Jan 2021 report: dealing with the minus signs in the determinant is the common error.	
		(3)
(b)	$\det \mathbf{M} = 2(-3) - 6 = -12$	B1
	$\frac{1}{-12}\begin{pmatrix} -3 & -3 \\ -2 & 2 \end{pmatrix}$	M1
	$\frac{1}{12}\begin{pmatrix} 3 & 3 \\ 2 & -2 \end{pmatrix}$	A1
		(3)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$\det \mathbf{A} = 6 - 5 = 1$ $\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$	M1 A1 (2)
(b)	$\mathbf{X} = \mathbf{BA}^{-1}$ $\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$ $\begin{pmatrix} 3 & -1 \\ 7 & -2 \end{pmatrix}$ *Jan 2021 report: evaluating $\mathbf{A}^{-1}\mathbf{B}$ is a common error.*	M1 M1 A1 (3)
(c)	Post-multiplying $\mathbf{XA} = \mathbf{B}$ by $\mathbf{A}^{-1}$ gives $\mathbf{X} = \mathbf{BA}^{-1}$; matrix multiplication is not commutative	B1 (1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\begin{pmatrix} 3 & -2 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 19 \end{pmatrix}$	B1 (1)
(b)	$\det = 12 + 10 = 22$ $\frac{1}{22} \begin{pmatrix} 4 & 2 \\ -5 & 3 \end{pmatrix}$	M1 A1 (2)
(c)	$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{22} \begin{pmatrix} 4 & 2 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 7 \\ 19 \end{pmatrix}$ $= \frac{1}{22} \begin{pmatrix} 66 \\ 22 \end{pmatrix}$ $x = 3, y = 1$	M1 A1 A1 (3)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$\mathbf{A}^2 = \begin{pmatrix} a^2 + 2 & 2a + 2b \\ a + b & 2 + b^2 \end{pmatrix}$ $a^2 + 2 = 11, a + b = 4, b^2 + 2 = 3$ $a = \pm 3, b = \pm 1$; only $a = 3, b = 1$ satisfies $a + b = 4$ $a = 3, b = 1$	M1 A1 M1 A1 (4)
(b)	$\mathbf{AB} = \begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}, \mathbf{BA} = \begin{pmatrix} 1 & 1 \\ 3 & 2 \end{pmatrix}$ Different, so $\mathbf{AB} \neq \mathbf{BA}$ *	M1 A1* (2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$p(p+3)-10=0$	M1
	$p^2+3p-10=(p+5)(p-2)=0$	A1
	$p=2$ or $p=-5$	A1
		(3)
(b)	$\det\mathbf{A}=18-10=8$	B1
	$\frac{1}{8}\begin{pmatrix}6&-5\\-2&3\end{pmatrix}$	M1
	$\frac{1}{8}\begin{pmatrix}6&-5\\-2&3\end{pmatrix}$	A1
	Jan 2020 report: $\frac{1}{\det}\mathbf{A}$ is not the inverse; substitute $p=3$ throughout.	
		(3)
(c)	$\mathbf{B}=\mathbf{A}^{-1}\begin{pmatrix}-2&11\\-4&10\end{pmatrix}$	M1
	$\frac{1}{8}\begin{pmatrix}8&16\\-8&8\end{pmatrix}$	A1
	$\begin{pmatrix}1&2\\-1&1\end{pmatrix}$	A1
		(3)
(d)	$\det(\mathbf{AB})=-20+44=24=8\times3$ *	B1
		(1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$(\mathbf{AB})\left(\mathbf{B}^{-1}\mathbf{A}^{-1}\right) = \mathbf{A}\left(\mathbf{BB}^{-1}\right)\mathbf{A}^{-1}$ $= \mathbf{AIA}^{-1} = \mathbf{AA}^{-1} = \mathbf{I}$ *	M1 A1* (2)
(b)	$\mathbf{A}^{-1} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$ $\mathbf{B}^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$	B1 B1 (2)
(c)	$\mathbf{B}^{-1}\mathbf{A}^{-1}$ in this order $\begin{pmatrix} 10 & -3 \\ -13 & 4 \end{pmatrix}$ $\mathbf{AB} = \begin{pmatrix} 4 & 3 \\ 13 & 10 \end{pmatrix}$, determinant 1, inverse agrees	M1 A1 B1 (3)
(d)	$\begin{pmatrix} x \\ y \end{pmatrix} = (\mathbf{AB})^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\begin{pmatrix} 10-6 \\ -13+8 \end{pmatrix}$ $x = 4,\ y = -5$	M1 M1 A1 (3)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$\mathbf{M}^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$	B1
	$\mathbf{M}^3 = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$	B1
	$\boxed{\mathbf{M}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}}$	B1
		(3)
(b)	$\det \mathbf{M} = 1$	M1
	$\boxed{\mathbf{M}^{-1} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}}$	A1
	$n = -1$ gives $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$: it fits *	A1*
		(3)
(c)	$\mathbf{X} = \mathbf{M}^{-1} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$	M1
	Correct order	M1
	$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$	A1
	$\boxed{\begin{pmatrix} 2 & -6 \\ 0 & 3 \end{pmatrix}}$	A1
		(4)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\mathbf{A}^2 = \begin{pmatrix} 14 & 10 \\ -5 & -1 \end{pmatrix}$	B1
	$\begin{pmatrix} 14 & 10 \\ -5 & -1 \end{pmatrix} - \begin{pmatrix} 20 & 10 \\ -5 & 5 \end{pmatrix} + \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$	M1
	$= \mathbf{0}$ *	A1*
		(3)
(b)	$\mathbf{A}^2 - 5\mathbf{A} = -6\mathbf{I}$, so $\mathbf{A}(5\mathbf{I} - \mathbf{A}) = 6\mathbf{I}$	M1
	Multiplies by $\mathbf{A}^{-1}$: $5\mathbf{I} - \mathbf{A} = 6\mathbf{A}^{-1}$	M1
	$\mathbf{A}^{-1} = \frac{1}{6}(5\mathbf{I} - \mathbf{A})$ *	A1*
		(3)
(c)	$\frac{1}{6}\begin{pmatrix} 1 & -2 \\ 1 & 4 \end{pmatrix}$	B1
	$\det \mathbf{A} = 6$ and the formula gives the same	B1
		(2)
(d)	$\mathbf{A}^3 = 5\mathbf{A}^2 - 6\mathbf{A} = 5(5\mathbf{A} - 6\mathbf{I}) - 6\mathbf{A}$	M1
	$19\mathbf{A} - 30\mathbf{I}$	A1
		(2)
	Total for Question 9: 10 marks	

Question 10

10 marks

Part	Scheme	Marks
(a)	$k^2 - 16 = 0$	M1
	$\boxed{k = \pm 4}$	A1
		(2)
(b)	$\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 8 & 2 \end{pmatrix}$, det $= -12$	B1
	$\mathbf{M}^{-1} = -\frac{1}{12}\begin{pmatrix} 2 & -2 \\ -8 & 2 \end{pmatrix}$	M1
	$-\frac{1}{12}\begin{pmatrix} 8-16 \\ -32+16 \end{pmatrix}$	M1
	$\boxed{x = \frac{2}{3},\ y = \frac{4}{3}}$	A1
		(4)
(c)	$k = 4$: $4x + 2y = 4$ and $8x + 4y = 8$	M1
	The second is twice the first: the same line, so $\boxed{\text{infinitely many solutions}}$	A1
	$k = -4$: $-4x + 2y = 4$ and $8x - 4y = 8$, i.e. $2x - y = -2$ and $2x - y = 2$	M1
	Parallel lines, so $\boxed{\text{no solutions}}$	A1
		(4)
		Total for Question 10: 10 marks

6 Chapter 6: Transformations Using Matrices

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1 (1)
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 (1)
(c)	$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1 (1)
(d)	$\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$	B1 (1)
(e)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$	B1 (1)
(f)	$(-5, -2)$	B1 (1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	Rotation about O 90° anticlockwise *Jan 2021 report: about a quarter of candidates described this matrix as a reflection.*	M1 A1 (2)
(b)	Reflection in the y-axis	B1 (1)
(c)	Rotation about O, $\cos\theta = \frac{1}{2}$, $\sin\theta = \frac{\sqrt{3}}{2}$ 60° anticlockwise	M1 A1 (2)
(d)	Enlargement, centre O, scale factor -3 (or rotation of 180° and enlargement scale factor 3)	B1 (1)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$12-(-10) = 22$ *Jan 2021 report: "$8+15$"-type sign errors in the determinant.*	B1 (1)
(b)	$22 \times 6 = 132$	B1 (1)
(c)	$(0,0)$; $(2,0) \rightarrow (6, 4)$ $(0,3) \rightarrow (-15, 12)$	B1 B1 (2)
(d)	Image area $\frac{1}{2}\lvert 6(12)-(-15)(4)\rvert = 66$ Original area 3, and $66 = 22 \times 3$ *	M1 A1* (2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	$\mathbf{P} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1
	$\mathbf{QP}$ (first transformation on the right)	M1
	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1
	$\boxed{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}$	A1
	Jan 2020 report: multiplying in the wrong order.	(4)
(b)	Reflection	M1
	$\boxed{\text{in the line } y = x}$	A1
		(2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	$\det\mathbf{M} = 1$, $\mathbf{M}^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$	M1
	$\mathbf{M}^{-1} \begin{pmatrix} 5 \\ 8 \end{pmatrix}$	M1
	$\boxed{A(2, 1)}$	A1
		(3)
(b)	$\det\mathbf{M} = 1$, so the area scale factor is 1	B1
		(1)
(c)	$\mathbf{M} \begin{pmatrix} t \\ t \end{pmatrix} = \begin{pmatrix} 3t \\ 5t \end{pmatrix}$	M1
	$\boxed{y = \frac{5}{3}x}$	A1
		(2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$\mathbf{A}\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}2p+2\\1\end{pmatrix}$ (correct order) $2p+2=12$: $\boxed{p=5}$	M1 A1 (2)
(b)	Base $OS = 7$, height = x-coordinate of $R = 2$ $\boxed{7}$ *Jan 2020 report: using the y-coordinate of R as the height gave $\frac{7}{2}$.*	M1 A1 (2)
(c)	$\det\mathbf{A} = 15+2 = 17$ $17\times 7 = \boxed{119}$ *Jan 2020 report: working from the images directly, ignoring "hence", scored no credit.*	M1 A1 (2)
(d)	$\boxed{(14, 21)}$	B1 (1)
(e)	$\begin{pmatrix}0&-1\\1&0\end{pmatrix}\mathbf{A}$ $\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}5&2\\-1&3\end{pmatrix}$ $\boxed{\begin{pmatrix}1&-3\\5&2\end{pmatrix}}$	M1 M1 A1 (3)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$\det \mathbf{A} = 3 + 2 = 5$ $5 \times 4 = \boxed{20}$ *Jan 2021 report: dividing by the determinant or squaring it is wrong.*	**M1** **A1** **(2)**
(b)	Rotation about O $\boxed{90^\circ \text{ clockwise}}$ (or 270° anticlockwise)	**M1** **A1** **(2)**
(c)	$\mathbf{C} = \mathbf{B}\mathbf{A}^{-1}$ $\mathbf{A}^{-1} = \frac{1}{5}\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ $\frac{1}{5}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$ $\boxed{\frac{1}{5}\begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}}$ *Jan 2021 report: $\mathbf{A}^{-1}\mathbf{B}$ is the common error.*	**M1** **M1** **M1** **A1** **(4)**
(d)	The transformation $\mathbf{A}$ followed by $\mathbf{C}$ has the same effect as the rotation $\mathbf{B}$	**B1** **B1** **(2)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$\mathbf{M} = \begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}$	B1 (1)
(b)	$\mathbf{M}^2 = \begin{pmatrix} 0.36+0.64 & 0.48-0.48 \\ 0.48-0.48 & 0.64+0.36 \end{pmatrix}$ $= \mathbf{I}$ * Applying T twice returns every point to where it started	M1 A1* B1 (3)
(c)	$\det\mathbf{M} = -0.36-0.64 = -1$ Areas are unchanged and the orientation is reversed (a reflection)	B1 B1 (2)
(d)	$\begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $0.6x+0.8y = x$ $0.8y = 0.4x$ $y = \frac{1}{2}x$	M1 A1 M1 A1 (4)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	B1 (1)
(b)	$2\mathbf{R} = \begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & 1 \end{pmatrix}$ * Rotation through 60° anticlockwise about O combined with an enlargement, centre O, scale factor 2	B1 B1 B1 (3)
(c)	$\mathbf{M}^2 = \begin{pmatrix} -2 & -2\sqrt{3} \\ 2\sqrt{3} & -2 \end{pmatrix}$ $\mathbf{M}^3 = \begin{pmatrix} -8 & 0 \\ 0 & -8 \end{pmatrix} = -8\mathbf{I}$ * Enlargement, centre O, scale factor -8 (rotation 180° with enlargement 8)	M1 A1* B1 (3)
(d)	64	B1 (1)
(e)	Rotation of $60n°$ must be a multiple of 360°: $n = 6$ $k = 64$	M1 A1 (2)
		Total for Question 9: 10 marks

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Question 10

10 marks

Part	Scheme	Marks
(a)	$\mathbf{T}$ is singular: $k^2 - 2 = 0$	M1
	$\boxed{k = \pm\sqrt{2}}$	A1
		(2)
(b)	$\mathbf{T}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \sqrt{2}x + 2y \\ x + \sqrt{2}y \end{pmatrix}$	M1
	The first coordinate is $\sqrt{2}$ times the second	M1
	$\boxed{y = \frac{x}{\sqrt{2}}}$ (or $x = \sqrt{2}y$)	A1
		(3)
(c)	$\det \mathbf{T} = 9 - 2 = 7$; original area 1	M1
	$\boxed{7}$	A1
		(2)
(d)	$\mathbf{T}^{-1} = \frac{1}{7}\begin{pmatrix} 3 & -2 \\ -1 & 3 \end{pmatrix}$	M1
	$\frac{1}{7}\begin{pmatrix} 24 - 10 \\ -8 + 15 \end{pmatrix}$	M1
	$\boxed{(2, 1)}$	A1
		(3)
	Total for Question 10: 10 marks	

7 Chapter 7: Series

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	$4\sum r - \sum 1$	M1
	$4 \times \frac{1}{2}n(n+1) - n$	A1
	$= 2n^2 + 2n - n = n(2n+1)$ *	A1*
	Jan 2021 report: $\sum_{r=1}^{n} 1 = n$, not 1.	(3)
(b)	$20 \times 41 =$ 820	B1
		(1)
(c)	$2n^2 + n - 465 = 0$: $(n-15)(2n+31) = 0$	M1
	$n = 15$	A1
		(2)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$\sum r^2 + \sum r$	M1
	$\frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1)$	A1
	$= \frac{1}{6}n(n+1)(2n+1+3)$	M1
	$= \frac{1}{6}n(n+1)(2n+4) = \frac{1}{3}n(n+1)(n+2)$ *	A1*
	Jan 2020 report: factorising out the common factor first is the most successful route.	(4)
(b)	$\frac{1}{3}(10)(11)(12)$	M1
	440	A1
		(2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	$f(30) - f(9)$, with $f(n) = \frac{1}{6}n(n+1)(2n+1)$	M1
	$9455 - 285$	A1
	$\boxed{9170}$	A1
	Jan 2020 report: $f(30) - f(10)$ *is the common error.*	(3)
(b)	$\frac{1}{4}\left(30^2\right)\left(31^2\right) - \frac{1}{4}\left(9^2\right)\left(10^2\right)$	M1
	$216\,225 - 2025$	A1
	$\boxed{214\,200}$	A1
		(3)
	Total for Question 3: 6 marks	

Question 4

6 marks

Part	Scheme	Marks
(a)	$\frac{1}{4}n^2(n+1)^2 - \frac{1}{2}n(n+1)$	M1
	$= \frac{1}{4}n(n+1)\left(n(n+1) - 2\right)$	M1
	$= \frac{1}{4}n(n+1)\left(n^2 + n - 2\right)$	A1
	$= \frac{1}{4}(n-1)n(n+1)(n+2)$ *	A1*
		(4)
(b)	$\frac{1}{4}(4)(5)(6)(7) = \boxed{210}$	B1
	$0 + 6 + 24 + 60 + 120 = 210$	B1
		(2)
	Total for Question 4: 6 marks	

Question 5

6 marks

Part	Scheme	Marks
(a)	$3\sum r^2 - \sum r + \sum 2$	M1
	$\frac{1}{2}n(n+1)(2n+1) - \frac{1}{2}n(n+1) + 2n$	A1
	$= \frac{1}{2}n(n+1)(2n) + 2n = n^2(n+1) + 2n$	M1
	$= n\left(n^2+n+2\right)$ *	A1*
	Jan 2021 report: the constant term sums to $2n$.	(4)
(b)	$2n\left(4n^2+2n+2\right) - n\left(n^2+n+2\right)$	M1
	$= n\left(7n^2+3n+2\right)$ *	A1*
		(2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$2\sum r^3 - \sum r^2$	M1
	$\frac{1}{2}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1)$	A1
	$= \frac{1}{6}n(n+1)\left(3n(n+1) - (2n+1)\right)$	M1
	$= \frac{1}{6}n(n+1)\left(3n^2+3n-2n-1\right)$	A1
	$= \frac{1}{6}n(n+1)\left(3n^2+n-1\right)$ *	A1*
		(5)
(b)	$f(20) - f(10)$	M1
	$85\,330 - 5665$	A1
	$\boxed{79\,665}$	A1
		(3)
(c)	$f(11) = 8206 < 10\,000$, $f(12) = 11\,518 > 10\,000$	M1
	$\boxed{n = 12}$	A1
		(2)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$\sum\left(r^2+5r+4\right) = \frac{1}{6}n(n+1)(2n+1)+\frac{5}{2}n(n+1)+4n$	**M1**
	$= \frac{1}{6}n[(n+1)(2n+1)+15(n+1)+24]$	**M1**
	$= \frac{1}{6}n\left(2n^2+18n+40\right)$	**A1**
	$= \frac{1}{3}n(n+4)(n+5)$ *	**A1***
		(4)
(b)	$\frac{1}{3}(2n)(2n+4)(2n+5)-\frac{1}{3}n(n+4)(n+5)$	**M1**
	$= \frac{1}{3}n[4(n+2)(2n+5)-(n+4)(n+5)]$	**M1**
	$= \frac{1}{3}n\left(7n^2+27n+20\right)$	**A1**
	$= \frac{1}{3}n(n+1)(7n+20)$ *	**A1***
	Jan 2021 report: substitute $2n$ into part (a) rather than starting again, and subtract, not add.	
		(4)
(c)	$n = 10$: $\frac{1}{3}(10)(11)(90)$	**M1**
	3300	**A1**
		(2)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$\left(\frac{1}{2}n(n+1)\right)^2 = \frac{1}{4}n^2(n+1)^2$	M1
	which equals $\sum r^3$ *	A1*
		(2)
(b)	$\frac{1}{2}n(n+1) = 210$: $n^2+n-420 = 0$	M1
	$(n+21)(n-20) = 0$	M1
	$\boxed{n = 20}$	A1
		(3)
(c)	$210^2 = \boxed{44\,100}$	B1
		(1)
(d)	Odd cubes = all cubes to 40 − even cubes to 40	M1
	Even cubes: $\sum(2r)^3 = 8\sum_{r=1}^{20} r^3 = 352\,800$	M1
	$\sum_{r=1}^{40} r^3 = 820^2 = 672\,400$	A1
	$\boxed{319\,600}$	A1
		(4)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{1}{6}n(n+1)(2n+1) + \frac{k}{2}n(n+1)$	M1
	Factor $\frac{1}{6}n(n+1)$	M1
	$\frac{1}{6}n(n+1)(2n+1+3k)$	A1
	Correct with no errors *	A1*
		(4)
(b)	$\frac{1}{6}(10)(11)(21+3k) = 660$	M1
	$21+3k = 36$	A1
	$\boxed{k = 5}$	A1
		(3)
(c)	f(20) − f(10), f$(n) = \frac{1}{6}n(n+1)(2n+16)$	M1
	$3920 - 660$	A1
	$\boxed{3260}$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$\sum\left(r^3+3r^2+2r\right)$	M1
	$\frac{1}{4}n^2(n+1)^2+\frac{1}{2}n(n+1)(2n+1)+n(n+1)$	A1
	$=\frac{1}{4}n(n+1)[n(n+1)+2(2n+1)+4]$	M1
	$=\frac{1}{4}n(n+1)\left(n^2+5n+6\right)$	A1
	$=\frac{1}{4}n(n+1)(n+2)(n+3)$ *	A1*
		(5)
(b)	$n(n+1)(n+2)(n+3)=3024=6\times7\times8\times9$	M1
	$\boxed{n=6}$: every term is positive, so the sum increases with n and can equal 756 only once	A1
		(2)
(c)	$\boxed{\frac{1}{5}n(n+1)(n+2)(n+3)(n+4)}$	B1
	$n=1$: $24=\frac{1}{5}(120)$	B1
	$n=2$: $24+120=144=\frac{1}{5}(720)$	B1
		(3)
		Total for Question 10: 10 marks

8 Chapter 8: Proof

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
	$n = 1$: LHS = 1, RHS = $\frac{1}{3}(1)(1)(3) = 1$: true	B1
	Assume true for $n = k$; adds $(2k+1)^2$: $\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$	M1
	$= \frac{1}{3}(2k+1)[k(2k-1)+3(2k+1)]$	M1
	$= \frac{1}{3}(2k+1)\left(2k^2+5k+3\right) = \frac{1}{3}(2k+1)(k+1)(2k+3)$	A1
	$= \frac{1}{3}(k+1)(2(k+1)-1)(2(k+1)+1)$: the formula with $n = k+1$	A1
	If true for $n = k$ then true for $n = k+1$; true for $n = 1$; so true for all $n \in \mathbb{Z}^+$ *	A1*
		(6)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
	Let f(n) = $6^n + 4$. f(1) = 10, which is divisible by 5	B1
	Assume f(k) divisible by 5; f($k+1$) = $6^{k+1} + 4$	M1
	f($k+1$) − f(k) = $6^{k+1} - 6^k = 5 \times 6^k$	M1
	f($k+1$) = f(k) + 5×6^k	A1
	Both terms are divisible by 5, so f($k+1$) is	A1
	If true for $n = k$ then true for $n = k+1$; true for $n = 1$; so true for all $n \in \mathbb{Z}^+$ *	A1*
	Summer 2024 report: make f($k+1$) *the subject, and show the* $n = 1$ *case is divisible by 5.*	
		(6)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
	$n=1$: $\begin{pmatrix} 3 & 0 \\ 3-1 & 1 \end{pmatrix} = \mathbf{M}$: true	B1
	Assume true for $n=k$: $\mathbf{M}^{k+1} = \begin{pmatrix} 3^k & 0 \\ 3^k-1 & 1 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3^{k+1}-3+2 & 1 \end{pmatrix}$	A1
	$= \begin{pmatrix} 3^{k+1} & 0 \\ 3^{k+1}-1 & 1 \end{pmatrix}$	A1
	This is the formula with $n=k+1$	A1
	Full conclusion *	A1*
		(6)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
	$n=1$: $3^0+1=2=u_1$: true	B1
	Assume $u_k = 3^{k-1}+1$	M1
	$u_{k+1} = 3\left(3^{k-1}+1\right)-2$	M1
	$= 3^k+3-2 = 3^k+1$	A1
	$= 3^{(k+1)-1}+1$: the formula with $n=k+1$	A1
	Full conclusion *	A1*
		(6)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	Assume $1+\cdots+k=\frac{1}{2}\left(k^2+k+2\right)$; add $k+1$ $\frac{1}{2}\left(k^2+k+2+2k+2\right)=\frac{1}{2}\left(k^2+3k+4\right)$ $=\frac{1}{2}\left((k+1)^2+(k+1)+2\right)$ *	**M1** **A1** **A1*** **(3)**
(b)	$n=1$: LHS = 1, RHS = $\frac{1}{2}(4)=2$ Not equal, so the claim is false *	**M1** **A1*** **(2)**
(c)	The inductive step only shows true for k implies true for $k+1$; with no true basis case, nothing starts the chain	**B1** **(1)**
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$n = 1: \begin{pmatrix} 2 & 1-2 \\ 0 & 1 \end{pmatrix} = \mathbf{M}$	B1
	$\mathbf{M}^{k+1} = \begin{pmatrix} 2^k & 1-2^k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ 0 & 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 2^{k+1} & -2^k+1-2^k \\ 0 & 1 \end{pmatrix}$	A1
	$= \begin{pmatrix} 2^{k+1} & 1-2^{k+1} \\ 0 & 1 \end{pmatrix}$	A1
	Full conclusion *	A1*
	Summer 2024 report: show the $n = 1$ substitution; give the full conclusion.	(5)
(b)	$\begin{pmatrix} 32 & -31 \\ 0 & 1 \end{pmatrix}$	B1
	Summer 2024 report: use part (a), not repeated multiplication.	(1)
(c)	$\det \mathbf{B} = \det \mathbf{N} \det \mathbf{T} = 32 \times 5 = 160$	M1
	Area of $R = \frac{480}{160}$	M1
	3	A1
	Summer 2024 report: multiplying the image area by the determinant instead of dividing was the most common error.	(3)
(d)	$n = 0: \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I} = \mathbf{M}^0$ *	B1
		(1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	$f(k+1) = 25 \times 5^{2k} - 1 = 25(f(k)+1) - 1$	M1
	$= 25f(k) + 24$ *	A1*
	$f(1) = 24$, divisible by 24	B1
	Assume $f(k)$ divisible by 24	M1
	Then $25f(k) + 24$ is divisible by 24	A1
	Full conclusion *	A1*
		(6)
(b)	$5^{2n+1} = 5\left(5^{2n} - 1\right) + 5$	M1
	Remainder 5	A1
		(2)
(c)	$n = 1$: $5 - 1 = 4$, not divisible by 24	M1
	No: a counterexample is enough	A1
		(2)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	$n = 1$: LHS $= 2$, RHS $= 2 + 0 = 2$	B1
	Assume true for $n = k$; adds $(k+1)2^{k+1}$	M1
	$2 + (k-1)2^{k+1} + (k+1)2^{k+1} = 2 + 2k \times 2^{k+1}$	A1
	$= 2 + k \times 2^{k+2}$	M1
	$= 2 + ((k+1) - 1)2^{(k+1)+1}$	A1
	Full conclusion *	A1*
		(6)
(b)	$2 + 9 \times 2^{11}$	M1
	18 434	A1
		(2)
(c)	$\left(2 + 19 \times 2^{21}\right) - 18\,434$	M1
	39 827 456	A1
		(2)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$u_2 = 13$	B1
	$u_3 = 29$	B1
		(2)
(b)	$n = 1$: $2^3 - 3 = 5 = u_1$	B1
	Assume $u_k = 2^{k+2} - 3$; $u_{k+1} = 2\left(2^{k+2} - 3\right) + 3$	M1
	$= 2^{k+3} - 3$	A1
	$= 2^{(k+1)+2} - 3$: the formula with $n = k + 1$	A1
	Full conclusion *	A1*
		(5)
(c)	$2^{n+2} > 1003$	M1
	$u_7 = 509$, $u_8 = 1021$	A1
	$n = 8$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$n = 1$: $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$: true	B1
	$\mathbf{M}^{k+1} = \begin{pmatrix} 2k+1 & -2k \\ 2k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$	M1
	$= \begin{pmatrix} 6k+3-4k & -4k-2+2k \\ 6k+2-4k & -4k-1+2k \end{pmatrix}$	A1
	$= \begin{pmatrix} 2(k+1)+1 & -2(k+1) \\ 2(k+1) & 1-2(k+1) \end{pmatrix}$	A1
	Full conclusion *	A1*
		(5)
(b)	$(2n+1)(1-2n)+4n^2 = 1-4n^2+4n^2 = 1$ *	B1
	Areas are unchanged by $\mathbf{M}^n$	B1
		(2)
(c)	$\mathbf{M}^n \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2n+1-2n \\ 2n+1-2n \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ *	M1
	Correct for all n	A1
	$\mathbf{M}^{10} \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$ $\boxed{(21, 20)}$	B1
		(3)
		Total for Question 10: 10 marks