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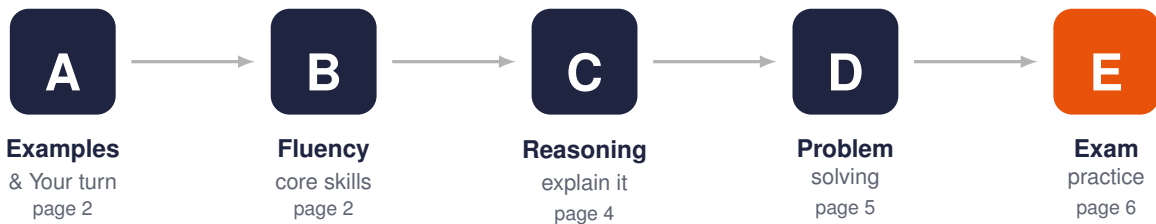
GCSE MATHEMATICS • GEOMETRY

Vectors

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Full worked solutions for the free Vectors worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Vectors worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

FOUNDATION

$$\text{A1} \quad \begin{pmatrix} 5 - (-3) \\ 2 - 6 \end{pmatrix} = \begin{pmatrix} 8 \\ -4 \end{pmatrix}$$

$$\text{A2} \quad 3\mathbf{p} = \begin{pmatrix} -3 \\ 9 \end{pmatrix} \text{ and } 2\mathbf{q} = \begin{pmatrix} 4 \\ -8 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\text{A3} \quad \mathbf{p} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} : 4 \text{ left, } 1 \text{ up} \quad \mathbf{q} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} : 3 \text{ right, } 3 \text{ down}$$

HIGHER

$$\text{A4} \quad \overrightarrow{AB} = \mathbf{b} - \mathbf{a}, \text{ so } \overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$$

$$\text{A5} \quad \overrightarrow{OP} = \mathbf{a} + \frac{3}{4}(\mathbf{b} - \mathbf{a}) = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$$

$$\text{A6} \quad 6\mathbf{a} + 9\mathbf{b} = \frac{3}{2}(4\mathbf{a} + 6\mathbf{b}), \text{ a multiple of it, so } \text{yes, parallel}$$

$$\text{A7} \quad \overrightarrow{QR} = 6\mathbf{a} - 2\mathbf{b} = 2(3\mathbf{a} - \mathbf{b}) = 2\overrightarrow{PQ}. \text{ They are parallel and share the point } Q, \text{ so } P, Q \text{ and } R \text{ are collinear}$$

B**Fluency solutions**One mark for each correct part. The notes in *italics* explain why each sequence was chosen.**FOUNDATION**

$$\text{B1} \quad (\text{a}) \begin{pmatrix} -1 \\ 6 \end{pmatrix} \quad (\text{b}) \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad (\text{c}) \begin{pmatrix} -5 \\ -4 \end{pmatrix} \quad (\text{d}) \begin{pmatrix} -4 \\ 7 \end{pmatrix} \quad \mathbf{b} - \mathbf{a} \text{ is the reverse of } \mathbf{a} - \mathbf{b}$$



B2 (a) $\begin{pmatrix} 6 \\ 15 \end{pmatrix}$ (b) $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} 4 \\ 10 \end{pmatrix} + \begin{pmatrix} -9 \\ 3 \end{pmatrix} = \begin{pmatrix} -5 \\ 13 \end{pmatrix}$ (d) $\frac{1}{2} \begin{pmatrix} -1 \\ 6 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 3 \end{pmatrix}$

B3 $\mathbf{p} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ $\mathbf{s} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

B4 (a) $m = 4$, $k = 3$ (b) $x = 4$, $y = 6$ (c) $2\mathbf{c} = \begin{pmatrix} -2 \\ 8 \end{pmatrix} : \mathbf{c} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ (d) $\begin{pmatrix} 6 \\ k \end{pmatrix} = 3 \begin{pmatrix} 2 \\ -1 \end{pmatrix} : k = -3$

B5 (a) $\sqrt{9+16} = 5$ (b) $\sqrt{25+144} = 13$ (c) $\sqrt{4+36} = \sqrt{40} = 6.32\dots : 6.3$

B6 (a) $\vec{AB} + \vec{BC} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ (b) $\vec{BC} + \vec{CD} = \begin{pmatrix} -6 \\ 5 \end{pmatrix}$ (c) all three: $\begin{pmatrix} -2 \\ 6 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$

HIGHER

B7 (a) $\mathbf{c}$ (b) $\mathbf{a} + \mathbf{c}$ (c) $\mathbf{c} - \mathbf{a}$ (d) $\vec{AM} = \vec{AO} + \vec{OM} = -\mathbf{a} + \frac{1}{2}(\mathbf{a} + \mathbf{c}) = \frac{1}{2}\mathbf{c} - \frac{1}{2}\mathbf{a}$

B8 (a) $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ (b) $\mathbf{a} + \frac{1}{4}(\mathbf{b} - \mathbf{a}) = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ (c) $\mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$
 (d) $\mathbf{a} + 2(\mathbf{b} - \mathbf{a}) = 2\mathbf{b} - \mathbf{a}$ Always start at O , go to A , then a fraction of $\vec{AB} = \mathbf{b} - \mathbf{a}$

B9 (a) $6\mathbf{a} + 9\mathbf{b} = 3(2\mathbf{a} + 3\mathbf{b})$: yes (b) $\mathbf{a} - 2\mathbf{b}$ would have to be multiplied by 3 for the $\mathbf{a}$ part and by -3 for the $\mathbf{b}$ part: no
 (c) $\vec{QR} = 3\vec{PQ}$ and they share Q : yes (d) $\vec{BC} = -2\vec{AB}$ and they share B : yes

B10 $\vec{OC} = \mathbf{b} - \mathbf{a}$, $\vec{OD} = -\mathbf{a}$, $\vec{OE} = -\mathbf{b}$
 (a) $\mathbf{b} - \mathbf{a}$ (b) $\vec{OC} - \vec{OB} = -\mathbf{a}$ (c) $\vec{OD} - \vec{OA} = -2\mathbf{a}$ (d) $\vec{OE} - \vec{OC} = \mathbf{a} - 2\mathbf{b}$



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

C1 Tom worked out $2 - 1$ at the bottom, but $2 - (-1) = 3$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

C2 They have the same length but opposite directions: $\vec{BA} = -\vec{AB}$

C3 $\begin{pmatrix} 2 \\ 6 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, so it is a multiple and they are parallel. $\begin{pmatrix} 3 \\ 9 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, so **yes**, it is parallel too.

C4 $\mathbf{b} = -\frac{3}{2}\mathbf{a}$: a multiple, so parallel. The multiplier is negative, so they point in opposite directions.

HIGHER

C5 Jo wrote half of $\vec{AB}$, which starts at A, not O. $\vec{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

C6 Parallel lines can be separate, side by side. For the points to be on one line, the lines must also share a point.

C7 AB is parallel to CD , and AB is $|k|$ times as long as CD . If k is negative, they point in opposite directions.

C8 (a) $\vec{OM} = \mathbf{a}$ and $\vec{ON} = \mathbf{b}$, so $\vec{MN} = \mathbf{b} - \mathbf{a}$ $\vec{AB} = 2\mathbf{b} - 2\mathbf{a}$

(b) $\vec{AB} = 2(\mathbf{b} - \mathbf{a}) = 2\vec{MN}$, so MN is parallel to AB and half its length.



D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

D1 $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ $\sqrt{2^2 + 6^2} = \sqrt{40} = 6.32 \dots$ **6.3** km

D2 $\vec{AD} = \vec{BC} = \begin{pmatrix} 6-5 \\ 7-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $D = (1+1, 2+4) = \mathbf{(2, 6)}$

D3 Top: $3 + 2k = 7$, so $k = 2$. Check the bottom: $6 - 5 \times 2 = -4$ **$k = 2$**

D4 $2m + n = 7$ and $m + 3n = 11$ $6m + 3n = 21$, so $5m = 10$ **$m = 2$** , **$n = 3$**

HIGHER

D5 (a) $\vec{AB} = \vec{AO} + \vec{OC} + \vec{CB} = -\mathbf{a} + \mathbf{c} + 2\mathbf{a} = \mathbf{a} + \mathbf{c}$ (b) $\vec{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{a} + \mathbf{c}) = \mathbf{\frac{3}{2}a + \frac{1}{2}c}$

D6 $\vec{AB} = \mathbf{b} - \mathbf{a}$ $\vec{AC} = 3\mathbf{b} - 2\mathbf{a} - \mathbf{a} = 3\mathbf{b} - 3\mathbf{a} = 3(\mathbf{b} - \mathbf{a}) = 3\vec{AB}$
 $\vec{AC}$ is a multiple of $\vec{AB}$ and they share the point A, so A, B and C are **collinear**

D7 $\vec{OP} = \mathbf{a} + t(\mathbf{b} - \mathbf{a}) = (1-t)\mathbf{a} + t\mathbf{b}$, so $t = \frac{2}{3}$: P is $\frac{2}{3}$ of the way from A to B **2 : 1**

D8 $\vec{OM} = \mathbf{a} + \frac{1}{2}\mathbf{c}$. On OM: $\vec{OX} = \lambda\mathbf{a} + \frac{\lambda}{2}\mathbf{c}$. On AC: $\vec{OX} = \mathbf{a} + \mu(\mathbf{c} - \mathbf{a}) = (1-\mu)\mathbf{a} + \mu\mathbf{c}$
 Compare: $\lambda = 1 - \mu$ and $\frac{\lambda}{2} = \mu$, so $\lambda = \frac{2}{3}$. X is $\frac{2}{3}$ of the way along OM **OX : XM = 2 : 1**



E**Exam practice solutions**

Marks are shown on the right of each line.

FOUNDATION*Foundation and Higher crossover*

E1 $\begin{pmatrix} 8 \\ -2 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

M1

$$\begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

A1

E2 (a) $\begin{pmatrix} 5 \\ 3 \end{pmatrix}$

B1

(b) $\begin{pmatrix} -5 \\ -3 \end{pmatrix}$

B1

E3 $3 + 2x = 11$

M1

$$x = 4$$

A1

$y - 2 = 5$: $y = 7$

A1

E4 $\begin{pmatrix} 6 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

M1

$$\sqrt{4^2 + 3^2}$$

M1

$$5$$

A1

E5 $\begin{pmatrix} 3 \\ -2 \end{pmatrix} = \frac{1}{2}\mathbf{p}$ and $\begin{pmatrix} -9 \\ 6 \end{pmatrix} = -\frac{3}{2}\mathbf{p}$; $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$ is not a multiple

M1

$$\begin{pmatrix} 3 \\ -2 \end{pmatrix} \text{ and } \begin{pmatrix} -9 \\ 6 \end{pmatrix}$$

A1

E6 (a) $\vec{AB} + \vec{BC} + \vec{CD}$

M1

$$\begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

A1

(b) $\sqrt{1 + 49} = 7.07 \dots$ 7.1

B1



HIGHER

- E7** $\vec{AB} = \mathbf{b} - \mathbf{a}$ B1
 $\vec{OP} = \mathbf{a} + \frac{3}{5}(\mathbf{b} - \mathbf{a})$ M1
 $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$ A1
-
- E8** $\vec{XY} = \vec{OY} - \vec{OX} = (\mathbf{a} + 3\mathbf{b}) - (3\mathbf{a} + \mathbf{b})$ M1
 $= -2\mathbf{a} + 2\mathbf{b}$ A1
 $= 2(\mathbf{b} - \mathbf{a})$, a multiple of $\mathbf{b} - \mathbf{a}$, so parallel C1
-
- E9** (a) $\vec{PQ} = 3\mathbf{a} + 3\mathbf{b}$ and $\vec{QR} = 2\mathbf{a} + 2\mathbf{b}$ M1
 $\vec{PQ} = \frac{3}{2}\vec{QR}$, so they are parallel M1
and they share Q , so P, Q, R are collinear C1
(b) $3:2$ B1
-
- E10** $\vec{AM} = \frac{1}{2}\mathbf{b} - \mathbf{a}$, so $\vec{OX} = \mathbf{a} + \frac{4}{5}(\frac{1}{2}\mathbf{b} - \mathbf{a}) = \frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$ M1 A1
 $\vec{ON} = \mathbf{a} + \frac{2}{3}(\mathbf{b} - \mathbf{a}) = \frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$ A1
 $\vec{ON} = \frac{5}{3}\vec{OX}$: parallel, sharing O , so O, X, N are collinear C1
 $OX : ON = 3 : 5$, so $OX : XN = 3 : 2$ A1
-

