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INTERNATIONAL A LEVEL MATHEMATICS • DECISION MATHEMATICS 1

Decision 1

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WDM11/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Algorithms** Flow charts, bubble sort, quick sort, bin packing, binary search	10	80	3	
2	**Graphs and Networks** Vocabulary, types of graph, degrees, adjacency and distance matrices	10	80	9	
3	**Algorithms on Graphs** Kruskal, Prim (networks and matrices), Dijkstra	10	80	14	
4	**Route Inspection** Eulerian graphs, pairing odd vertices, start and finish points	10	80	19	
5	**The Travelling Salesman Problem** Classical and practical problems, upper and lower bounds	10	80	25	
6	**Critical Path Analysis** Activity networks, event times, floats, Gantt charts, scheduling	10	80	30	
7	**Linear Programming** Formulation, feasible regions, objective lines, integer solutions	10	80	36	

Question booklet Free courses More worksheets

How to use this mark scheme

The same marking rules Pearson examiners use for WDM11.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

"Solutions relying entirely on calculator technology are not acceptable." When a WDM11 question says this, a correct final answer with no algebraic working scores **no marks**. Exact answers (surds, multiples of π) are required when asked for; otherwise give 3 significant figures, with arguments in radians.

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1 Chapter 1: Algorithms

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	Pass 1 correct: 19 25 12 30 7 16 21 4	M1
	Pass 2 correct: 25 19 30 12 16 21 7 4	A1
	Passes 3 and 4 correct	A1
	Pass 5 gives 30 25 21 19 16 12 7 4; pass 6 makes no swaps, so the sort is complete	A1
	Summer 2022 report: a final pass with no swaps is needed even when the list already looks sorted.	(4)
(b)	7 comparisons, 5 swaps	B1
		(1)
(c)	7	B1
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	Pivot 11 (middle-right of 10 items): 8 6 11 23 41 15 32 27 19 36	M1
	Next pivots 6 and 32: 6 8 11 23 15 27 19 32 41 36	A1
	Pivots 8, 27, 36: 6 8 11 23 15 19 27 32 36 41	A1
	Pivots 15, 41 then 19, then 23	M1
	6 8 11 15 19 23 27 32 36 41	A1
	Jan 2020 report: choose pivots consistently (middle-right throughout) and keep every item.	(5)
(b)	Every item has been chosen as a pivot (or: sort complete stated)	B1
		(1)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	$\frac{77}{20} = 3.85$ 4 crates	M1 A1 (2)
(b)	Crate 1: 8 4 6 Crate 2: 15 5; Crate 3: 12 7 Crate 4: 9 11 *Jan 2016 report: first fit means the first crate the parcel fits in, not the most recently used crate.*	M1 A1 A1 (3)
(c)	It uses 4 crates, equal to the lower bound	B1 (1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	A: 37, 74, 148, 296, 592, 1184 B: 22, 11, 5, 2, 1, 0 Odd?: no, yes, yes, no, yes C: 0, 74, 222, 814 Output 814 *Jan 2025 report: the answer must be written on the output line.*	M1 A1 A1 A1 A1 (5)
(b)	It multiplies A by B ($37 \times 22 = 814$)	B1 (1)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	Middle $\left[\frac{1+11}{2}\right] = 6$: Fofana; Kapoor is after, so keep 7 to 11	M1
	$\left[\frac{7+11}{2}\right] = 9$: Ito; keep 10 to 11	A1
	$\left[\frac{10+11}{2}\right] = 11$ (rounded up): Lopez; keep 10	A1
	Position 10: Kapoor, found	A1
		(4)
(b)	6 Fofana, then $\left[\frac{1+5}{2}\right] = 3$ Chen, then $\left[\frac{1+2}{2}\right] = 2$ Begum, then 1 Adams	M1
	The list is now empty, so Baker is not in the list	A1
		(2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	Pivot 35 (middle-right of 12): 35 18 31 12 26 9 22 14 27 7 20 16	M1
	Pivot 22: 35 31 26 27 <u>22</u> 18 12 9 14 7 20 16	A1
	Pivots 26, 14: 35 31 27 26 22 18 20 16 14 12 9 7	A1
	Pivots 27, 20, 9 then 31, 16, 12, 7 then 18: 35 31 27 26 22 20 18 16 14 12 9 7	A1
	Summer 2022 and 2023 reports: the order asked for (here descending) must be used.	
		(4)
(b)	Disc 1: 35 14; Disc 2: 31 18	M1
	Disc 3: 27 22; Disc 4: 26 20	A1
	Disc 5: 16 12 9 7	A1
		(3)
(c)	$\frac{237}{50} = 4.74$, so lower bound 5	M1
	5 discs used, so it is optimal	A1
		(2)
(d)	Large items are placed first, when there is most space, leaving small items to fill the gaps	B1
		(1)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	11 fitted on Card 2 with 19 and 8 so $n \geqslant 19+8+11=38$ *	**M1** **A1*** **(2)**
(b)	15 did not fit on Card 2 when it held 19 and 8: $19+8+15=42>n$, so $n \leqslant 41$ Other checks: 6 not on Card 1 ($37+6=43>n$), 8 not on Card 1 ($45>n$), both satisfied $\boxed{n=41}$ (so $38 \leqslant n \leqslant 41$) *Oct 2021 report: each item that did or did not fit gives an inequality for n.*	**M1** **M1** **A1** **(3)**
(c)	Sorted: 24 19 15 13 11 8 6 Card 1: 24 15; Card 2: 19 13 8 $\boxed{\text{Card 3: 11 6}}$	**B1** **M1** **A1** **(3)**
(d)	$\frac{96}{40}=2.4$, lower bound 3 Both allocations use 3 cards, the minimum possible	**M1** **A1** **(2)**
		Total for Question 7: 10 marks

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Question 8

10 marks

Part	Scheme	Marks
(a)	$a = 168$, $b = 120$: $q = 1$, $r = 48$, No	M1
	$a = 120$, $b = 48$: $q = 2$, $r = 24$, No	A1
	$a = 48$, $b = 24$: $q = 2$, $r = 0$, Yes	A1
	Loop returns to the q box each time (not to the input box)	A1
	Output 24	A1
		(5)
(b)	It finds the highest common factor of a and b	B1
		(1)
(c)	First pass: $q = 0$, $r = 120$, then $a = 168$, $b = 120$ and the rest is as part (a); output 24	M1
	It needs one extra pass, which simply swaps a and b	A1
		(2)
(d)	$r = 21$, then 14, then 7, then 0	M1
	Output 7	A1
		(2)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	Pivot Tanaka (middle of 9): all other names before it	M1
	Pivot Diallo: Bell Diallo Okafor Singh Moreau Kowalski Hughes Rossi Tanaka	A1
	Pivots Bell, Kowalski: Bell Diallo Hughes Kowalski Okafor Singh Moreau Rossi Tanaka	A1
	Pivots Hughes, Moreau, then Singh, then Rossi, then Okafor	M1
	Bell, Diallo, Hughes, Kowalski, Moreau, Okafor, Rossi, Singh, Tanaka	A1
		(5)
(b)	$\left[\frac{1+9}{2}\right] = 5$: Moreau; keep 6 to 9	M1
	$\left[\frac{6+9}{2}\right] = 8$: Singh; keep 6 to 7	A1
	$\left[\frac{6+7}{2}\right] = 7$: Rossi, found	A1
		(3)
(c)	Moreau (5), Singh (8), Rossi (7), then 6 Okafor; Patel is after Okafor	M1
	No names remain, so Patel is not in the list	A1
		(2)
		Total for Question 9: 10 marks

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Question 10

10 marks

Part	Scheme	Marks
(a)	Pass 1: 14 21 9 17 11 25 8 19 6 4; 9 comparisons, 6 swaps	M1
	Pass 2: 21 14 17 11 25 9 19 8 6 4; 8, 5	A1
	Passes 3 to 5 correct (7, 3; 6, 2; 5, 2)	A1
	Pass 6: 25 21 19 17 14 11 9 8 6 4; 4, 2	A1
	Pass 7: 3 comparisons, 0 swaps, so the sort is complete	A1
		(5)
(b)	$9+8+\cdots+1 = \boxed{45}$	B1
		(1)
(c)	Trolley 1: 14 9 6	M1
	Trolley 2: 21 8; Trolley 3: 17 11	A1
	Trolley 4: 25 4; Trolley 5: 19	A1
		(3)
(d)	$\frac{134}{30} = 4.47$, lower bound 5, so 5 trolleys is optimal	B1
		(1)
		Total for Question 10: 10 marks

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2 Chapter 2: Graphs and Networks

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	A 2, B 3, C 4	B1
	D 4, E 3, F 2	B1
		(2)
(b)	Sum of degrees = 18 = 2× number of arcs (handshaking lemma)	M1
	9 arcs	A1
		(2)
(c)	e.g. $A–C–D–F$ (no vertex repeated)	B1
	e.g. $A–B–D–F–E–C–A$	B1
		(2)
	Total for Question 1: 6 marks	

Question 2

6 marks

Part	Scheme	Marks
(a)	Arcs PQ, PT, QR, RS, RT	M1
	Two separate arcs between P and S	A1
	All 7 arcs, none extra	A1
		(3)
(b)	Not simple: there are two arcs between P and S (the entry 2)	B1
	Connected: every vertex can be reached from P	B1
		(2)
(c)	$\frac{1}{2} \times 14 = 7$	B1
		(1)
	Total for Question 2: 6 marks	

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Question 3

6 marks

Part	Scheme	Marks
(a)	$\frac{1}{2} \times 7 \times 6 =$ 21	B1 (1)
(b)	Each of the 3 vertices joined to each of the 4, and no other arcs 12 arcs	B1 B1 (2)
(c)	Sum of degrees = 11, which is odd but the sum must be 2× number of arcs, which is even	M1 A1 (2)
(d)	8 (a tree)	B1 (1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	At least five arcs correct All seven arcs AB 7, AD 5, AE 9, BC 4, BE 6, CD 8, DE 3 No extra arcs, all weights shown	M1 A1 A1 (3)
(b)	42 km	B1 (1)
(c)	A–B–C (11 km) and A–D–C (13 km) A–B–C is shorter	M1 A1 (2)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	Degrees: X has 1 and 3 of degree 3; Y has Q and S of degree 3	M1
	$1 \to Q, 2 \to R, 3 \to S, 4 \to T, 5 \to P$ (or an equivalent)	A1
	Checks that all six arcs map: $12 \to QR$, $23 \to RS$, $34 \to ST$, $45 \to TP$, $51 \to PQ$, $13 \to QS$	M1
	All arcs correspond, so X and Y are isomorphic	A1
		(4)
(b)	Z has a vertex (Y) of degree 1	M1
	X has none, so the degrees cannot be matched	A1
		(2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	Volunteers on one side, jobs on the other	M1
	Arcs $A1$, $A3$, $B2$, $B3$, $B5$, $C1$, $D1$, $D4$ only	A1
		(2)
(b)	8 arcs	B1
	sum of degrees 16	B1
		(2)
(c)	C can only do job 1, so C must have it	B1
	Then A takes 3, D takes 4, B takes 2 or 5	M1
	$A3, B2, C1, D4$ (or $B5$)	A1
		(3)
(d)	Connected: every vertex is joined by a path (through 1 and 3)	B1
		(1)
(e)	$K_{4,5}$ has $4 \times 5 = 20$ arcs	M1
	12 more	A1
		(2)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	Sum = 13, odd	M1
	The sum of degrees must be even (2× arcs), so impossible	A1
		(2)
(b)	A simple graph (no loops or multiple arcs)	M1
	Degrees correct, e.g. VA, VB, VC, VD, CD, CA, DB	A1
	7 arcs	B1
		(3)
(c)	A vertex of degree 4 in a simple graph on 5 vertices is joined to every other vertex	M1
	So each degree-1 vertex is joined to all three degree-4 vertices	M1
	giving it degree at least 3, a contradiction	A1
		(3)
(d)	5 arcs	B1
	sum 10	B1
		(2)
	Total for Question 7: 10 marks	

Question 8

10 marks

Part	Scheme	Marks
(a)	Nine arcs drawn	M1
	Directions correct for at least seven	A1
	All correct: AB, AC, BC, BD, CD, CE, DA, DE, EB	A1
		(3)
(b)	The matrix is not symmetric, e.g. there is a street $A \to B$ but not $B \to A$	B1
		(1)
(c)	Out: 2, 2, 2, 2, 1	B1
	In: 1, 2, 2, 2, 2	B1
	Each total is 9 streets	B1
		(3)
(d)	$A \to C \to E$	B1
	$A \to B \to D \to E$, $A \to C \to D \to E$	M1
	and $A \to B \to C \to E$; no others	A1
		(3)
	Total for Question 8: 10 marks	

Question 9

10 marks

Part	Scheme	Marks
(a)	10 arcs	B1
	4 arcs in a spanning tree	B1
	Fix the start: 4! = 24 orders of the other vertices	M1
	each cycle counted twice (reversed): $24 \div 2 = 12$ *	A1*
		(4)
(b)	$ABCEDA$ 30, $ABECDA$ 29	M1
	$ABEDCA$ 38, $ACBEDA$ 38	A1
	All four found, no others	A1
	Shortest A–B–E–C–D–A, length 29	A1
		(4)
(c)	F has degree 1	M1
	A cycle through F must enter and leave by different arcs, so needs degree at least 2	A1
		(2)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	At least seven arcs correct	M1
	All ten arcs with weights	A1
	No extra arcs	A1
		(3)
(b)	A 3, B 3, C 4, D 4, E 3, F 3	B1
	Sum $20 = 2 \times 10$, so 10 corridors *	B1
		(2)
(c)	There are four odd vertices (A, B, E, F), so the network is not Eulerian	B1
	Each repeated corridor can make two odd vertices even, so at least 2	M1
	2, e.g. AB and EF	A1
		(3)
(d)	55 m	B1
	A spanning tree has 5 arcs: remove 5	B1
		(2)
		Total for Question 10: 10 marks

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3 Chapter 3: Algorithms on Graphs

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	GA 3 ✓, GF 4 ✓	M1
	EF 7 ✓, FA 8 ×	A1
	GD 10 ✓, DE 11 ×	A1
	GE 12 ×, CD 14 ✓, AB 17 ✓; 6 arcs, so stop	A1
		(4)
(b)	Tree GA, GF, EF, GD, CD, AB drawn	B1
		(1)
(c)	55 (thousand dollars)	B1
	Jan 2016 report: add only the accepted arcs, not the rejected ones.	
		(1)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	AB 15	M1
	BC 16	A1
	CD 4	A1
	DE 11, DF 17: order AB, BC, CD, DE, DF	A1
		(4)
(b)	63	B1
		(1)
(c)	No need to check for cycles (or: it can be applied to a matrix; arcs need not be sorted first)	B1
		(1)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	AC 18	M1
	CB 15	A1
	CE 19	A1
	EF 17, BD 20: order AC, CB, CE, EF, BD	A1
	Jan 2025 report: number the columns and list the arcs; start at the vertex given.	(4)
(b)	Tree drawn with these five arcs	B1
	89 km	B1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	S 1, 0	M1
	B 2, 7; C 3, 12 (15, 12)	A1
	A 4, 13; E 5, 15 (21, 15)	A1
	D 6, 19 (24, 19); T 7, 30 (34, 30)	A1
	Shortest time 30 minutes	A1
		(5)
(b)	S–A–D–T	B1
		(1)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	BD 3 ✓, DF 5 ✓ BE 6 ✓, DE 6 × CE 7 ✓, EF 8 ×, AB 10 ✓; 5 arcs, so stop	M1 A1 A1 (3)
(b)	BE and DE both have weight 6; only one can be used, as either completes a cycle with arcs already chosen Second tree uses the other of these two arcs	B1 B1 (2)
(c)	31 for both	B1 (1)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	A 1, 0 D 2, 14 B 3, 17; C 4, 22 E 5, 24 F 6, 25 (33, 25) G 7, 35 (45, 35); H 8, 37 (38, 37) *Jan 2025 report: working values must be complete and in the order found.*	M1 A1 A1 A1 A1 A1 (6)
(b)	$A–C–F–H$ 37	B1 B1 (2)
(c)	H can now be reached only from D, G: $14+24=38$, $35+4=39$; the final values there are unchanged $A–D–H$, 38	M1 A1 (2)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	AC 27	M1
	CE 21, AG 29	A1
	ED 30	A1
	DB 24, DF 26: order AC, CE, AG, ED, DB, DF	A1
		(4)
(b)	157 m	B1
		(1)
(c)	The arcs would have to be sorted from the table, and checked for cycles, which is awkward from a matrix	B1
		(1)
(d)	Include AF (51) first, then apply Kruskal	M1
	CE 21, BD 24, DF 26	A1
	AC 27, AG 29: six arcs, so stop	A1
	178 m	A1
		(4)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	DE 5 ✓	M1
	AC 6 ✓, CD 7 ✓	A1
	CE 8 ×, BC 9 ✓	A1
	AB 10 ×, EF 11 ✓; 5 arcs, so stop	A1
	Weight 38	A1
		(5)
(b)	F joins the tree by BF (x) or EF (11); the rest of the tree is DE, AC, CD, BC (27)	M1
	$x < 11$	A1
	Weight $27 + x$	A1
		(3)
(c)	$x = 11$	B1
	BF and EF are then equal, and either can join F to the tree	B1
		(2)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	A 1, 0	M1
	E 2, 10; B 3, 12	A1
	F 4, 13 (19, 13)	A1
	G 5, 26 (36, 26); C 6, 29	A1
	D 7, 34; J 8, 45 (49, 47, 45)	A1
	Route $A–B–C–D–J$	A1
	45 km	A1
		(7)
(b)	A to E from part (a): 10, by $A–E$	M1
	E to J: 39, by $E–F–G–J$	M1
	$A–E–F–G–J$, 49 km	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	FG 4 ✓, BC 5 ✓	M1
	BE 6 ✓, AB 7 ✓	A1
	DG 9 ✓, DE 10 ✓; 6 arcs, so stop	A1
	41 km	A1
		(4)
(b)	A 1, 0; B 2, 7	M1
	C 3, 12 (21, 12); E 4, 13	A1
	D 5, 18; F 6, 26 (29, 26); G 7, 27 (33, 27)	A1
	Route $A–B–D–G$	A1
	27 km	A1
		(5)
(c)	A minimum spanning tree minimises the total length of all the roads, not the distance between two particular places (the route uses BD, which is not in the tree)	B1
		(1)
		Total for Question 10: 10 marks

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4 Chapter 4: Route Inspection

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	B and D have odd degree (3), so the network is not Eulerian	B1
	Shortest path from B to D: B–C–D, length 14 (not the direct arc BD, 19)	M1
	Repeat BC and CD	A1
		(3)
(b)	110 + 14 = 124	B1
		(1)
(c)	A route from A to A using every arc once and the repeated arcs twice	M1
	e.g. A–B–C–B–D–C–E–F–G–C–D–A	A1
		(2)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	Odd vertices identified	B1
	Odd vertices A, C, E, G	M1
	$AC + EG$: total 43 [AC=20 (AGC), EG=23]; $AE + CG$: total 41 [AE=29 (AFE), CG=12]	A1
	$AG + CE$: total 26 [AG=8, CE=18 (CDE)]	A1
	Repeat AG, CD, DE	A1
	Jan 2025 report: find the total for every pairing, not just the pairings themselves.	(5)
(b)	114 + 26 = 140 km	B1
		(1)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	Exactly two vertices, B and C, have odd degree (semi-Eulerian) Start at B and finish at C (or the reverse)	B1 B1 (2)
(b)	136 m	B1 (1)
(c)	Repeat the shortest path from B to C B–C, 11 136 + 11 = 147 m	M1 A1 A1 (3)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Every vertex has even degree, so the network is Eulerian	B1 (1)
(b)	A closed route using all 9 arcs once e.g. A–B–C–E–B–D–E–F–C–A 148	M1 A1 A1 (3)
(c)	B has degree 4 each visit uses two arcs, so 2 times	M1 A1 (2)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	Odd vertices A, B, C, E; three pairings considered	M1
	$AB + CE$: total 35 [AB=21, CE=14]; $AC + BE$: total 43 [AC=19 (AEC), BE=24]	A1
	$AE + BC$: total 17 [AE=5, BC=12]	A1
	$122 + 17 =$ 139 km (closed route)	(3)
(b)	E and the finish are left unpaired; the other two are repeated: finish A repeats 12; finish B repeats 19; finish C repeats 21	M1
	Finish at A	A1
	$122 + 12 =$ 134 km	A1
	Jan 2025 report: state the finishing vertex and justify it.	(3)
	Total for Question 5: 6 marks	

Question 6

10 marks

Part	Scheme	Marks
(a)	Odd vertices A, B, E, G; three pairings considered	M1
	$AB + EG$: total 39 [AB=19, EG=20]; $AE + BG$: total 71 [AE=36 (AGE), BG=35 (BAG)]	A1
	$AG + BE$: total 41 [AG=16, BE=25 (BDE)]	A1
	Repeat AB, EG	A1
	$158 + 39 =$ 197 km	A1
		(5)
(b)	Leave two odd vertices unpaired as the ends; repeat the shortest path between the other two	M1
	Options: ends B, E: repeat 16; ends E, G: repeat 19; ends A, B: repeat 20; ends A, G: repeat 25; ends A, E: repeat 35; ends B, G: repeat 36	A1
	Start at B, finish at E (or reverse), 174 km	A1
		(3)
(c)	Options from G: finish E, repeat 19; finish A, repeat 25; finish B, repeat 36	M1
	Finish at E, 177 km	A1
		(2)
	Total for Question 6: 10 marks	

Question 7

10 marks

Part	Scheme	Marks
(a)	Odd vertices E and F only	B1
	Shortest E to F: E–F, 10	M1
	$158 + 10 = \boxed{168}$	A1
		(3)
(b)	B and G become odd, so the odd vertices are B, E, F, G	B1
	New total $158 + 17 = 175$	B1
	Odd vertices B, E, F, G; three pairings considered	M1
	$BE + FG$: total 52 [BE=13 (BAE), FG=39 (FCG)]; $BF + EG$: total 53 [BF=23 (BAEF), EG=30 (EABG)]	A1
	$BG + EF$: total 27 [BG=17, EF=10]	A1
	$175 + 27 = \boxed{202}$	A1
		(6)
(c)	It is 34 longer: the extra path (17) plus a change in the repeated paths (27 instead of 10)	B1
		(1)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	Total $77 + x$	B1
	Odd vertices A, B, C, D	B1
	$AD + BC = x + 12$ (BC via A); $AB + CD = 7 + 16 = 23$ (CD via F)	M1
	$AC + BD = 5 + \min(x + 7, 28)$ (never less than $x + 12$ or 23)	A1
		(4)
(b)	$x + 12 \leqslant 23$	M1
	$\boxed{x \leqslant 11}$: route $77 + x + x + 12 = 89 + 2x$	A1
	$\boxed{x \geqslant 11}$: repeat AB and CD, route $100 + x$	A1
		(3)
(c)	$100 + x = 116$ gives $x = 16 \geqslant 11$, valid	M1
	$89 + 2x = 116$ gives $x = 13.5$, not $\leqslant 11$, so rejected	M1
	$\boxed{x = 16}$	A1
		(3)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	A 1, 0	**M1**
	B 2, 12; D 3, 18	**A1**
	C 4, 22	**A1**
	F 5, 23; E 6, 24	**A1**
	G 7, 27 (44, 39, 27); H 8, 38 (41, 38)	**A1**
		(5)
(b)	Distances from A read from part (a); other pairs by inspection	**M1**
	$AB + FH$: total 33 [AB=12, FH=21 (FEH)]; $AF + BH$: total 55 [AF=23 (ABF), BH=32 (BFEH)]	**A1**
	$AH + BF$: total 49 [AH=38 (AEH), BF=11]	**A1**
	Repeat $\boxed{AB, FE, EH}$	**A1**
	Jan 2025 report: use the Dijkstra values from part (a) for the distances from A.	
		(4)
(c)	$177 + 33 = \boxed{210}$	**B1**
		(1)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	Odd vertices A, B, C, F; three pairings considered	M1
	$AB + CF$: total 38 [AB=21, CF=17 (CDF)]; $AC + BF$: total 46 [AC=31 (ABDC), BF=15 (BDF)]	A1
	$AF + BC$: total 46 [AF=36 (ABDF), BC=10 (BDC)]	A1
	Repeat AB, CD, DF	A1
	$118 + 38 =$ 156	A1
		(5)
(b)	A stays odd as the start; the finish is one of the other odd vertices and the remaining two are paired	M1
	finish F: repeat 10; finish C: repeat 15; finish B: repeat 17	A1
	Finish at F, repeating BD, DC	A1
	$118 + 10 =$ 128	A1
	Jan 2025 report: state the finishing vertex, not only the repeated arcs.	
		(4)
(c)	D has degree 4, plus 2 from the repeated streets (degree 6): 3 times	B1
		(1)
		Total for Question 10: 10 marks

5 Chapter 5: The Travelling Salesman Problem

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	Row and column A given (7, 5, 11, 17); row B: 7, –, 2, 8, 10	M1
	Row C: 5, 2, –, 6, 12	A1
	Rows D, E: D: 11, 8, 6, –, 9; E: 17, 10, 12, 9, –	A1
	Symmetric, with $BD = 8$ (via C), $CE = 12$ (via B)	A1
		(4)
(b)	Classical: each vertex is visited **exactly** once before returning to the start	B1
	Practical: each vertex is visited **at least** once before returning to the start	B1
	Jan 2025 report: both must mention returning to the start.	(2)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$A–C–F–D–B–E$	M1
	returns to A: $A–C–F–D–B–E–A$	A1
	$18+22+9+23+67+41 = 180$	A1
		(3)
(b)	$D–F–B–C–A–E–D$	M1
	156	A1
		(2)
(c)	156, the smaller value	B1
		(1)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	Weight $9+6+14+11+23=63$ $2\times 63=$ 126 km	**M1** **A1** **(2)**
(b)	Doubled-tree walk, e.g. A–D–E–D–B–D–A–C–F–C–A Shortcut $B\rightarrow C$ directly (39) instead of B–D–A–C (46) Saving 7 119 km	**M1** **M1** **A1** **A1** **(4)**
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Residual minimum spanning tree on B to F: BF 21, FC 20, CE 32, ED 7 Weight 80; the two least arcs from A are 11 and 13 $80+11+13=$ 104 km	**M1** **M1** **A1** **(3)**
(b)	Residual tree on A, C to F: AF 13, AC 16, CE 32, ED 7 (68), plus $11+21$ 100 km	**M1** **A1** **(2)**
(c)	104, the greater value	**B1** **(1)**
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	Best lower bound 117, best upper bound 128 $117 \leqslant T \leqslant 128$ *Jan 2025 report: a strict inequality for the upper bound was the most common error.*	M1 A1 (2)
(b)	Each deletion gives a different residual tree and different pair of arcs Every lower bound is valid, so the greatest is the closest to the optimal value	B1 B1 (2)
(c)	120 lies in the interval, so it could be optimal It is not shown to be optimal, as it is greater than the best lower bound (117)	B1 B1 (2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	The table satisfies the triangle inequality (every entry is a shortest distance), so no vertex need be revisited	B1 (1)
(b)	AF 36, FE 11 EC 7, FD 16, CB 19 Weight 89 Upper bound $2 \times 89 =$ 178 km	M1 A1 A1 B1 (4)
(c)	A–F–E–C–B–D–A 157 km *Jan 2025 report: the route must return to A and be stated.*	M1 A1 (2)
(d)	Residual tree BC, CE, EF, FD: 53; plus 36 + 38 127 km	M1 A1 (2)
(e)	$127 \leqslant T \leqslant 157$	B1 (1)
		Total for Question 6: 10 marks

Question 7

10 marks

Part	Scheme	Marks
(a)	At least ten entries correct	M1
	$AD = 22$, $AF = 29$, $CE = 20$, $CF = 15$ (all via D or C)	A1
	All correct: BD 33, BE 45, BF 40, EF 19, others direct	A1
		(3)
(b)	$A–C–D–F–E–B–A$ on the table	M1
	$14+8+7+19+45+27$	M1
	120 km	A1
	Actual roads: $A–C–D–F–D–E–D–C–B–A$	A1
		(4)
(c)	Residual tree DF 7, CD 8, DE 12, BC 25: 52	M1
	Add the two least from A: $14+22$	M1
	88 km	A1
		(3)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	From A: $A–B–D–C–E–F–A$	M1
	182	A1
	From B: $B–D–A–C–E–F–B$	M1
	176	A1
		(4)
(b)	Delete A: tree BD, BC, CE, EF (75) + $35+43$	M1
	153	A1
	Delete B: tree AD, AC, CE, EF (116) + $11+35$	M1
	162	A1
		(4)
(c)	$162 \leqslant T \leqslant 176$	B1
	Neither is known to be optimal: 176 is greater than the best lower bound	B1
		(2)
		Total for Question 8: 10 marks

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Question 9

10 marks

Part	Scheme	Marks
(a)	Residual tree on B to F: CE 7, EF 11, FD 16, BC 19	M1
	Weight 53	A1
	Arcs from A: 45, 47, x, 40, 36; the least is 36 (as $x > 36$)	M1
	Second least is $\min(x, 40)$: $53 + 36 + \min(x, 40) = 89 + \min(x, 40)$ *	A1*
		(4)
(b)	From A the least entry is 36 (F), since $x > 36$	B1
	Then E 11, C 7, B 19, D 46, back to A by x	M1
	$36 + 11 + 7 + 19 + 46 + x = 119 + x$ *	A1*
		(3)
(c)	$119 + x = 157$, so $\boxed{x = 38}$ (which satisfies $36 < x < 45$)	B1
	Lower bound $89 + 38 = 127$	M1
	$\boxed{127 \leqslant T \leqslant 157}$	A1
		(3)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	AE 12, EF 14, FG 7	M1
	ED 17, GC 21, CB 11	A1
	Weight $\boxed{82}$	A1
		(3)
(b)	Initial upper bound $2 \times 82 = 164$	B1
	Walk A–E–F–G–C–B–C–G–F–E–D–E–A; shortcut $B \to D$ (63) instead of B–C–G–F–E–D (70)	M1
	Saving 7: $\boxed{157}$	A1
		(3)
(c)	D–E–A–F–G–C–B–D	M1
	$\boxed{150}$	A1
		(2)
(d)	Residual tree 65, plus $17 + 26$: $\boxed{108}$	M1
	$\boxed{108 \leqslant T \leqslant 150}$	A1
		(2)
		Total for Question 10: 10 marks

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6 Chapter 6: Critical Path Analysis

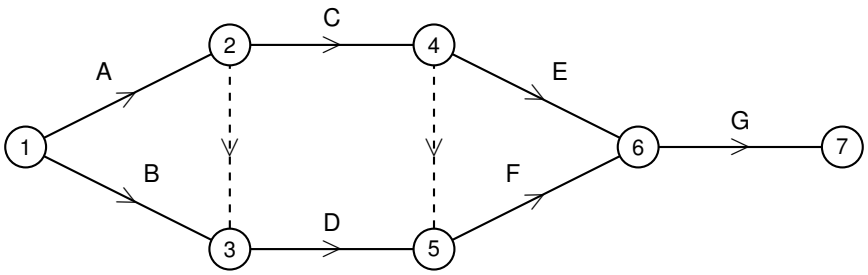
Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
	A and B start together at a single start event; C and D follow A	M1
	A dummy from the end of A to the end of B, so that D follows both A and B	A1
	E follows C only	A1
	A dummy from the end of C to the end of D, so that F follows C and D	A1
	G follows E and F; a single end event	A1
	Exactly two dummies, arrows on every activity and dummy, dummies dotted	A1
	Jan 2025 report: put arrows on the middle of activities and dummies, draw dummies dotted, and finish at a single end event.	
		(6)
		Total for Question 1: 6 marks

One correct network:



Question 2

6 marks

Part	Scheme	Marks
(a)	Early times: 1: 0, 2: 5, 3: 3, 4: 11, 5: 13, 6: 18	M1
	All early times correct	A1
	Late times: 1: 0, 2: 5, 3: 6, 4: 11, 5: 13, 6: 18	M1
	All late times correct	A1
	Jan 2025 report: top boxes increase going forward; bottom boxes decrease going back.	
		(4)
(b)	A, C, G, H	B1
	18 days	B1
		(2)
		Total for Question 2: 6 marks

Question 3

6 marks

Part	Scheme	Marks
(a)	Float = late finish − early start − duration	M1
	C: $6-0-4$; F: $18-6-8$; I: $15-4-7$	A1
	C 2, F 4, I 4	A1
		(3)
(b)	B–H–K	B1
		(1)
(c)	C has float 2, which is less than 3	M1
	The project is delayed by 1 hour, to 23 hours	A1
		(2)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	Critical activities A, C, G, J on one row, end to end from 0 to 18	M1
	At least four non-critical bars starting at their earliest start times	M1
	All bars correct	A1
	Floats shown (for example, shaded or dotted) and correct	A1
	Jan 2025 report: use a ruler and shade the floats clearly.	(4)
(b)	An activity must be happening if latest start $\leqslant 7$ and earliest finish $\geqslant 8$	M1
	C, D, E, H	A1
		(2)
		Total for Question 4: 6 marks

Question 5

6 marks

Part	Scheme	Marks
(a)	A, B –; C, D: A	B1
	E: B, D	B1
	F, G: C; H: C, E	B1
	I: F, G, H	B1
		(4)
(b)	The dummy from the end of C: H depends on C (and E), but F and G do not depend on E (logic)	B1
	The dummy at the end of G: F and G share start and end events, so a dummy keeps each activity uniquely described by its events (uniqueness)	B1
		(2)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	Dummy so that E follows A and B, while D follows A only	M1
	Dummy so that F follows B and C, but not A (so B has its own end event)	A1
	Dummy so that G and H follow both D and E	A1
	Uniqueness dummy, as G and H start and end together	A1
	I follows F; single end event	A1
	Exactly five dummies, arrows on all arcs	A1
	Jan 2025 report: the uniqueness dummy was the one most often forgotten.	
		(6)
(b)	Forward and backward passes completed on their network	M1
	Event times consistent with their network (correct network: end event 6)	A1
	Critical activities A, E, G	A1
	6 days	A1
		(4)
		Total for Question 6: 10 marks

One correct network, with event times:

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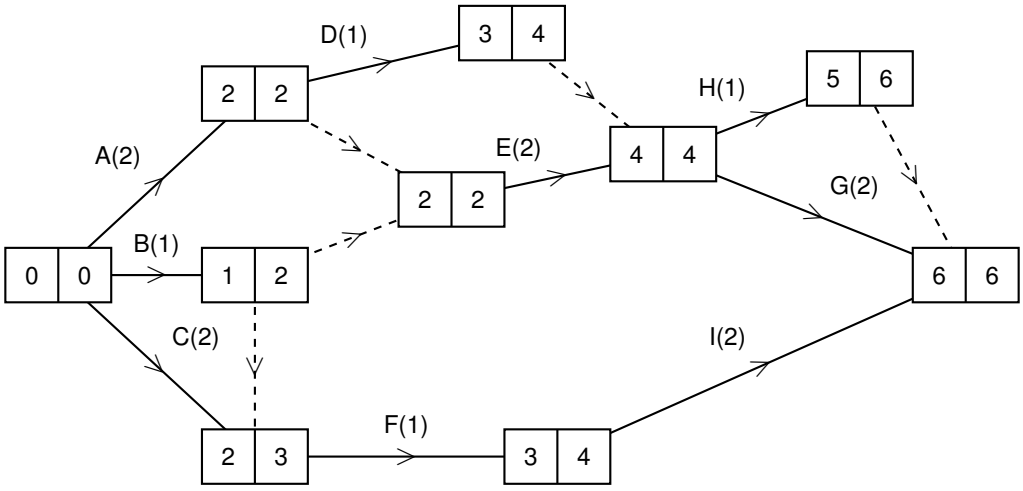

Question 7

10 marks

Part	Scheme	Marks
(a)	G and I depend on B, C and E, but F depends on C only so C must end at its own event, joined to the start of G and I by the dummy	**M1** **A1** **(2)**
(b)	Early times: 1: 0, 2: 6, 3: 5, 4: 13, 6: 14, 5: 9, 7: 17, 8: 18, 9: 24 All correct Late times: 1: 0, 2: 8, 3: 5, 4: 15, 6: 14, 5: 11, 7: 19, 8: 18, 9: 24 All correct	**M1** **A1** **M1** **A1** **(4)**
(c)	$\boxed{C–F–J–L}$	**B1** **(1)**
(d)	$19 - 13 - 4 = \boxed{2}$	**B1** **(1)**
(e)	$\frac{70}{24} = 2.92$ $\boxed{3 \text{ workers}}$	**M1** **A1** **(2)**
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	E: $15-3-10$; F: $15-9-3$; H: $16-5-8$; I: $20-13-5$	M1
	E 2, F 3, H 3, I 2	A1
	Two critical paths: A–C and B–D both take 9 days to event 4, then G–J	B1
		(3)
(b)	Critical activities A, B, C, D, G, J shown	M1
	At least two non-critical bars correct	M1
	All non-critical bars correct	A1
	Floats shown and shaded	A1
		(4)
(c)	Day 13 is the interval $12 < t < 13$	B1
	E, G, H	M1
	3 workers	A1
	Jan 2025 report: use a strict inequality for the time interval.	
		(3)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	$\frac{34}{17} = 2.00$	M1
	2 workers	A1
		(2)
(b)	Two workers, every activity once, durations correct	M1
	Precedences respected (e.g. E not before C and I finish)	M1
	Worker 1: A 0–5; C 5–9; E 9–12; G 12–17	A1
	Worker 2: B 0–3; I 3–5; D 5–11; F 11–15; H 15–17	A1
	Finishes at 17 days	A1
		(5)
(c)	D has total float 2	M1
	The extra 3 days exceed the float by 1	M1
	Minimum completion time becomes 18 days	A1
		(3)
		Total for Question 9: 10 marks

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Question 10

10 marks

Part	Scheme	Marks
(a)	Early times: 1: 0, 2: 4, 3: 6, 4: 3, 5: 11, 6: 12, 7: 17	**M1**
	Late times: 1: 0, 2: 4, 3: 6, 4: 4, 5: 11, 6: 13, 7: 17	**M1**
	All correct	**A1**
	Critical activities $\boxed{A, B, D, E, H}$ (two critical paths)	**A1**
	$\boxed{17 \text{ hours}}$	**A1**
		(5)
(b)	Path B–F–I has length $6+x+4 = 10+x$; the critical paths have length 17	**M1**
	F critical when $10+x \geqslant 17$: $\boxed{x \geqslant 7}$	**A1**
	Completion time $\boxed{10+x}$ hours	**A1**
		(3)
(c)	Float $= 17-(10+x)$	**M1**
	$\boxed{7-x}$	**A1**
		(2)
		Total for Question 10: 10 marks

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7 Chapter 7: Linear Programming

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
(a)	x is the number of trays of muffins and y the number of trays of scones made each day	B1 (1)
(b)	Maximise $P = 12x + 9y$ *Jan 2025 report: "maximise" must be stated; "maximum" is not accepted.*	B1 (1)
(c)	Oven: $20x + 15y \leqslant 360$ $4x + 3y \leqslant 72$ Flour: $3x + 2y \leqslant 50$ Ratio: $x \leqslant 3y$; and $x,\ y \geqslant 0$	M1 A1 B1 B1 (4)
		Total for Question 1: 6 marks

Question 2

6 marks

Part	Scheme	Marks
(a)	$x + 2y = 16$ drawn correctly $3x + 2y = 24$ drawn correctly $x = 2y$ drawn correctly Correct region labelled R, unwanted regions shaded clearly	B1 B1 B1 B1 (4)
(b)	Three vertices correct $(0, 0), (6, 3), (4, 6), (0, 8)$	M1 A1 (2)
		Total for Question 2: 6 marks

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Question 3

6 marks

Part	Scheme	Marks
(a)	Objective line with gradient $-\frac{3}{4}$ drawn (e.g. through $(4,0)$ and $(0,3)$)	M1
	Line moved parallel, away from the origin	M1
	Optimal vertex where $2x+3y=30$ meets $5x+2y=40$	A1
		(3)
(b)	Solves $2x+3y=30$ and $5x+2y=40$	M1
	$\left(\frac{60}{11}, \frac{70}{11}\right)$	A1
		(2)
(c)	$P=\frac{460}{11}$	B1
		(1)
		Total for Question 3: 6 marks

Question 4

6 marks

Part	Scheme	Marks
(a)	All three lines drawn correctly	B1
	Unbounded region above all three lines labelled R	B1
		(2)
(b)	Vertices $(0,12)$, $(2,6)$, $(6,2)$, $(12,0)$ found by simultaneous equations	M1
	At least two vertices correct	A1
	C: 24, 22, 34, 60	M1
	$C=22$ at $x=2$, $y=6$	A1
		(4)
		Total for Question 4: 6 marks

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Question 5

6 marks

Part	Scheme	Marks
(a)	Solves $3x+5y=45$ and $5x+2y=40$	M1
	$\left(\frac{110}{19}, \frac{105}{19}\right)$	A1
		(2)
(b)	Tests integer points near the vertex, e.g. $(5,5)$, $(5,6)$, $(6,4)$, $(6,5)$	M1
	Checks each against the constraints: $(6,5)$ and $(5,6)$ are feasible; $(6,6)$ is not	M1
	$P(5,6)=50$, $P(6,5)=49$	A1
	$x=5$, $y=6$, $P=50$	A1
		(4)
		Total for Question 5: 6 marks

Question 6

10 marks

Part	Scheme	Marks
(a)	$0.8x+1.2y$	B1
	Maximise $P=0.8x+1.2y$	B1
	Jan 2025 report: do not multiply the objective through (for example by 10).	
		(2)
(b)	Fraction of the day for A: $\frac{x}{400}$; for B: $\frac{y}{300}$	M1
	$\frac{x}{400}+\frac{y}{300}\leqslant 1$	A1
	Multiplying by 1200: $3x+4y\leqslant 1200$ *	A1*
		(3)
(c)	$y\geqslant 0.3(x+y)$	M1
	$7y\geqslant 3x$	A1
	$x\geqslant 120$	B1
	Jan 2025 report: convert percentages, and simplify to integer coefficients with the correct inequality direction.	
		(3)
(d)	Volumes of liquid can take any value, so x and y are continuous	B1
	e.g. all the drink made can be sold, or the profit per litre is constant	B1
		(2)
		Total for Question 6: 10 marks

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Question 7

10 marks

Part	Scheme	Marks
(a)	$x+y \geqslant 6$, $2x-y \leqslant 8$	**B1**
	$x+3y \leqslant 27$, $2y-x \leqslant 10$	**B1**
		(2)
(b)	Objective line or vertex method; vertices by simultaneous equations	**M1**
	Maximum at $\ell_2 \cap \ell_3$: $\left(\frac{51}{7}, \frac{46}{7}\right)$	**A1**
	$P_{\max} = \frac{240}{7}$	**A1**
	Minimum at $\ell_1 \cap \ell_2$: $\left(\frac{14}{3}, \frac{4}{3}\right)$	**A1**
	$P_{\min} = \frac{40}{3}$	**A1**
		(5)
(c)	Tests integer points in R near $\left(\frac{14}{3}, \frac{4}{3}\right)$ (on the feasible side of ℓ_1 and ℓ_2)	**M1**
	$(4,2)$ is in R; $(5,1)$ breaks $2x-y \leqslant 8$	**M1**
	$P = 14$ at $(4,2)$	**A1**
	Jan 2025 report: an integer point for a minimum must satisfy every constraint.	
		(3)
		Total for Question 7: 10 marks

Question 8

10 marks

Part	Scheme	Marks
(a)	Lines $x+2y=20$ and $3x+2y=36$ correct	**B1**
	$y=8$ and $x=10$ correct	**B1**
	R labelled	**B1**
		(3)
(b)	At least four correct	**M1**
	$(0,0)$, $(10,0)$, $(10,3)$, $(8,6)$, $(4,8)$, $(0,8)$	**A1**
		(2)
(c)	$(8,6)$ is where $x+2y=20$ (gradient $-\frac{1}{2}$) meets $3x+2y=36$ (gradient $-\frac{3}{2}$)	**M1**
	Objective gradient $-k$ must lie between them	**M1**
	$\frac{1}{2} \leqslant k \leqslant \frac{3}{2}$	**A1**
		(3)
(d)	$P=8+6$	**M1**
	14	**A1**
		(2)
		Total for Question 8: 10 marks

Question 9

10 marks

Part	Scheme	Marks
(a)	x chairs and y tables per week; maximise $P=30x+45y$	**B1**
	$3x+5y \leqslant 45$, $4x+3y \leqslant 48$	**B1**
	$x \geqslant 2y$; $x,\ y \geqslant 0$ (integers)	**B1**
		(3)
(b)	Two lines correct	**B1**
	All three lines correct	**B1**
	R labelled	**B1**
		(3)
(c)	Objective line, gradient $-\frac{2}{3}$, moved away from O: last vertex $\left(\frac{105}{11}, \frac{36}{11}\right)$	**M1**
	Not integers, so test nearby integer points in R	**M1**
	$(8,4)$ gives 420; $(9,3)$ gives 405; $(10,2)$ gives 390	**A1**
	8 chairs and 4 tables, profit £420	**A1**
		(4)
		Total for Question 9: 10 marks

Question 10

10 marks

Part	Scheme	Marks
(a)	$6x+2y=25$ and $2x+4y=21$ correct $x+y=8$ correct Unbounded region labelled R	**B1** **B1** **B1** **(3)**
(b)	Objective line ($5x+4y=k$) moved towards O: optimal where $6x+2y=25$ meets $x+y=8$ $\left(\frac{9}{4}, \frac{23}{4}\right)$ Minimum cost £34.25	**M1** **A1** **A1** **(3)**
(c)	Tests feasible integer points near the vertex: $(3,5)$ gives 35, $(2,6)$ is not feasible, $(4,4)$ gives 36 3 kg of A and 5 kg of B, £35	**M1** **A1** **(2)**
(d)	Gradient $-\frac{c}{4}$ between -3 and -1 $4 \leqslant c \leqslant 12$	**M1** **A1** **(2)**
		Total for Question 10: 10 marks

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