eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 3

Further Pure 3

Chapter 3: Differentiation

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM03/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM03 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Hyperbolic Functions** Definitions, graphs, inverse functions, identities and equations	10	80	separate file	
2	**Further Coordinate Systems** Ellipses and hyperbolas, foci and directrices, tangents, normals and loci	10	80	separate file	
3	**Differentiation** Hyperbolic and inverse trigonometric and hyperbolic functions	10	80	2	
4	**Integration** Substitutions, reduction formulae, arc length and surface area	10	80	separate file	
5	**Vectors** Vector product, triple products, lines and planes, distances and angles	10	80	separate file	
6	**Further Matrix Algebra** 3×3 matrices, inverses, eigenvalues and eigenvectors, diagonalisation	10	80	separate file	

Full mark scheme | Free courses | More worksheets

Name ..... Class ..... Date .....

CHAPTER 3 • KEY SKILLS

3 Differentiation

Specification FP3: Differentiation • 10 questions • 80 marks • about 96 minutes

1 Hyperbolic derivatives

$\sinh \to \cosh$, $\cosh \to \sinh$,

$\tanh \to \operatorname{sech}^2$, $\operatorname{sech} \to -\operatorname{sech}\tanh$,

$\operatorname{cosech} \to -\operatorname{cosech}\coth$,

$\coth \to -\operatorname{cosech}^2$

2 Chain, product, quotient

$\frac{\mathrm{d}}{\mathrm{d}x}\tanh^2(2x) = 2\tanh 2x \times 2\operatorname{sech}^2 2x$:
do not lose the inner derivative.

3 Inverse trigonometric

$\arcsin x \to \frac{1}{\sqrt{1-x^2}}$,

$\arccos x \to -\frac{1}{\sqrt{1-x^2}}$,

$\arctan x \to \frac{1}{1+x^2}$

4 Inverse hyperbolic

$\operatorname{arsinh} x \to \frac{1}{\sqrt{x^2+1}}$,

$\operatorname{arcosh} x \to \frac{1}{\sqrt{x^2-1}}$,

$\operatorname{artanh} x \to \frac{1}{1-x^2}$

5 Deriving them

Write $x = \mathrm{f}(y)$, find $\frac{\mathrm{d}x}{\mathrm{d}y}$, use an identity to write it in x, then take the reciprocal.

6 Stationary points

Set $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$; equations such as $2\sinh x + \cosh x = 0$ give $\tanh x = -\frac{1}{2}$

7 Exact values

Use $\cosh x = \frac{1}{\sqrt{1-\tanh^2 x}}$ or logarithms to give exact coordinates.

8 Differential equations

Find y' and y'', then substitute; clearing a square root first often helps.

9 Valid solutions

Check domains: $|x| < 1$ for artanh, $x \geqslant 1$ for arcosh; reject values outside them.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Hyperbolic derivatives	•			•		•	•			
2 Chain, product, quotient	•	•	•			•	•	•	•	
3 Inverse trigonometric		•			•			•		
4 Inverse hyperbolic			•			•			•	•
5 Deriving them		•	•			•				
6 Stationary points				•			•			•
7 Exact values				•		•	•			•
8 Differential equations					•				•	
9 Valid solutions						•		•		•
Interleaved: hyperbolic identities and logarithms (Chapter 1)			•	•		•	•			•
Interleaved: tangents, stationary points, second derivative (P2, P3)				•		•	•			•
Interleaved: trigonometric identities (P3)		•			•			•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM03 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write every "h" clearly. When deriving an inverse derivative, show $x = \mathrm{f}(y)$, the derivative in y, the identity used and the final form in x.

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

1. Differentiate with respect to x

(a) $\cosh 3x \sinh 2x$

(2)

(b) $\tanh^2(2x)$

(2)

(c) $\ln(\cosh x)$

(1)

(d) $\operatorname{sech} x$

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 1 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2. **(a)** Given that $y = \arcsin x$, $-1 < x < 1$, show that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

(3)

(b) Find $\dfrac{d}{dx}(\arctan 3x)$

(1)

(c) Find $\dfrac{d}{dx}\left(\arccos x^2\right)$, $0 < x < 1$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3. **(a)** Given that $y = \text{artanh}\, x$, $|x| < 1$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{1-x^2}$

(3)

(b) Find $\dfrac{\mathrm{d}}{\mathrm{d}x}(\text{arsinh}\, 2x)$

(1)

(c) Find $\dfrac{\mathrm{d}}{\mathrm{d}x}(x\,\text{arcosh}\, x)$, $x > 1$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 3 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4. The curve C has equation $y = 2\cosh x + \sinh x$

(a) Find the exact coordinates of the stationary point of C.

(4)

(b) Determine the nature of the stationary point.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 4 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5. The function $y = \arcsin x$ is defined for $-1 < x < 1$. By finding its first and second derivatives, prove that y is a solution of the differential equation

$$\left(1 - x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x\frac{\mathrm{d}y}{\mathrm{d}x}$$

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 5 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6. **(a)** Given that $y = \text{arsech}\left(\frac{x}{2}\right)$, $0 < x < 2$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{x\sqrt{4-x^2}}$

(5)

(b) Find the exact value of x, $0 < x < 1$, at which the gradient of $y = \text{artanh}\, x$ is $\frac{4}{3}$, explaining why any other value is rejected.

(2)

(c) Find an equation of the tangent to $y = \text{arsech}\left(\frac{x}{2}\right)$ at the point where $x = 1$, giving the y-intercept as an exact expression involving a logarithm.

(3)

DO NOT WRITE IN THIS AREA

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 6 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7. The curve C has equation $y = \mathrm{e}^{-x}\cosh 2x$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-x}(2\sinh 2x - \cosh 2x)$

(2)

(b) Show that the x-coordinate of the stationary point of C is $\frac{1}{4}\ln 3$

(3)

(c) Find the exact y-coordinate of the stationary point, in the form $k \times 3^n$

(3)

(d) Determine the nature of the stationary point.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 7 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8. **(a)** By differentiating, show that $\arctan x + \arctan \frac{1}{x}$ is constant for $x > 0$, and find the constant.

(4)

(b) Show that, for $|x| < 1$, $\frac{\mathrm{d}}{\mathrm{d}x} \arcsin\left(\frac{2x}{1+x^2}\right) = \frac{2}{1+x^2}$

(4)

(c) Hence show that $\arcsin\left(\frac{2x}{1+x^2}\right) = 2\arctan x$ for $|x| < 1$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

9.

(a) Given that $y = \text{arsinh}\, x$, show that $\left(1+x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\dfrac{\mathrm{d}y}{\mathrm{d}x}=0$

(5)

(b) Given instead that $y = (\text{arsinh}\, x)^2$, show that $\left(1+x^2\right)\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}+x\dfrac{\mathrm{d}y}{\mathrm{d}x}=2$

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 9 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

10.

$$f(x) = 2\operatorname{arcosh} x - x, \qquad x \geqslant 1$$

(a) Find the exact coordinates of the stationary point of $y = f(x)$, and show that it is a maximum.

(6)

(b) Find an equation of the tangent to $y = f(x)$ at the point where $x = \frac{5}{3}$, giving exact values.

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 3 IS 80 MARKS

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk