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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 3

Further Pure 3

Chapter 4: Integration

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM03/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM03 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Hyperbolic Functions** Definitions, graphs, inverse functions, identities and equations	10	80	separate file	
2	**Further Coordinate Systems** Ellipses and hyperbolas, foci and directrices, tangents, normals and loci	10	80	separate file	
3	**Differentiation** Hyperbolic and inverse trigonometric and hyperbolic functions	10	80	separate file	
4	**Integration** Substitutions, reduction formulae, arc length and surface area	10	80	2	
5	**Vectors** Vector product, triple products, lines and planes, distances and angles	10	80	separate file	
6	**Further Matrix Algebra** 3×3 matrices, inverses, eigenvalues and eigenvectors, diagonalisation	10	80	separate file	

Full mark scheme | Free courses | More worksheets

Name ..... Class ..... Date .....

CHAPTER 4 • KEY SKILLS

4 Integration

Specification FP3: Integration • 10 questions • 80 marks • about 96 minutes

1 Hyperbolic integrals

$\sinh x \to \cosh x$, $\cosh x \to \sinh x$, $\tanh x \to \ln\cosh x$; use $\cosh^2 x = \frac{1}{2}(\cosh 2x + 1)$

2 Standard results

$\frac{1}{\sqrt{a^2+x^2}} \to \operatorname{arsinh}\frac{x}{a}$;

$\frac{1}{\sqrt{x^2-a^2}} \to \operatorname{arcosh}\frac{x}{a}$;

$\frac{1}{a^2+x^2} \to \frac{1}{a}\arctan\frac{x}{a}$;

$\frac{1}{\sqrt{a^2-x^2}} \to \arcsin\frac{x}{a}$

3 Completing the square

$x^2+4x+13 = (x+2)^2+9$; with $-x^2$, write $a^2-(x-p)^2$. Never take a minus sign out of a square root.

4 Coefficient of x^2

$\sqrt{64-(2x-1)^2}$: with $u = 2x-1$, $\mathrm{d}x = \frac{1}{2}\mathrm{d}u$, so keep the factor $\frac{1}{2}$

5 Substitutions

$x = a\sinh u$ for $\sqrt{a^2+x^2}$, $x = a\cosh u$ for $\sqrt{x^2-a^2}$, $x = a\sin u$ for $\sqrt{a^2-x^2}$; change the limits.

6 Reduction formulae

Integrate by parts to relate I_n to I_{n-1} or I_{n-2}; bracket carefully, then work down to I_0 or I_1

7 Arc length

$s = \int \sqrt{1+y'^2}\,\mathrm{d}x$ or $\int \sqrt{\dot{x}^2+\dot{y}^2}\,\mathrm{d}t$.

Simplify the derivative before squaring.

8 Surface area

Rotating about the x-axis:

$S = 2\pi\int y\,\mathrm{d}s$. "Total" surface area adds any end discs.

9 Exact answers

Give logs and surds exactly: $\operatorname{arsinh}1 = \ln\left(1+\sqrt{2}\right)$; write the h in arsinh, arcosh, artanh.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Hyperbolic integrals	•			•	•			•		•
2 Standard results		•	•				•		•	
3 Completing the square			•						•	
4 Coefficient of x^2									•	
5 Substitutions				•			•	•		
6 Reduction formulae						•				
7 Arc length					•		•			•
8 Surface area							•			•
9 Exact answers	•	•	•	•	•	•	•	•	•	•
Interleaved: integration by parts and substitution (P4)				•		•	•	•		
Interleaved: hyperbolic functions and identities (Chapter 1)	•	•		•	•		•	•		•
Interleaved: parametric differentiation and volumes (P4)							•			•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM03 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Complete the square before reaching for the formula book, change the limits with every substitution, and read whether the question asks for a curved or a total surface area.

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1.

(a) Find $\int \cosh^2 x \, \mathrm{d}x$

(2)

(b) Find the exact value of $\int_0^{\ln 2} \sinh x \, \mathrm{d}x$

(2)

(c) Find $\int \tanh x \, \mathrm{d}x$

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Using standard integrals from the formula book, evaluate exactly

(a) $\int_0^3 \frac{1}{\sqrt{9+x^2}}\,\mathrm{d}x$, as a logarithm,

(2)

(b) $\int_0^2 \frac{1}{4+x^2}\,\mathrm{d}x$

(2)

(c) $\int_3^5 \frac{1}{\sqrt{x^2-9}}\,\mathrm{d}x$, giving your answer as a logarithm.

(2)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. By completing the square, find

(a) $\int \frac{1}{\sqrt{x^2+4x+13}}\,\mathrm{d}x$

(3)

(b) $\int \frac{1}{\sqrt{8+2x-x^2}}\,\mathrm{d}x$

(3)

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Question 3 continued

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4. Use the substitution $x = 2\sinh u$ to find the exact value of

$$\int_0^2 \sqrt{4+x^2}\,\mathrm{d}x$$

giving your answer in the form $a\ln\left(1+\sqrt{2}\right)+b\sqrt{2}$

(6)

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Question 4 continued

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5. **(a)** Find the length of the arc of the curve $y = \cosh x$ from $x = 0$ to $x = \ln 3$

(3)

(b) Find the length of the arc of the curve $y = \frac{2}{3}x^{\frac{3}{2}}$ from $x = 0$ to $x = 3$

(3)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6.

$$I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, \mathrm{d}x, \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$, $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$

(5)

(b) Hence find the exact value of I_4, giving your answer in the form $a\pi^3 + b\pi + c$

(5)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. The curve C has parametric equations $x = t^2$, $y = 2t$, $0 \leqslant t \leqslant 1$

(a) Show that the length of C is $\sqrt{2} + \ln\left(1 + \sqrt{2}\right)$

(5)

(b) A bowl is modelled by spinning C completely about the x-axis. Calculate the exact area of the bowl's curved surface.

(5)

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Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8.

$$I = \int_4^5 \sqrt{x^2 - 16}\,\mathrm{d}x$$

(a) Use the substitution $x = 4\cosh\theta$ to show that $I = 16\int_0^{\ln 2} \sinh^2\theta\,\mathrm{d}\theta$

(3)

(b) Hence show that $I = \frac{15}{2} - 8\ln 2$

(5)

(c) Interpret I as an area, and check that its value is reasonable.

(2)

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Question 8 continued

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Question 8 continued

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9.

(a) Find $\int \frac{1}{\sqrt{63+4x-4x^2}}\,\mathrm{d}x$

(5)

(b) Find the exact value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{4x^2+4x+5}\,\mathrm{d}x$

(5)

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Question 9 continued

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Question 9 continued

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10. The curve C has equation $y = 2\cosh\left(\frac{x}{2}\right)$, $0 \leqslant x \leqslant 2\ln 3$

(a) Find the length of C.

(3)

(b) The curve is rotated through 2π radians about the x-axis. Show that the area of the curved surface generated is $4\pi\ln 3 + \frac{80\pi}{9}$

(5)

(c) Find the exact **total** surface area of the solid formed.

(2)

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Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 4 IS 80 MARKS

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