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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 1 (WFM01)

Further Pure 1 Practice Paper 2

Edexcel IAL Further Pure Mathematics 1

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM01/01) • Calculator allowed

A full 75-mark paper, harder than a typical series, with chapters combined in each question and a separate mark scheme

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 9 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	A matrix with a parameter and an inverse	7	
2	A cubic: a numerical root, then complex roots	8	
3	Complex roots and a new quadratic	8	
4	A series and an equation in n	9	
5	A tangent to a parabola and the directrix	9	
6	A combined transformation and area	9	
7	A complex equation and modulus-argument form	8	
8	Proof by induction for a matrix power	8	
9	A normal that meets the hyperbola again	9	
	Total	**75**	

Rough grade guide (out of 75): A **58** • B **51** • C **44**

Based on recent WFM01 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1.

$$\mathbf{M} = \begin{pmatrix} k & 2 \\ 3 & k+1 \end{pmatrix}$$

where k is a constant.

(a) Find the values of k for which $\mathbf{M}$ is singular.

(3)

(b) Given that $\mathbf{M}$ is non-singular, find $\mathbf{M}^{-1}$ in terms of k

(2)

Given that $k = 4$, the transformation represented by $\mathbf{M}$ maps the point P onto the point $(10, 18)$

(c) Use your answer to part (b) to find the coordinates of P

(2)

(Total for Question 1 is 7 marks)

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2.

$$f(x) = x^3 - 2x^2 + 5x - 10$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.5, 2.5]

(2)

(b) Taking 1.5 as a first approximation to α, apply the Newton–Raphson process once to $f(x)$ to find a second approximation to α, giving your answer to 3 decimal places.

(3)

(c) Show that $\alpha = 2$ exactly, and hence find the other two roots of the equation $f(x) = 0$

(3)

(Total for Question 2 is 8 marks)

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3. The roots of the equation

$$3x^2 - 2x + 4 = 0$$

are α and β

(a) Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$

(1)

(b) Without solving the equation, find the exact value of $\alpha^3 + \beta^3$

(3)

(c) Find a quadratic equation, with integer coefficients, whose roots are

$$\frac{\alpha}{\beta} \quad \text{and} \quad \frac{\beta}{\alpha}$$

(4)

(Total for Question 3 is 8 marks)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(an+b)$$

where a and b are integers to be found.

(4)

(b) Hence find the value of n for which

$$\sum_{r=1}^{n} r(2r-1) = \sum_{r=1}^{n} \left(r^2+40\right)$$

(5)

(Total for Question 4 is 9 marks)

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5.

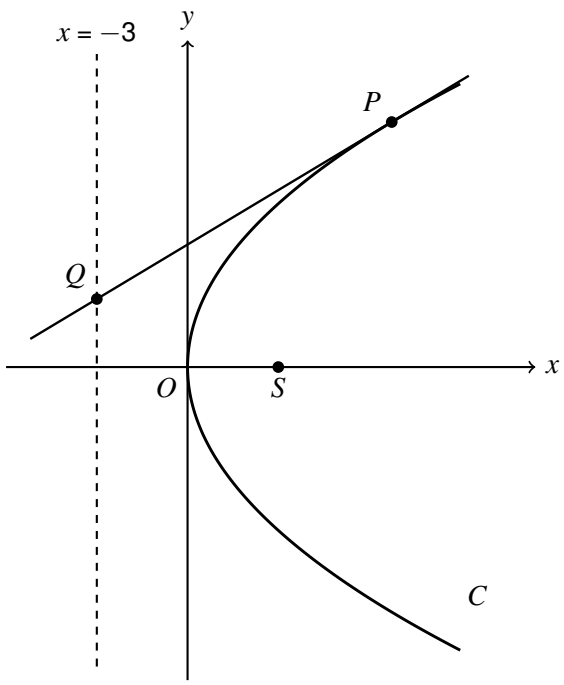

Figure 1

Figure 1 shows part of the parabola C with equation $y^2 = 12x$. The point S is the focus of C and the line with equation $x = -3$ is the directrix of C.

The point $P\left(3p^2, 6p\right)$, $p > 0$, lies on C. The tangent to C at P meets the directrix at the point Q

(a) Using calculus, find an equation of the tangent to C at P

(3)

(b) Find, in terms of p, the coordinates of Q

(2)

Given that the distance SP is 15

(c) find the value of p

(2)

(d) find the area of the triangle PSQ

(2)

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Question 5 continued

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(Total for Question 5 is 9 marks)

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6. The transformation R is a reflection in the line $y = -x$

(a) Write down the matrix $\mathbf{R}$ that represents R

(1)

$$\mathbf{S} = \begin{pmatrix} k & -1 \\ 1 & 2 \end{pmatrix}$$

where k is a positive constant. The transformation represented by $\mathbf{S}$ is followed by the transformation R, and the combined transformation is represented by the matrix $\mathbf{C}$

(b) Find $\mathbf{C}$ in terms of k

(2)

(c) Find $\mathbf{C}^{-1}$ in terms of k

(2)

A triangle T has area 3 cm^2. The image of T under the transformation represented by $\mathbf{C}$ has area 33 cm^2

(d) Find the value of k

(2)

(e) Using this value of k, find the coordinates of the point that is mapped by $\mathbf{C}$ onto the point $(-8, -7)$

(2)

(Total for Question 6 is 9 marks)

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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The complex number z satisfies

$$3z - 2z^* = \frac{-12 + 8\mathrm{i}}{1 + \mathrm{i}}$$

where z^* is the complex conjugate of z

(a) Find z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real.

(4)

(b) Express z in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving r and θ as exact values.

(2)

(c) Find z^2, and show z and z^2 on a single Argand diagram.

(2)

(Total for Question 7 is 8 marks)

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8.

$$\mathbf{M} = \begin{pmatrix} 3 & 2 \\ 0 & 1 \end{pmatrix}$$

(a) Prove by induction that, for all positive integers n,

$$\mathbf{M}^n = \begin{pmatrix} 3^n & 3^n - 1 \\ 0 & 1 \end{pmatrix}$$

(6)

(b) Hence find $(\mathbf{M}^n)^{-1}$ in terms of n

(2)

(Total for Question 8 is 8 marks)

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9.

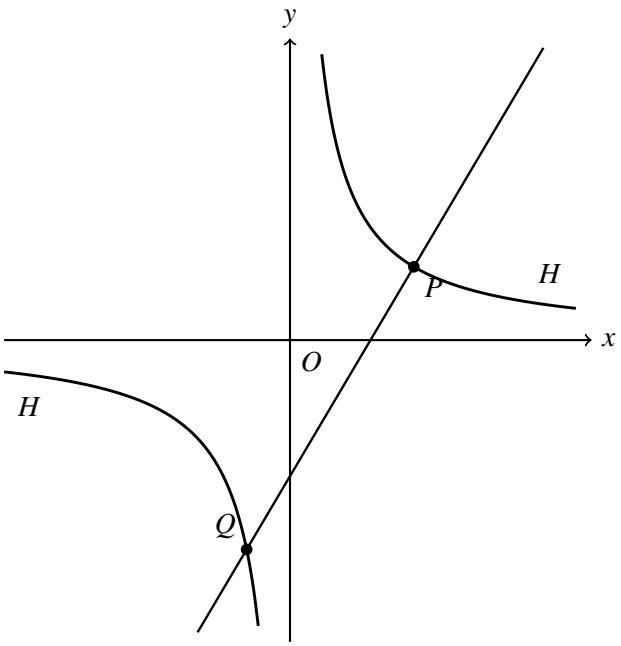

Figure 2

The rectangular hyperbola H has equation $xy = 9$. The point $P\left(3p, \frac{3}{p}\right)$, $p > 0$, lies on H. The normal to H at P meets H again at the point Q, as shown in Figure 2.

(a) Show that an equation of the normal to H at P is

$$p^3x - py = 3p^4 - 3$$

(4)

(b) Find, in terms of p, the coordinates of Q

(4)

(c) Write down the coordinates of Q when P is the point $\left(6, \frac{3}{2}\right)$

(1)

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Question 9 continued

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(Total for Question 9 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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