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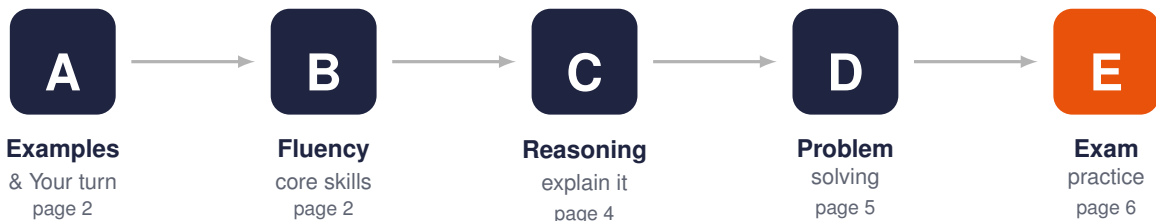
GCSE MATHEMATICS • NUMBER

Powers and Roots

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • Calculator only where shown

Full worked solutions for the free Powers and Roots worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Powers and Roots worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

Question sheet

Number revision engine

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A**Your turn answers**

Compare each line of your working with the solution.

FOUNDATION

A1 $4 \times 4 \times 4 = 64$, so $\sqrt[3]{64} = 4$ $3^4 = 3 \times 3 \times 3 \times 3 = 81$ $4 + 81 = \boxed{85}$

A2 (a) $11^2 = 121$ and $(-11)^2 = 121$ $\boxed{x = 11 \text{ or } x = -11}$ (b) $(-3) \times (-3) \times (-3) = -27$ $\boxed{x = -3}$

A3 $7.3^3 - 12.5 = 389.017 - 12.5 = 376.517$ $\sqrt{376.517} = 19.404\dots$ $\boxed{19.4}$

HIGHER

A4 $180 = 36 \times 5$ $\sqrt{180} = \sqrt{36} \times \sqrt{5} = \boxed{6\sqrt{5}}$

A5 $\sqrt{8} = 2\sqrt{2}$ $\sqrt{98} = 7\sqrt{2}$ $2\sqrt{2} + 7\sqrt{2} = \boxed{9\sqrt{2}}$

A6 $\frac{22}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}} = \frac{22\sqrt{11}}{11} = \boxed{2\sqrt{11}}$

A7 $(3 - \sqrt{2})(3 - \sqrt{2}) = 9 - 3\sqrt{2} - 3\sqrt{2} + 2 = \boxed{11 - 6\sqrt{2}}$

A8 Multiply top and bottom by $\sqrt{6} - 1$ Bottom: $(\sqrt{6} + 1)(\sqrt{6} - 1) = 6 - 1 = 5$ Top: $10(\sqrt{6} - 1)$
 $\frac{10(\sqrt{6} - 1)}{5} = 2(\sqrt{6} - 1) = \boxed{2\sqrt{6} - 2}$

B**Fluency solutions**

One mark for each correct part unless stated. The notes in italics explain why each sequence was chosen.

FOUNDATION

B1 (a) $\boxed{49}$ (b) $\boxed{49}$ (c) $\boxed{-49}$ (d) $\boxed{0.49}$

In (c) there is no bracket, so only the 7 is squared and the answer stays negative. In (d), 0.7×0.7 has two decimal places.



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B2 (a) 8 (b) -8 (c) 1000 (d) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

A negative number cubed is negative. Cubing a number between 0 and 1 makes it smaller.

B3 (a) 8 (b) 4 (c) $\frac{1}{2}$ (d) 0.3

64 is both a square number and a cube number. The square root of a number between 0 and 1 is bigger than the number.

B4 (a) 32 (b) 81 (c) $1\,000\,000$ (d) 1

B5 (a) $x = 6$ or $x = -6$ (b) $x = 5$ (c) $x = 1.5$ or $x = -1.5$ (d) $x = -10$

B6 (a) 50.653 (b) $14.1421 \dots$ 14.14 (c) $-3.68403 \dots$ -3.684

B7 $2^5 = 32$ $5^2 = 25$ $3^3 = 27$ $\sqrt{900} = 30$ $5^2, 3^3, \sqrt{900}, 2^5$

HIGHER

B8 (a) $\sqrt{25} \times \sqrt{2} = 5\sqrt{2}$ (b) $\sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ (c) $\sqrt{9} \times \sqrt{7} = 3\sqrt{7}$ (d) $3 \times 2\sqrt{2} = 6\sqrt{2}$

B9 (a) $\sqrt{36} = 6$ (b) $\sqrt{50} = 5\sqrt{2}$ (c) $\sqrt{36} = 6$ (d) $4 \times 3 = 12$

B10 (a) $2\sqrt{3} + 3\sqrt{3} = 5\sqrt{3}$ (b) $5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$
 (c) $3 \times 2\sqrt{5} + 3\sqrt{5} = 6\sqrt{5} + 3\sqrt{5} = 9\sqrt{5}$ (d) $2\sqrt{2} + 4\sqrt{2} - \sqrt{2} = 5\sqrt{2}$

B11 (a) $\frac{6\sqrt{3}}{3} = 2\sqrt{3}$ (b) $\frac{10\sqrt{2}}{2} = 5\sqrt{2}$ (c) $\frac{\sqrt{5}}{5}$ (d) $\frac{9\sqrt{3}}{2 \times 3} = \frac{9\sqrt{3}}{6} = \frac{3\sqrt{3}}{2}$

B12 (a) $3\sqrt{2} + 2$ (b) $3 - \sqrt{5} + 3\sqrt{5} - 5 = -2 + 2\sqrt{5}$
 (c) $16 - 4\sqrt{3} - 4\sqrt{3} + 3 = 19 - 8\sqrt{3}$ (d) $7 - 2\sqrt{7} + 2\sqrt{7} - 4 = 3$

In (d) the brackets differ only in sign, so the surd terms cancel. This is the idea behind rationalising a two-term denominator.



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

C1 $(-5)^2 = (-5) \times (-5)$, and a negative times a negative is positive. $(-5)^2 = 25$

C2 $\sqrt{9+16} = \sqrt{25} = 5$ $\sqrt{9} + \sqrt{16} = 3+4 = 7$ **No**

C3 $2^6 = 64$ $6^2 = 36$ **2^6 is greater**

C4 **64** $64 = 8^2$ and $64 = 4^3$

C5 (a) $6^2 = 36$ and $7^2 = 49$, and 40 is between 36 and 49, so $\sqrt{40}$ is **between 6 and 7**

(b) For example $\sqrt[3]{\frac{1}{8}} = \frac{1}{2}$, and $\frac{1}{2}$ is bigger than $\frac{1}{8}$. (Any number between 0 and 1 works, and so does any number below -1 .)

HIGHER

C6 (a) $\sqrt{18} = 3\sqrt{2}$, so $\sqrt{18} + \sqrt{2} = 3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$. But $\sqrt{20} = 2\sqrt{5}$, which is different. You cannot add the numbers under the roots.

(b) $4\sqrt{2} = \sqrt{16} \times \sqrt{2} = \sqrt{32}$

C7 He replaced $\sqrt{3}$ with 3. He should multiply top and bottom by $\sqrt{3}$: $\frac{6\sqrt{3}}{3} = \sqrt{3}$

C8 $\sqrt{12} = 2\sqrt{3}$, so $\sqrt{12} - \sqrt{3} = \sqrt{3}$ $(\sqrt{3})^2 = 3$ **3, a whole number**

C9 (a) $5\sqrt{3} = \sqrt{25} \times \sqrt{3} = \sqrt{75}$ **$a = 75$** (b) $\sqrt{27} = 3\sqrt{3}$, so $\sqrt{b} = 2\sqrt{3} = \sqrt{12}$ **$b = 12$**

C10 (a) Multiply top and bottom by $\sqrt{n+1} - \sqrt{n}$. The bottom becomes $(n+1) - n = 1$, so the fraction equals $\sqrt{n+1} - \sqrt{n}$. **as required**

(b) Using (a) with $n = 1, 2$ and 3 : $(\sqrt{2} - 1) + (\sqrt{3} - \sqrt{2}) + (\sqrt{4} - \sqrt{3})$

Everything cancels except $\sqrt{4} - 1 = 2 - 1 = \mathbf{1}$



D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

D1 Side = $\sqrt{169} = 13$ m Perimeter = $4 \times 13 =$ **52 m**

D2 Side = $\sqrt[3]{343} = 7$ cm One face = $7^2 = 49$ cm² $6 \times 49 =$ **294 cm²**

D3 (a) $1+8+27+64 = 100 = 10^2$ (b) The square roots go 3, 6, 10, so the next is 15. $100+125 = 225 = 15^2$
225

D4 $\frac{30}{4.9} = 6.1224\dots$ $\sqrt{6.1224\dots} = 2.4743\dots$ **2.47 seconds**

D5 Side of floor = $\sqrt{12.96} = 3.6$ m = 360 cm Tiles along one side: $360 \div 30 = 12$ $12^2 =$ **144 tiles**

HIGHER

D6 (a) $\sqrt{27} \times \sqrt{12} = \sqrt{324} =$ **18 cm²**

(b) $\sqrt{27} = 3\sqrt{3}$ and $\sqrt{12} = 2\sqrt{3}$ $2 \times (3\sqrt{3} + 2\sqrt{3}) = 2 \times 5\sqrt{3} =$ **$10\sqrt{3}$ cm**

D7 $(\sqrt{5})^2 + (2\sqrt{5})^2 = 5 + 20 = 25$ $\sqrt{25} =$ **5 cm**

D8 Length = $\frac{12}{\sqrt{5}-1}$ Multiply top and bottom by $\sqrt{5}+1$: $\frac{12(\sqrt{5}+1)}{5-1} = \frac{12(\sqrt{5}+1)}{4} = 3(\sqrt{5}+1)$
 $(3\sqrt{5}+3)$ cm

D9 side² + side² = 10², so side² = 50 side = $\sqrt{50} =$ **$5\sqrt{2}$ cm** Area = side² = **50 cm²**

D10 $x\sqrt{2} - x = 2$ $x(\sqrt{2} - 1) = 2$ $x = \frac{2}{\sqrt{2}-1}$
Multiply top and bottom by $\sqrt{2}+1$: $x = \frac{2(\sqrt{2}+1)}{2-1} =$ **$2\sqrt{2}+2$**



E

Exam practice solutions

Marks are shown on the right of each line.

FOUNDATION

Foundation and Higher crossover

- E1 (a) 81 B1
 (b) 3 B1
 (c) 16×5 M1
 80 A1
- E2 (a) $4.3 + 17.576$ M1
 21.876 A1
 (b) 21.9 B1 ft
- E3 (a) 36 B1
 (b) 27 B1
 (c) 64 B1
- E4 (a) $x = 14$ or $x = -14$ B1 for 14, B1 for -14
 (b) -6 B1

HIGHER

- E5 $\sqrt{28} = 2\sqrt{7}$ M1
 $\sqrt{112} = \sqrt{16} \times \sqrt{7} = 4\sqrt{7}$ M1
 $2\sqrt{7} + 4\sqrt{7} = 6\sqrt{7}$ A1
- E6 $\frac{18}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{18\sqrt{6}}{6}$ M1
 $3\sqrt{6}$ A1
- E7 $(3 + \sqrt{8})(3 + \sqrt{8}) = 9 + 3\sqrt{8} + 3\sqrt{8} + 8$ M1
 $= 17 + 6\sqrt{8}$ M1
 $\sqrt{8} = 2\sqrt{2}$, so $6\sqrt{8} = 12\sqrt{2}$, giving $17 + 12\sqrt{2}$ as required A1



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E8	Multiply top and bottom by $5 + \sqrt{3}$	<i>M1</i>
	Bottom: $25 - 3 = 22$	<i>M1</i>
	$\frac{22(5 + \sqrt{3})}{22} = \boxed{5 + \sqrt{3}}$	<i>A1</i>
E9	$\sqrt{45} = 3\sqrt{5}$, so the top is $3\sqrt{5} - \sqrt{5} = 2\sqrt{5}$	<i>M1</i>
	Multiply top and bottom by $\sqrt{5} - 1$	<i>M1</i>
	Top: $2\sqrt{5}(\sqrt{5} - 1) = 10 - 2\sqrt{5}$ Bottom: $5 - 1 = 4$	<i>M1</i>
	$\frac{10 - 2\sqrt{5}}{4} = \frac{5 - \sqrt{5}}{2} \quad \boxed{a = 5, b = 2}$	<i>A1</i>

