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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 2

Further Pure 2

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM02/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Inequalities** Rational and modulus inequalities, critical values, graphs	10	80	2	
2	**Series** Method of differences, partial fractions, telescoping sums	10	80	28	
3	**Complex Numbers** Exponential form, de Moivre's theorem, trigonometric identities, nth roots	10	80	54	
4	**Further Argand Diagrams** Loci, regions and transformations of the complex plane	10	80	80	
5	**First-Order Differential Equations** Integrating factors, substitutions, general and particular solutions	10	80	106	
6	**Second-Order Differential Equations** Auxiliary equations, complementary functions, particular integrals, substitutions	10	80	132	
7	**Maclaurin and Taylor Series** Higher derivatives, standard and compound series, Taylor series, series solutions	10	80	158	
8	**Polar Coordinates** Polar curves, areas, tangents parallel and perpendicular to the initial line	10	80	184	

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CHAPTER 1 • KEY SKILLS

1 Inequalities

Specification FP2: Inequalities • 10 questions • 80 marks • about 96 minutes

1 Never multiply blindly

Do not multiply by an expression whose sign is unknown. Multiply by its square, or collect everything on one side.

2 Single fraction

Write as $\frac{f(x)}{g(x)} > 0$: the critical values are the zeros of the numerator and of the denominator.

3 Signs or sketch

Test the sign in each interval, or sketch $y = f(x)/g(x)$ (or both sides), to choose the regions.

4 Excluded values

A value making a denominator zero is never included, even with $\leqslant$ or $\geqslant$

5 Modulus cases

$|f(x)| = g(x)$ gives $f(x) = g(x)$ or $-f(x) = g(x)$. Check each root in the original: some are invalid.

6 Use the graph

Sketch both sides: the solution is where one graph is above the other, including regions where one side is negative.

7 Equal curves first

Setting the two sides equal gives the crossing points directly; add the asymptotes as further critical values.

8 Parameters

"No solution" or "always true" conditions come from a discriminant $b^2 - 4ac < 0$

9 Notation

Give intervals such as $x < -5$ or $-2 < x < 1$, or $\{x : x < -5\} \cup \{x : -2 < x < 1\}$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Never multiply blindly	•	•			•		•		•	•
2 Single fraction	•			•	•		•			•
3 Signs or sketch	•	•		•	•	•	•	•		•
4 Excluded values	•	•		•	•		•	•		•
5 Modulus cases			•			•		•		•
6 Use the graph			•			•		•		•
7 Equal curves first						•	•	•		
8 Parameters									•	
9 Notation	•	•	•	•	•	•	•	•	•	•
Interleaved: quadratics, cubics and the discriminant (P1, P2)		•		•		•		•	•	
Interleaved: the modulus function and graphs (P3)			•			•		•		•
Interleaved: algebraic fractions (P3)	•				•		•	•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Find every critical value (crossings and asymptotes), check modulus roots in the original, then sketch to choose the regions. Use $<$ next to a value that makes a denominator zero.

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1. **(a)** Use algebra to find the set of values of x for which $\frac{x+1}{x-2} < 3$

(5)

(b) Write down the set of values of x for which $\frac{x+1}{x-2} \leqslant 3$

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Use algebra to find the set of values of x for which

$$\frac{2}{x-1} \geqslant x$$

(6)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. **(a)** On the same axes, sketch the graphs of $y = |x - 3|$ and $y = 2x$.

(2)

(b) Hence, or otherwise, solve the inequality $|x - 3| > 2x$

(4)

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Question 3 continued

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(Total for Question 3 is 6 marks)

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4.

(a) Find the set of values of x for which $\dfrac{x^2-4}{x+1} > 0$

(4)

(b) Explain why $x = -1$ cannot be included in the solution of $\dfrac{x^2-4}{x+1} \geqslant 0$

(1)

(c) Write your answer to part (a) using set notation.

(1)

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Question 4 continued

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(Total for Question 4 is 6 marks)

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5. Showing all your algebraic working, determine the values of x that satisfy

$$\frac{1}{x+2} > \frac{2}{x-1}$$

giving your answer in set notation.

(6)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6. **(a)** On the same axes, sketch the graphs of $y = |x^2 - 4|$ and $y = 3x$, showing the coordinates of any points where the first graph meets the axes.

(3)

(b) Find the x-coordinates of the points where the graphs meet, explaining why any other roots of your equations are rejected.

(5)

(c) Hence solve the inequality $|x^2 - 4| < 3x$

(2)

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. **(a)** Use algebra to find the set of values of x for which $\frac{x+1}{x-3} \geqslant \frac{x}{x+2}$

(6)

(b) Hence write down the set of values of x for which $\frac{x+1}{x-3} < \frac{x}{x+2}$

(2)

(c) Explain why multiplying both sides of the inequality in part (a) by $(x-3)(x+2)$ is not a valid method.

(2)

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8. **(a)** On the same axes, sketch the graphs of $y = |x - 2|$ and $y = \frac{6}{x+3}$, $x \neq -3$, showing any asymptotes.

(3)

(b) Using algebra, find the set of values of x for which $|x - 2| \geqslant \frac{6}{x+3}$

(7)

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Question 8 continued

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9. **(a)** Solve the inequality $\frac{3x}{x^2+2} \geqslant 1$

(3)

(b) Explain why it was valid to multiply both sides by x^2+2 in part (a).

(1)

(c) Find the set of values of the constant k, where $k > 0$, for which the inequality $\frac{3x}{x^2+2} \geqslant k$ has no real solutions.

(4)

(d) Hence state the range of the function $f(x) = \frac{3x}{x^2+2}, x \in \mathbb{R}$

(2)

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Question 9 continued

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(Total for Question 9 is 10 marks)

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10. **(a)** By considering the cases $x \geqslant 0$ and $x < 0$, find the set of values of x for which $\dfrac{|x|}{x-1} < 2$

(6)

(b) Hence write down the set of values of x for which $\dfrac{|x|}{x-1} \geqslant 2$

(2)

(c) Find the set of values of x for which $\dfrac{|x|}{x-1} < -1$

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 1 IS 80 MARKS

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CHAPTER 2 • KEY SKILLS

2 Series

Specification FP2: Method of differences • 10 questions • 80 marks • about 96 minutes

1 Partial fractions

Write the fractions in full, e.g. $\frac{2}{r(r+1)} = \frac{2}{r} - \frac{2}{r+1}$, not just the constants.

2 Write out the terms

List the first three and last three rows of $f(r) - f(r+1)$ and cross out the cancelling terms.

3 What survives

A gap of 1 leaves one term at each end; a gap of 2 leaves two at each end; a gap of 3 leaves three.

4 Keep factors outside

Leave a constant such as $\frac{1}{8}$ outside the sum until after cancelling.

5 Simplify

Combine the remaining terms over the lowest common denominator and factorise.

6 Other limits

$\sum_{r=n+1}^{2n} = S(2n) - S(n)$, using your formula with $2n$ in place of n.

7 Sum to infinity

Let $n \to \infty$ in $S(n)$: terms such as $\frac{1}{n+1} \to 0$.

8 Second differences

With three partial fractions whose coefficients add to 0, terms cancel across three rows; list enough rows to see the pattern.

9 Not only fractions

Any $f(r) - f(r+1)$ telescopes: surds after rationalising, or $r^3 - (r-1)^3$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Partial fractions	•	•		•	•	•	•	•		•
2 Write out the terms	•	•	•	•	•	•	•	•	•	•
3 What survives	•	•		•	•	•	•	•	•	•
4 Keep factors outside		•			•	•	•			•
5 Simplify	•	•		•	•	•	•	•		•
6 Other limits				•		•		•	•	
7 Sum to infinity	•				•		•	•	•	•
8 Second differences							•			•
9 Not only fractions			•					•	•	
Interleaved: partial fractions (P4)	•	•		•	•	•	•			•
Interleaved: standard sums of r and r^2 (FP1)			•							
Interleaved: surds, inequalities and limits (P1)							•	•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Show the partial fractions, enough rows at the start and end to show the cancelling, the surviving terms, and the simplification. A correct answer with no rows shown loses marks.

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1. **(a)** Express $\frac{2}{r(r+1)}$ in partial fractions.

(1)

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} \frac{2}{r(r+1)} = \frac{2n}{n+1}$

(3)

(c) Find the exact value of $\sum_{r=1}^{20} \frac{2}{r(r+1)}$

(1)

(d) Write down the sum to infinity of the series.

(1)

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(Total for Question 1 is 6 marks)

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2.

$$u_r = \frac{1}{r(r+2)}$$

(a) Find constants A and B such that $u_r = \frac{A}{r} + \frac{B}{r+2}$

(2)

(b) By writing out the first few and last few terms, prove that the sum of the first n terms of u_r is

$$\frac{n(3n+5)}{4(n+1)(n+2)}$$

(4)

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Question 2 continued

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3. **(a)** Show that $r^3 - (r-1)^3 = 3r^2 - 3r + 1$

(2)

(b) Hence, using the method of differences, show that $\sum_{r=1}^{n} \left(3r^2 - 3r + 1\right) = n^3$

(2)

(c) Use part (b) and the result $\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$ to deduce that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$

(2)

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Question 3 continued

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(Total for Question 3 is 6 marks)

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4.

(a) Using the method of differences, show that $\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \frac{n}{2(n+2)}$

(4)

(b) Hence find, in its simplest form, $\sum_{r=n+1}^{2n} \frac{1}{(r+1)(r+2)}$

(2)

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5. **(a)** Show that $\sum_{r=1}^{n} \frac{3}{r(r+3)} = \frac{11}{6} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$

(5)

(b) Hence find the sum to infinity of the series.

(1)

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Question 5 continued

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6. **(a)** Express $\frac{4}{(4r-1)(4r+3)}$ in partial fractions.

(2)

(b) Hence show that $\sum_{r=1}^{n} \frac{4}{(4r-1)(4r+3)} = \frac{4n}{3(4n+3)}$

(4)

(c) Hence show that $\sum_{r=n+1}^{3n} \frac{4}{(4r-1)(4r+3)} = \frac{8n}{3(4n+1)(4n+3)}$

(4)

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7. **(a)** Show that $\frac{1}{r(r+1)(r+2)} = \frac{1}{2}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$

(3)

(b) Hence find $\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$, giving your answer in the form $\frac{1}{4} - g(n)$

(4)

(c) Find the smallest value of n for which this sum is within 0.001 of its sum to infinity.

(3)

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Question 7 continued

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8. **(a)** Show that $\frac{1}{r^2} - \frac{1}{(r+1)^2} = \frac{2r+1}{r^2(r+1)^2}$

(2)

(b) Hence show that $\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2} = \frac{n(n+2)}{(n+1)^2}$

(4)

(c) Find the exact value of $\sum_{r=10}^{\infty} \frac{2r+1}{r^2(r+1)^2}$

(2)

(d) Explain why every partial sum of this series is less than 1

(2)

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Question 8 continued

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9. **(a)** By rationalising the denominator, show that $\frac{1}{\sqrt{r}+\sqrt{r+1}} = \sqrt{r+1} - \sqrt{r}$, and hence show that $\sum_{r=1}^{n} \frac{1}{\sqrt{r}+\sqrt{r+1}} = \sqrt{n+1} - 1$

(5)

(b) Find the value of n for which the sum is 9

(2)

(c) Explain why this series has no sum to infinity.

(1)

(d) Find the value of $\sum_{r=4}^{24} \frac{1}{\sqrt{r}+\sqrt{r+1}}$

(2)

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Question 9 continued

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Question 9 continued

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10. **(a)** Express $\dfrac{r+3}{r(r+1)(r+2)}$ in partial fractions.

(3)

(b) Hence show that $\displaystyle\sum_{r=1}^{n} \frac{r+3}{r(r+1)(r+2)} = \frac{5}{4} - \frac{2n+5}{2(n+1)(n+2)}$

(5)

(c) Find the sum to infinity of the series, and the value of n for which the sum to n terms first exceeds 1.2

(2)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 2 IS 80 MARKS

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CHAPTER 3 • KEY SKILLS

3 Complex Numbers

Specification FP2: Further complex numbers • 10 questions • 80 marks • about 96 minutes

1 Exponential form

$z = re^{i\theta} = r(\cos\theta + i\sin\theta)$, with $-\pi < \theta \leqslant \pi$ for the principal argument.

2 Multiply and divide

$r_1e^{i\theta_1} \times r_2e^{i\theta_2} = r_1r_2e^{i(\theta_1+\theta_2)}$; for division divide the moduli and subtract the arguments.

3 de Moivre

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$, so $\left(re^{i\theta}\right)^n = r^ne^{in\theta}$

4 Multiple angles

Expand $(c+is)^n$ with binomial coefficients; real part gives $\cos n\theta$, imaginary part $\sin n\theta$; then use $s^2 = 1 - c^2$

5 Powers of cos and sin

With $z = e^{i\theta}$: $z^n + z^{-n} = 2\cos n\theta$ and $z^n - z^{-n} = 2i\sin n\theta$. Expand $(z+z^{-1})^n$ and pair the terms.

6 nth roots

$z^n = re^{i\theta}$ gives $z = r^{\frac{1}{n}}e^{i\frac{\theta+2k\pi}{n}}$, $k = 0, 1, \ldots, n-1$: n roots.

7 Geometry of roots

The roots are equally spaced on a circle of radius $r^{\frac{1}{n}}$: a regular polygon, and their sum is 0

8 Solving with identities

Turn a polynomial in x into $\cos n\theta = k$ with $x = \cos\theta$; list every solution, then choose the one asked for.

9 Hence and working

Use the result just proved. A calculator-only solution of the polynomial scores nothing.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Exponential form	•	•			•			•		•
2 Multiply and divide	•							•		
3 de Moivre	•	•	•			•	•			
4 Multiple angles			•			•	•			
5 Powers of cos and sin				•					•	
6 nth roots					•			•		•
7 Geometry of roots					•			•		•
8 Solving with identities			•			•	•			•
9 Hence and working			•	•		•	•		•	•
Interleaved: binomial expansion (P2)			•	•		•	•		•	
Interleaved: trigonometric equations and integration (P3, P4)			•	•		•	•		•	
Interleaved: complex numbers and Argand diagrams (FP1)	•	•			•			•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write out the full binomial expansion with its coefficients, then extract the real or imaginary terms. When solving, find all the angles first, then select the value the question asks for.

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1. The complex numbers $z = 2e^{\frac{\pi}{3}i}$ and $w = 3e^{-\frac{\pi}{4}i}$

(a) Find zw in the form $re^{i\theta}$

(1)

(b) Find $\frac{z}{w}$ in the form $re^{i\theta}$

(1)

(c) Write z in the form $a+bi$, giving exact values.

(1)

(d) Show that z^6 is real, and find its value.

(2)

(e) Find w^2 in the form $re^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(1)

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Question 1 continued

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2. **(a)** Use de Moivre's theorem to find $\left(\sqrt{3}+\mathrm{i}\right)^8$ in the form $a+b\mathrm{i}$, where a and b are exact.

(4)

(b) Find the smallest positive integer n for which $\left(\sqrt{3}+\mathrm{i}\right)^n$ is real and positive.

(2)

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Question 2 continued

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3. **(a)** Use de Moivre's theorem to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(4)

(b) Hence find the three roots of $8x^3 - 6x - 1 = 0$, giving your answers to 3 decimal places.

(2)

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Question 3 continued

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4.

(a) By considering $\left(z+\frac{1}{z}\right)^4$, where $z=\cos\theta+\mathrm{i}\sin\theta$, show that

$$\cos^4\theta=\frac{1}{8}(\cos 4\theta+4\cos 2\theta+3)$$

(4)

(b) Hence find the exact value of $\int_0^{\frac{\pi}{2}}\cos^4\theta\,\mathrm{d}\theta$

(2)

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Question 4 continued

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5. **(a)** Solve the equation $z^3 = 8i$, giving your answers in the form $re^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(4)

(b) Show the roots on an Argand diagram, and describe the shape formed by joining them.

(2)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6. **(a)** Use de Moivre's theorem to show that $\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

(5)

(b) Hence find the smallest positive root of the equation $32x^5 - 40x^3 + 10x - 1 = 0$, giving your answer to 3 significant figures.

(4)

(c) Explain why the equation in part (b) has exactly five real roots.

(1)

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Question 6 continued

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Question 6 continued

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7. **(a)** Use de Moivre's theorem to show that $\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$

(5)

(b) Hence solve, for $0 < \theta < \pi$, the equation $\sin 5\theta = \sin\theta$, giving your answers as exact multiples of π.

(5)

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Question 7 continued

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Question 7 continued

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8. **(a)** Solve the equation $z^5 = 16 + 16\sqrt{3}\,\mathrm{i}$, giving your answers in the form $re^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(4)

(b) The points representing the roots are joined to form a pentagon. Find its exact area, and give its value to 3 significant figures.

(3)

(c) Show that the sum of the five roots is zero.

(2)

(d) Write down the product of the five roots.

(1)

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Question 8 continued

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Question 8 continued

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9. **(a)** Show that $32\cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10$

(5)

(b) Hence find the exact value of $\int_0^{\frac{\pi}{2}} \cos^6\theta \, d\theta$

(3)

(c) Find the exact value of $\int_0^{\frac{\pi}{4}} \cos^6\theta \, d\theta$

(2)

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Question 9 continued

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10. **(a)** By writing $w = z^3$, solve $w^2 + w + 1 = 0$, giving the roots in the form $e^{i\theta}$

(2)

(b) Hence find the six roots of $z^6 + z^3 + 1 = 0$ in the form $e^{i\theta}$, where $-\pi < \theta \leqslant \pi$

(4)

(c) By pairing conjugate roots, express $z^6 + z^3 + 1$ as a product of three quadratic factors with real coefficients.

(3)

(d) Deduce that $\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9} = 0$

(1)

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Question 10 continued

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TOTAL FOR CHAPTER 3 IS 80 MARKS

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CHAPTER 4 • KEY SKILLS

4 Further Argand Diagrams

Specification FP2: Loci and transformations • 10 questions • 80 marks • about 96 minutes

1 Circles

$|z-a|=r$: centre a, radius r. Write $z-(3-2\mathrm{i})$ for $z-3+2\mathrm{i}$ to read off the centre $(3,-2)$.

2 Perpendicular bisectors

$|z-a|=|z-b|$: the perpendicular bisector of ab. Square both sides with $z=x+\mathrm{i}y$ for its equation.

3 Half-lines

$\arg(z-a)=\theta$: a half-line from a (not including a) at angle θ to the positive real direction.

4 $|z-a|=k|z-b|$

For $k\neq 1$ this is a circle: square, collect, divide by the x^2 coefficient and complete the square.

5 Arcs

$\arg\dfrac{z-a}{z-b}=\theta$: an arc from a to b on which the chord subtends θ. Test a point to choose the arc.

6 Regions

Draw the boundary, decide strict or not, and shade the correct side, e.g. the side nearer a for $|z-a|\leqslant|z-b|$

7 Greatest and least

Distance from O to the centre $\pm$ the radius gives the extreme $|z|$; tangents from O give the extreme $\arg z$

8 Transformations

Make z the subject, substitute into the locus of z, then write $w=u+\mathrm{i}v$ and use moduli.

9 Identify the image

Equal coefficients of u^2 and v^2 give a circle: complete the square for the centre, then square root for the radius.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Circles	•			•		•		•		•
2 Perpendicular bisectors		•					•			
3 Half-lines	•							•		
4 $\lvert z-a\rvert=k\lvert z-b\rvert$			•			•				•
5 Arcs					•				•	
6 Regions		•						•		•
7 Greatest and least			•	•				•	•	
8 Transformations						•	•			•
9 Identify the image			•			•	•		•	•
Interleaved: circles and coordinate geometry (P2)	•	•	•	•	•	•	•	•	•	•
Interleaved: complex numbers and arguments (FP1)	•			•	•		•	•	•	
Interleaved: radians, sectors and triangles (P1)				•				•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Label every sketch with centres, intercepts and excluded points, and make clear which part is the locus or region. For a transformation, rearrange for z first and check it by substituting back.

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1. **(a)** On an Argand diagram, sketch the locus of points satisfying $|z - 3 + 2i| = 4$

(2)

(b) On the same diagram, sketch the locus of points satisfying $\arg(z + 1) = \frac{\pi}{4}$

(2)

(c) Determine, using algebra, whether the two loci meet.

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The point P represents the complex number z on an Argand diagram, where $|z-3-4\mathrm{i}| = |z+1+\mathrm{i}|$

(a) Show that the locus of P has Cartesian equation $8x+10y = 23$

(4)

(b) Shade on an Argand diagram the region for which $|z-3-4\mathrm{i}| \leqslant |z+1+\mathrm{i}|$

(2)

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Question 2 continued

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3. **(a)** Show that the locus of points satisfying $|z-6| = 2|z|$ is a circle, and find its centre and radius.

(5)

(b) Write down the greatest value of $|z|$ for points on this locus.

(1)

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Question 3 continued

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(Total for Question 3 is 6 marks)

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4. The complex number z satisfies $|z - 4 - 3i| = 2$

(a) Find the greatest and least values of $|z|$.

(3)

(b) Find the greatest value of $\arg z$, giving your answer in radians to 3 significant figures.

(3)

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Question 4 continued

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5. **(a)** Sketch the locus of points satisfying $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{2}$

(3)

(b) Find a Cartesian equation of the locus, stating any restrictions on x and y.

(3)

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Question 5 continued

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6. The transformation T from the z-plane to the w-plane is given by $w = \frac{z+1}{z-1}, z \neq 1$

(a) Show that $z = \frac{w+1}{w-1}$

(2)

(b) Show that the image under T of the circle $|z| = 2$ is a circle, and find its centre and radius.

(6)

(c) Show that the image under T of the imaginary axis is the circle $|w| = 1$

(2)

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. The point P represents z in the z-plane, where $|z| = |z - 4|$

(a) Sketch the locus of P, and find its Cartesian equation.

(3)

The transformation T is given by $w = \frac{1}{z}$, $z \neq 0$

(b) Show that the image under T of the locus of P is a circle with centre $\frac{1}{4}$ and radius $\frac{1}{4}$

(6)

(c) State the point on this circle that is not the image of any point P, and explain why.

(1)

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Question 7 continued

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(Total for Question 7 is 10 marks)

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8. The region R is the set of points satisfying $|z-3| \leqslant 3$ and $0 \leqslant \arg z \leqslant \frac{\pi}{4}$

(a) Shade R on an Argand diagram, labelling the points where the boundaries meet.

(4)

(b) Find the exact area of R.

(4)

(c) Find the greatest value of $|z|$ for z in R.

(2)

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Question 8 continued

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9. The point P represents z on an Argand diagram, where $\arg\left(\frac{z-4\text{i}}{z+4}\right) = \frac{\pi}{4}$

(a) Sketch the locus of P.

(3)

(b) Show that the locus of P is an arc of a circle with centre $-4+4\text{i}$, and find its radius.

(4)

(c) Find the exact greatest value of $|z|$ for points on the locus.

(3)

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10. The transformation T from the z-plane to the w-plane is given by $w = \dfrac{3}{z-2}, z \neq 2$

(a) Show that the image under T of the circle $|z| = 1$ is the circle $|w+2| = 1$

(6)

(b) Find the image under T of the region $|z| < 1$, giving a reason.

(2)

(c) Describe the image under T of the real axis in the z-plane.

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 4 IS 80 MARKS

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CHAPTER 5 • KEY SKILLS

5 First-Order Differential Equations

Specification FP2: First-order differential equations • 10 questions • 80 marks • about 96 minutes

1 Standard form

Write $\frac{dy}{dx} + P(x)y = Q(x)$: divide by the coefficient of $\frac{dy}{dx}$ first.

2 Integrating factor

$e^{\int P\,dx}$, simplified: $e^{2\ln x} = x^2$, $e^{-\int \tan x\,dx} = \cos x$

3 Multiply through

Every term is multiplied by the integrating factor; the left side becomes $\frac{d}{dx}(\text{IF} \times y)$

4 Integrate the right

Integrate IF $\times$ Q(x), often by parts; watch the signs and coefficients.

5 The constant

Add c when you integrate, then divide everything by the IF: $y = \ldots + ce^{-2x}$, not $\ldots + c$

6 Particular solutions

Substitute the condition into the general solution to find c, then write "$y = \ldots$".

7 Substitutions

From $z = y^{-1}$: $\frac{dz}{dx} = -y^{-2}\frac{dy}{dx}$. Replace every y and $\frac{dy}{dx}$, showing all steps.

8 Reverse the substitution

Put the original variable back: if $z = \frac{1}{y}$ then $y = \frac{1}{z}$, taking the reciprocal of the whole expression.

9 Interpreting

Use the solution to find greatest or least values, limits as $x \to \infty$ and asymptotes.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Standard form	•	•	•	•	•	•		•	•	
2 Integrating factor	•	•	•	•	•	•		•	•	
3 Multiply through	•	•	•	•	•	•		•	•	
4 Integrate the right			•	•	•	•				
5 The constant	•	•	•	•	•	•	•	•	•	•
6 Particular solutions			•			•	•	•	•	•
7 Substitutions					•	•	•			•
8 Reverse the substitution					•	•	•			•
9 Interpreting								•	•	•
Interleaved: integration by parts and trigonometric integrals (P3, P4)			•	•	•	•				
Interleaved: separating variables (P4)							•			•
Interleaved: differentiation, stationary points and asymptotes (P2, P3)								•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Standard form, integrating factor, multiply every term, integrate, add c, then divide everything by the integrating factor. In a substitution "show that", show each replacement.

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1. Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \mathrm{e}^{-x}$$

(6)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Using an integrating factor, solve

$$x\frac{dy}{dx} + 2y = x^2, \qquad x > 0$$

expressing y as a function of x.

(6)

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Question 2 continued

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3.

$$\frac{dy}{dx} - y\tan x = \cos x, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(a) Find the general solution of the differential equation.

(4)

(b) Find the particular solution for which $y = 0$ when $x = 0$

(2)

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Question 3 continued

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4. Solve the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = \ln x, \qquad x > 0$$

giving y in terms of x.

(6)

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5.

$$\frac{dy}{dx} + y = xy^2 \qquad \text{(I)}$$

(a) Show that the substitution $z = \frac{1}{y}$ transforms equation (I) into $\frac{dz}{dx} - z = -x$

(3)

(b) Hence find the general solution of equation (I), giving y in terms of x.

(3)

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6.

$$\frac{dy}{dx} + 2y = 2xy^3 \qquad \text{(I)}$$

(a) Show that the substitution $z = y^{-2}$ transforms equation (I) into $\frac{dz}{dx} - 4z = -4x$ (II)

(5)

(b) Solve equation (II), and hence find the general solution of equation (I) in the form $y^2 = f(x)$

(4)

(c) Find the particular solution of (I) for which $y = 2$ when $x = 0$

(1)

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Question 6 continued

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7.

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}, \qquad x > 0,\ y > 0$$

(a) By writing $y = vx$, with v depending on x, reduce this to the separable equation $x\frac{dv}{dx} = \frac{1}{v}$

(4)

(b) Solve for v, and deduce an expression for y^2 in terms of x and an arbitrary constant.

(4)

(c) Determine the solution curve through the point (1, 2)

(2)

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Question 7 continued

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8. The speed $v\,\text{m s}^{-1}$ of a model car at time t seconds satisfies

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{v}{t+1} = 2, \qquad t \geqslant 0$$

and $v = 5$ when $t = 0$

(a) Show that $v = t + 1 + \dfrac{4}{t+1}$

(6)

(b) Find the least speed of the car and the time at which it occurs.

(3)

(c) Describe how the speed changes for large values of t.

(1)

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Question 8 continued

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9.

$$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=4x$$

(a) Find the general solution of the differential equation.

(5)

(b) Find the particular solution whose graph passes through (0, 3)

(2)

(c) For this particular solution, show that $y=2+\frac{1}{x^2+1}$, and hence state the range of values of y.

(3)

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Question 9 continued

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10.

$$\frac{dy}{dx} = (y - x)^2 + 1$$

(a) Show that the substitution $u = y - x$ transforms the equation into $\frac{du}{dx} = u^2$

(3)

(b) Hence find the general solution of the original equation, giving y in terms of x.

(4)

(c) Find the particular solution for which $y = 1$ when $x = 0$, and state the equation of the vertical asymptote of its graph.

(3)

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TOTAL FOR CHAPTER 5 IS 80 MARKS

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CHAPTER 6 • KEY SKILLS

6 Second-Order Differential Equations

Specification FP2: Second-order differential equations • 10 questions • 80 marks • about 96 minutes

1 Auxiliary equation

For $ay'' + by' + cy = 0$ solve $am^2 + bm + c = 0$

2 Complementary function

Distinct α, β: $Ae^{\alpha x} + Be^{\beta x}$. Repeated α: $(A + Bx)e^{\alpha x}$. $p \pm qi$: $e^{px}(A\cos qx + B\sin qx)$

3 Particular integral

Match the right-hand side: $ax + b$; λe^{kx}; $p\cos kx + q\sin kx$ (both terms, even if only one appears).

4 When the PI clashes

If the trial form is in the CF, multiply by x (by x^2 for a repeated root): $\lambda x e^{2x}$, $\lambda x^2 e^{2x}$

5 Finding constants

Differentiate the trial PI twice, substitute, and equate coefficients of each type of term.

6 General solution

$y = \text{CF} + \text{PI}$, written as "$y = \ldots$" with two arbitrary constants.

7 Conditions

Use y and $\dfrac{dy}{dx}$ at the given points; differentiate the whole general solution first.

8 Substitution $x = e^t$

$x\dfrac{dy}{dx} = \dfrac{dy}{dt}$ and $x^2\dfrac{d^2y}{dx^2} = \dfrac{d^2y}{dt^2} - \dfrac{dy}{dt}$, by the chain rule. Return to x at the end: $e^{2t} = x^2$

9 Behaviour

As $x \to \infty$, terms with e^{-x} vanish; for a solution to stay finite, some constants must be 0

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Auxiliary equation	•	•	•	•	•	•	•	•	•	•
2 Complementary function	•	•	•	•	•	•	•	•	•	•
3 Particular integral		•	•		•	•	•			•
4 When the PI clashes				•				•		•
5 Finding constants		•	•	•	•	•		•		•
6 General solution		•	•	•	•	•	•	•	•	•
7 Conditions			•				•	•	•	•
8 Substitution $x = e^t$						•			•	
9 Behaviour							•	•		•
Interleaved: differentiation, chain and product rules (P3)				•		•	•	•	•	•
Interleaved: complex roots of quadratics (FP1)	•		•		•		•		•	
Interleaved: simultaneous equations and exponentials (P1, P3)		•	•		•	•	•	•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the CF first, then choose the PI form, checking it against the CF. Give the final answer as "$y =$" in the original variable, with the exponentials simplified.

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1. For each equation below, solve the auxiliary equation and write down y in terms of x and two arbitrary constants.

(a) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 5\dfrac{\mathrm{d}y}{\mathrm{d}x} + 6y = 0$

(2)

(b) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\dfrac{\mathrm{d}y}{\mathrm{d}x} + 4y = 0$

(2)

(c) $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + 5y = 0$

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Using a particular integral of the form $ax + b$, solve completely

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 4x$$

(6)

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Question 2 continued

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3.

$$\frac{d^2y}{dx^2} + 9y = 10e^x$$

(a) Obtain y in terms of x and two arbitrary constants.

(4)

(b) Hence determine the solution with $y = 2$ and $\frac{dy}{dx} = 4$ at $x = 0$

(2)

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(Total for Question 3 is 6 marks)

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4.

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 3e^{2x}$$

(a) Explain why a particular integral of the form λe^{2x} cannot be used.

(1)

(b) Find the general solution of the differential equation.

(5)

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5. A trigonometric forcing term produces the equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 10\sin x$$

Choosing a suitable trigonometric particular integral, express y in terms of x and two arbitrary constants.

(6)

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Question 5 continued

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6.

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3, \qquad x > 0 \qquad \text{(I)}$$

(a) Given that $x = e^t$, show that $x\frac{dy}{dx} = \frac{dy}{dt}$ and $x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dt^2} - \frac{dy}{dt}$

(4)

(b) Hence show that (I) becomes $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = e^{3t}$ (II)

(1)

(c) Find the general solution of (II).

(3)

(d) Hence find the general solution of (I), giving y in terms of x.

(2)

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. The displacement x metres of a particle from a fixed point at time t seconds satisfies

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10$$

(a) Find the general solution of the differential equation.

(5)

(b) Given that the particle starts from rest at the fixed point, find x in terms of t.

(4)

(c) Describe the motion of the particle for large values of t.

(1)

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Question 7 continued

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8.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 6e^{2x}$$

(a) Explain why neither λe^{2x} nor $\lambda x e^{2x}$ can be a particular integral.

(2)

(b) Find the general solution of the differential equation.

(4)

(c) Find the particular solution for which $y = 1$ and $\frac{dy}{dx} = 0$ when $x = 0$

(3)

(d) Show that this particular solution is positive for all values of x.

(1)

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9.

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4xy = 0, \qquad x > 0 \qquad \text{(I)}$$

(a) By writing $y = \frac{z}{x}$, where z depends on x, show that (I) reduces to $\frac{d^2z}{dx^2} + 4z = 0$

(5)

(b) Solve for z, and deduce y in terms of x and two constants.

(3)

(c) Find the particular solution of (I) for which $y = \frac{4}{\pi}$ when $x = \frac{\pi}{4}$ and $y = -\frac{2}{\pi}$ when $x = \frac{\pi}{2}$

(2)

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Question 9 continued

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10.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4x - 2 + 3e^x$$

(a) Find the complementary function.

(2)

(b) Find a particular integral, explaining your choice of form.

(5)

(c) Find the particular solution for which $y = 1$ when $x = 0$ and y remains finite as $x \to -\infty$

(3)

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TOTAL FOR CHAPTER 6 IS 80 MARKS

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CHAPTER 7 • KEY SKILLS

7 Maclaurin and Taylor Series

Specification FP2: Maclaurin and Taylor series • 10 questions • 80 marks • about 96 minutes

1 Higher derivatives

Use the product and chain rules, and simplify each derivative before differentiating again.

2 Maclaurin series

$f(x) =$

$$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

Find the values at 0 first, then substitute.

3 Standard series

Use e^x, $\sin x$, $\cos x$, $\ln(1+x)$ and $(1+x)^n$ from the formula book, with their ranges of validity.

4 Compound functions

Substitute (e.g. $2x$ for x), use $\ln\frac{a}{b} = \ln a - \ln b$, or multiply series, discarding powers beyond the one asked for.

5 Taylor series

About $x = a$:

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

6 Differential equations

Differentiate the equation itself to get higher derivatives, then substitute the known values at the point.

7 Using a relation

If $y^{(4)} = -4y$ then $y^{(5)} = -4y'$ and $y^{(6)} = -4y''$: no need to differentiate again.

8 Approximations

Substitute a small value; percentage error $= \dfrac{|\text{approx} - \text{true}|}{\text{true}} \times 100$

9 Limits

Replace each function by its series, cancel the lowest power of x, then let $x \to 0$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Higher derivatives	•		•		•	•	•			•
2 Maclaurin series	•					•				•
3 Standard series		•		•				•	•	
4 Compound functions		•		•				•	•	
5 Taylor series			•		•		•			
6 Differential equations					•		•			
7 Using a relation						•				•
8 Approximations			•				•	•		
9 Limits				•					•	
Interleaved: differentiation rules (P3)	•		•		•	•	•			•
Interleaved: logarithms and binomial expansion (P2, P4)	•	•						•	•	•
Interleaved: differential equations (Chapters 5, 6)					•		•			

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. List the values $f(a)$, $f'(a)$, $f''(a)$, … before writing the series, show the factorials, and finish with "$y = \ldots$". When told "hence", use the result just found.

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1.

$$f(x) = \ln(\cos x), \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(a) Find $f'(x)$, $f''(x)$, $f'''(x)$ and $f''''(x)$.

(4)

(b) Deduce that, for small values of x, $\ln(\cos x) \approx -\frac{1}{2}x^2 - \frac{1}{12}x^4$

(2)

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Question 1 continued

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2. **(a)** Using standard series, find the series expansion of $\ln\left(\frac{1+2x}{1-x}\right)$ in ascending powers of x, up to and including the term in x^3

(4)

(b) State the range of values of x for which the expansion is valid.

(2)

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3. **(a)** Find the Taylor series of $\tan x$ about $x = \frac{\pi}{4}$, up to and including the term in $\left(x - \frac{\pi}{4}\right)^2$

(4)

(b) Use your series to estimate $\tan 0.8$, giving your answer to 5 decimal places.

(2)

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Question 3 continued

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4. **(a)** Using standard series, show that $e^x \sin x = x + x^2 + \frac{1}{3}x^3 + \cdots$, and show that the coefficient of x^4 is zero.

(4)

(b) Hence find $\lim_{x \to 0} \frac{e^x \sin x - x}{x^2}$

(2)

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Question 4 continued

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5. The function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = x^2 + y^2$, with $y = 1$ when $x = 0$

(a) Find the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 0$

(4)

(b) Hence find a series solution for y in ascending powers of x, up to and including the term in x^3

(2)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6.

$$y = e^x \cos x$$

(a) Show that $\dfrac{d^4y}{dx^4} = -4y$

(4)

(b) Hence find the values of $\dfrac{d^5y}{dx^5}$ and $\dfrac{d^6y}{dx^6}$ when $x = 0$

(2)

(c) Write down the expansion of $e^x \cos x$ in ascending powers of x as far as the term in x^5, showing the value of each derivative used.

(4)

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7.

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

At $x = 1$, $y = 1$ and $\frac{dy}{dx} = 0$

(a) Find the value of $\frac{d^2y}{dx^2}$ at $x = 1$

(1)

(b) Show that $\frac{d^3y}{dx^3} = -2\frac{dy}{dx} - x\frac{d^2y}{dx^2}$

(2)

(c) Find the value of $\frac{d^4y}{dx^4}$ at $x = 1$

(2)

(d) Hence find a series solution for y in ascending powers of $(x-1)$, up to and including the term in $(x-1)^4$

(3)

(e) Use your series to estimate the value of y at $x = 1.2$, giving your answer to 4 decimal places.

(2)

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8.

$$f(x) = \frac{e^{2x}}{1-x}$$

(a) By multiplying two standard series, show that $f(x) = 1 + 3x + 5x^2 + \frac{19}{3}x^3 + \cdots$

(4)

(b) State the range of values of x for which the expansion is valid.

(1)

(c) Use the series with $x = 0.1$ to estimate $f(0.1)$, and find the percentage error in this estimate.

(3)

(d) Use the series to estimate $\int_0^{0.1} f(x)\,dx$

(2)

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9. Use series expansions to find the values of the following limits, showing your working.

(a) Show that $1 - \cos 2x = 2x^2 - \frac{2}{3}x^4 + \cdots$

(2)

(b) $\lim_{x \to 0} \frac{x \sin x}{1 - \cos 2x}$

(3)

(c) $\lim_{x \to 0} \frac{e^x - 1 - x}{x \sin x}$

(3)

(d) $\lim_{x \to 0} \frac{\ln(1 + x) - x}{x^2}$

(2)

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Question 9 continued

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10.

$$y = \ln\left(1 + e^{x}\right)$$

(a) Show that $\frac{dy}{dx} = \frac{e^{x}}{1 + e^{x}}$ and that $\frac{d^2y}{dx^2} = \frac{dy}{dx}\left(1 - \frac{dy}{dx}\right)$

(3)

(b) Hence find the Maclaurin series of y, up to and including the term in x^4

(5)

(c) By showing that $\ln(1 + e^{x}) - \frac{x}{2} = \ln\left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right)$, explain why your series has no terms in x^3 or x^5

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 7 IS 80 MARKS

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CHAPTER 8 • KEY SKILLS

8 Polar Coordinates

Specification FP2: Polar coordinates • 10 questions • 80 marks • about 96 minutes

1 Converting

$x = r\cos\theta$, $y = r\sin\theta$, $r^2 = x^2 + y^2$. Multiply through by r when the equation has $\cos\theta$ or $\sin\theta$ alone.

2 Sketching

Only values with $r \geqslant 0$ are drawn. Find r at $\theta = 0, \frac{\pi}{2}, \pi, \ldots$ and where $r = 0$ (the curve meets the pole).

3 Intersections

Set the two expressions for r equal, or use a given length OP; give the points as (r, θ)

4 Area

$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\,\mathrm{d}\theta$, with limits from where the region starts and ends, not automatically 0 to 2π

5 Squaring

Expand r^2 fully, then $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ and $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$ before integrating.

6 Combining regions

Split at intersection points; subtract or add triangles, using $\frac{1}{2}r^2\sin\theta\cos\theta$ or $\frac{1}{2}\times$ base $\times$ height

7 Parallel tangents

Tangent parallel to the initial line: $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0$, where $y = r\sin\theta$

8 Perpendicular tangents

Tangent perpendicular to the initial line: $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0$, where $x = r\cos\theta$

9 Exact answers

Leave areas in terms of π and surds; reject angles that give $r < 0$ or lie outside the given interval.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Converting	•					•		•		
2 Sketching		•	•				•		•	
3 Intersections						•	•			
4 Area		•	•			•	•	•	•	•
5 Squaring		•	•			•	•			•
6 Combining regions						•	•		•	
7 Parallel tangents				•					•	•
8 Perpendicular tangents					•			•		
9 Exact answers	•	•	•	•	•	•	•	•	•	•
Interleaved: trigonometric identities and integration (P3, P4)		•	•	•	•	•	•		•	•
Interleaved: Newton–Raphson (FP1)								•		
Interleaved: circles, parabolas and coordinate geometry (P1, P2)	•					•		•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.
Must learn. Mark the region on the figure and decide its limits before integrating. Square r fully (three terms for $(a+b\cos\theta)^2$), and draw any triangle you need to add or subtract.

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1. **(a)** Find a Cartesian equation of the curve with polar equation $r = 4\cos\theta$, and describe the curve.

(3)

(b) Find a polar equation of the curve with Cartesian equation $y = x^2$, in the form $r = f(\theta)$

(2)

(c) Find the polar coordinates (r, θ) of the point with Cartesian coordinates $\left(-1, \sqrt{3}\right)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$

(1)

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Question 1 continued

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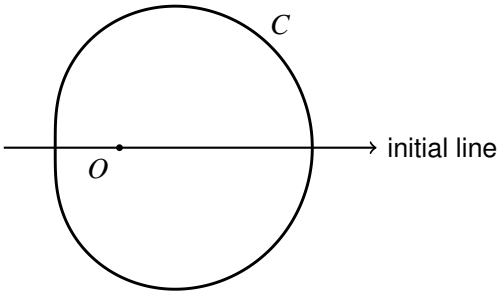

Figure 1

Figure 1 shows the curve C with polar equation $r = 2 + \cos\theta$, $0 \leqslant \theta < 2\pi$

(a) Show that $r^2 = \frac{9}{2} + 4\cos\theta + \frac{1}{2}\cos 2\theta$

(2)

(b) Hence find the exact area of the region enclosed by C.

(4)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. The curve with polar equation $r = 3\sin 2\theta$ has a loop for $0 \leqslant \theta \leqslant \frac{\pi}{2}$

(a) Sketch this loop, stating the value of θ at which r is greatest.

(2)

(b) Find the exact area enclosed by the loop.

(4)

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Question 3 continued

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4. The curve C has polar equation $r = 1 + \cos\theta$, $0 \leqslant \theta \leqslant \pi$

(a) Show that, at points where the tangent to C is parallel to the initial line, $2\cos^2\theta + \cos\theta - 1 = 0$

(4)

(b) Hence find the polar coordinates of the point on C, other than the pole, at which the tangent is parallel to the initial line.

(2)

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Question 4 continued

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5. For $0 \leqslant \theta \leqslant \frac{2\pi}{3}$, a limacon is traced out by $r = 1 + 2\cos\theta$

(a) By writing $x = r\cos\theta$, prove that $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -\sin\theta(1 + 4\cos\theta)$

(3)

(b) Locate, in polar coordinates, every point of this arc where the tangent is perpendicular to the initial line, with angles to 3 significant figures.

(3)

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Question 5 continued

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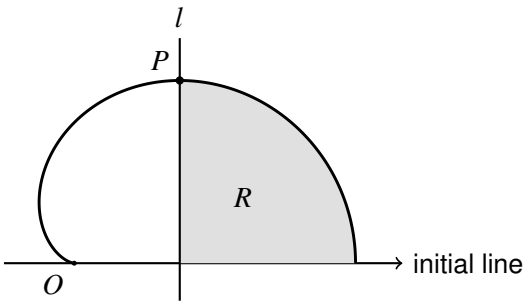

Figure 2

Figure 2 shows part of the curve C with polar equation $r = 3(1+\cos\theta)$, $0 \leqslant \theta \leqslant \pi$, and the line l with polar equation $r = \frac{9}{4}\sec\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The line l meets C at the point P. The region R, shaded, is bounded by C, l and the initial line.

(a) Show that P has polar coordinates $\left(\frac{9}{2}, \frac{\pi}{3}\right)$

(3)

(b) Find the exact area of R, giving your answer in the form $p\pi + q\sqrt{3}$, where p and q are rational.

(7)

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Question 6 continued

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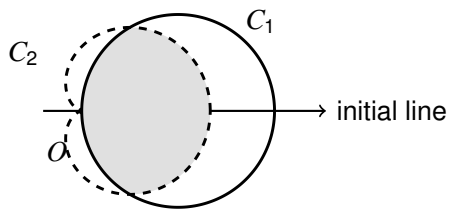

Figure 3

Figure 3 shows the curves C_1 and C_2 with polar equations $r = 3\cos\theta$, $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$, and $r = 1 + \cos\theta$, $0 \leqslant \theta < 2\pi$, respectively. The shaded region lies inside both curves.

(a) Find the polar coordinates of the points, other than the pole, where the curves meet.

(3)

(b) Find the exact area of the shaded region.

(7)

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Question 7 continued

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8. The curve C is the spiral with polar equation $r = \theta$, $0 \leqslant \theta \leqslant \pi$

(a) Find the exact area of the region bounded by C and the initial line produced backwards through O.

(3)

(b) Show that, at a point where the tangent to C is perpendicular to the initial line, $\theta \tan \theta = 1$

(3)

(c) Use the Newton–Raphson process once, with $\theta_0 = 0.85$, applied to $f(\theta) = \cos\theta - \theta \sin\theta$, to find this value of θ to 3 decimal places, and find the x-coordinate of the point.

(4)

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Question 8 continued

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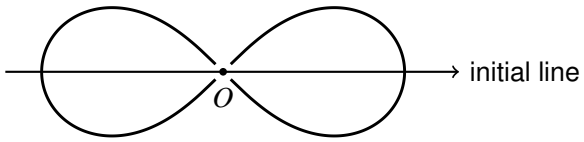

Figure 4

Figure 4 shows the curve with polar equation $r^2 = 4\cos 2\theta$

(a) Explain why there are no points on the curve for $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$

(2)

(b) Find the total area enclosed by the curve.

(4)

(c) Find the polar coordinates of the point on the curve, with $0 < \theta < \frac{\pi}{4}$, at which the tangent is parallel to the initial line.

(4)

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Question 9 continued

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10. The curve C has polar equation $r = a + 2\cos\theta$, $0 \leqslant \theta < 2\pi$, where $a > 2$. The area enclosed by C is 18π

(a) Show that $a = 4$

(3)

(b) Show that the tangents to C parallel to the initial line touch C where $\cos\theta = \frac{\sqrt{3}-1}{2}$, and find the value of r at these points in the form $p + \sqrt{q}$

(5)

(c) Find, to 3 significant figures, the distance between these two tangents.

(2)

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TOTAL FOR CHAPTER 8 IS 80 MARKS

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