



eClassroom

A-LEVEL MATHEMATICS • PURE 1

Algebraic Expressions

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Algebraic Expressions worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Algebraic Expressions worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

$$\text{A1} \quad (3a^2b)^3 = 27a^6b^3 \quad \frac{27a^6b^3 \times 2a^{-1}b}{9a^4} = \frac{54a^5b^4}{9a^4} = \boxed{6ab^4}$$

$$\text{A2} \quad (2x-1)(x+4) = 2x^2 + 7x - 4 \quad (2x^2 + 7x - 4)(x-3) = 2x^3 - 6x^2 + 7x^2 - 21x - 4x + 12 = \boxed{2x^3 + x^2 - 25x + 12}$$

$$\text{A3} \quad \left(\frac{4}{25}\right)^{\frac{3}{2}} = \left(\frac{2}{5}\right)^3 = \boxed{\frac{8}{125}}$$

STRETCH

$$\text{A4} \quad (x^2 - 4)(x^2 - 9), \text{ then each is a difference of two squares: } \boxed{(x-2)(x+2)(x-3)(x+3)}$$

$$\text{A5} \quad \frac{x^2}{3x^3} - \frac{6x^{\frac{1}{2}}}{3x^3} = \boxed{\frac{1}{3}x^{-1} - 2x^{-\frac{5}{2}}}$$

$$\text{A6} \quad \text{Multiply by } 4 - \sqrt{2}; \text{ the bottom is } 16 - 2 = 14 \quad \frac{12 - 3\sqrt{2} - 4\sqrt{2} + 2}{14} = \frac{14 - 7\sqrt{2}}{14} = \boxed{1 - \frac{1}{2}\sqrt{2}}$$

$$\text{A7} \quad 2^{2x} \times 2^{3x-3} = 2^5, \text{ so } 5x - 3 = 5 \quad \boxed{x = \frac{8}{5}}$$

B**Fluency solutions**One mark for each correct part. The notes in *italics* explain why each sequence was chosen.**CORE**

$$\text{B1} \quad (\text{a}) \boxed{x^8} \quad (\text{b}) \boxed{x^{15}} \quad (\text{c}) \boxed{x^{-2}} \quad (\text{d}) \frac{32x^{15}}{4x^5} = \boxed{8x^{10}} \quad \text{Add, multiply, subtract: the three laws.}$$

$$\text{B2} \quad (\text{a}) \boxed{4} \quad (\text{b}) \boxed{\frac{1}{4}} \quad (\text{c}) 4^2 = \boxed{16} \quad (\text{d}) \frac{1}{8^3} = \boxed{\frac{1}{512}} \quad \text{The bottom of the fraction is the root; the minus sign flips it.}$$



B3 (a) $x^2 - 25$ (b) $4x^2 - 12x + 9$ (c) $x^3 + 1$ (d) $x^3 - 6x^2 + 12x - 8$

B4 (a) $4\sqrt{3}$ (b) $4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3}$ (c) $\sqrt{576} = 24$ (d) $\sqrt{4} = 2$

B5 (a) $\frac{6\sqrt{3}}{3} = 2\sqrt{3}$ (b) $\frac{4(\sqrt{5}+1)}{5-1} = \sqrt{5}+1$ (c) $\frac{(\sqrt{7}+\sqrt{3})^2}{7-3} = \frac{10+2\sqrt{21}}{4} = \frac{5+\sqrt{21}}{2}$

B6 (a) $3x(2x-3)$ (b) $x(x-4)(x+4)$ (c) $x(2x-1)(x+3)$ (d) $y(x-2y)(x+2y)$

Always take out the common factor first, then look at what is left.

STRETCH

B7 (a) $x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$ (b) $x\sqrt{x} = x^{\frac{3}{2}}: 3x^{\frac{1}{2}} - 2x^{-\frac{3}{2}}$

(c) $\frac{4+4\sqrt{x}+x}{x} = 4x^{-1} + 4x^{-\frac{1}{2}} + 1$ (d) $\frac{5}{2}x^{-1} - x^{-\frac{3}{2}}$

B8 (a) $9 - 2 = 7$ (b) $20 - 4\sqrt{5} + 1 = 21 - 4\sqrt{5}$ (c) $6 + 2\sqrt{12} + 2 = 8 + 4\sqrt{3}$

(d) $x(4 - \sqrt{3}) = 2$, so $x = \frac{2(4 + \sqrt{3})}{16 - 3} = \frac{8 + 2\sqrt{3}}{13}$

B9 (a) $2^x = 2^{-3}: x = -3$ (b) $2x - 1 = 3: x = 2$ (c) $3^{2x} = 3^{\frac{3}{2}}: x = \frac{3}{4}$ (d) $2^{2x} = 2^{3-3x}: x = \frac{3}{5}$

B10 (a) $2^x \times 2^3 = 8y$ (b) $(2^x)^2 = y^2$ (c) $\frac{(2^x)^3}{8} = \frac{y^3}{8}$ (d) $\frac{1}{2 \times (2^x)^2} = \frac{1}{2y^2}$

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 Squaring a bracket means multiplying it by itself, so there are two middle terms of $3x$ $(x+3)(x+3) = x^2 + 6x + 9$

C2 You cannot split a root over a sum: $\sqrt{16+9} = \sqrt{25} = 5$, not 7



C3 $2\sqrt{3} \times \sqrt{3} - 4\sqrt{3} + \sqrt{3} - 2 = 6 - 3\sqrt{3} - 2 = 4 - 3\sqrt{3}$ as required

C4 Both mean the cube root of 8^2 , because $\frac{2}{3} = 2 \times \frac{1}{3}$. Taking the root first keeps the numbers small: $(\sqrt[3]{8})^2 = 2^2 = \boxed{4}$

STRETCH

C5 $(3 - \sqrt{5})^2 = 14 - 6\sqrt{5}$ still contains a surd. Multiply by the conjugate $3 + \sqrt{5}$: the bottom becomes $9 - 5 = 4$

$\frac{3 + \sqrt{5}}{4}$

C6 $(\sqrt{5} + 2)(\sqrt{5} - 2) = 1$, so $\frac{1}{\sqrt{5} + 2} = \sqrt{5} - 2$ and $\frac{1}{\sqrt{5} - 2} = \sqrt{5} + 2$. Adding gives $2\sqrt{5}$ as required

C7 $(\sqrt{a} + \sqrt{b})^2 = a + 2\sqrt{a}\sqrt{b} + b$. Since a and b are positive, $\sqrt{a}\sqrt{b} = \sqrt{ab} > 0$, so $(\sqrt{a} + \sqrt{b})^2 > a + b$

C8 (a) $\frac{1}{2 + \sqrt{3}} = \frac{2 - \sqrt{3}}{4 - 3} = 2 - \sqrt{3}$, so $x + \frac{1}{x} = 4$ as required

(b) $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$, so $x^2 + \frac{1}{x^2} = 16 - 2 = \boxed{14}$

(c) $\left(x + \frac{1}{x}\right)^3 = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$, so $x^3 + \frac{1}{x^3} = 64 - 3 \times 4 = \boxed{52}$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1 \text{ cm}^2$ as required

(b) Volume = $1 \times \sqrt{12} = \sqrt{4 \times 3} = \boxed{2\sqrt{3}} \text{ cm}^3$, so $k = 2$

D2 (a) $AC^2 = (\sqrt{5} + 1)^2 + (\sqrt{5} - 1)^2 = (6 + 2\sqrt{5}) + (6 - 2\sqrt{5}) = 12$, so $AC = \sqrt{12} = 2\sqrt{3}$ as required

(b) $\frac{1}{2}(\sqrt{5} + 1)(\sqrt{5} - 1) = \frac{1}{2} \times 4 = \boxed{2} \text{ cm}^2$

(c) $(\sqrt{5} + 1) + (\sqrt{5} - 1) + 2\sqrt{3} = \boxed{(2\sqrt{5} + 2\sqrt{3})} \text{ cm}$



D3 $4^t = 2 \times 8^{t-1}$, so $2^{2t} = 2^1 \times 2^{3(t-1)} = 2^{3t-2}$
 $2t = 3t - 2$ $t = 2$ Check: $4^2 = 16$ and $2 \times 8^1 = 16$

D4 (a) A quadratic in x^2 : $x^4 - x^2 - 12 = (x^2 - 4)(x^2 + 3)$, and $x^2 - 4$ is a difference of two squares.
 $(x - 2)(x + 2)(x^2 + 3)$
 $x^2 + 3$ has no real factors, because $x^2 + 3 > 0$ for every x
 (b) On the x -axis, $y = (x - 2)(x + 2)(x^2 + 3) = 0$, so $x = \pm 2$ $A(-2, 0)$, $B(2, 0)$

STRETCH

D5 (a) Area = $\frac{1}{2} \times QR \times PM$, so $PM = \frac{2(10 + 6\sqrt{3})}{4 + 2\sqrt{3}} = \frac{10 + 6\sqrt{3}}{2 + \sqrt{3}}$
 $= \frac{(10 + 6\sqrt{3})(2 - \sqrt{3})}{4 - 3} = 20 - 10\sqrt{3} + 12\sqrt{3} - 18 = 2 + 2\sqrt{3}$ as required
 (b) $PM > 5$ means $2\sqrt{3} > 3$. Both sides are positive, so square: $12 > 9$, which is true.

D6 (a) $(a + b\sqrt{3})^2 = a^2 + 3b^2 + 2ab\sqrt{3} = 37 + 20\sqrt{3}$. Compare the rational and surd parts: $a^2 + 3b^2 = 37$ and $2ab = 20$, so $ab = 10$
 (b) Positive integer pairs with $ab = 10$: try $a = 5$, $b = 2$: $25 + 12 = 37$ $a = 5$, $b = 2$

D7 (a) Constant terms: $a \times (-2) \times 1 = c$. x^2 terms: $(x - 2)(2x + 1) = 2x^2 - 3x - 2$, and $(x + a)(2x^2 - 3x - 2)$ has x^2 coefficient $2a - 3 = -1$ $a = 1$
 (b) $(x + 1)(2x^2 - 3x - 2) = 2x^3 - x^2 - 5x - 2$ $b = -5$, $c = -2$

D8 (a) $\frac{(\sqrt{3} + \sqrt{2})^2}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})} = \frac{3 + 2\sqrt{6} + 2}{3 - 2} = 5 + 2\sqrt{6}$ as required
 (b) In the same way, $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{(\sqrt{3} - \sqrt{2})^2}{1} = 5 - 2\sqrt{6}$ $(5 + 2\sqrt{6}) - (5 - 2\sqrt{6}) = 4\sqrt{6}$



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1

(a) 25

B1

(b) $\frac{8x^{\frac{3}{2}}}{4x^{-1}}$

M1: correct power of a bracket

$2x^{\frac{5}{2}}$

A1

(c) $25^{x+1} = 25 \times (5^x)^2$

M1: uses $25^x = (5^x)^2$

$25y^2$

A1

E2

(a) $(3\sqrt{5} - 2)(\sqrt{5} + 4) = 15 + 12\sqrt{5} - 2\sqrt{5} - 8$
 $= 7 + 10\sqrt{5}$

M1: four terms

as required

A1*

(b) $QR = \frac{7 + 10\sqrt{5}}{\sqrt{5} + 2} \times \frac{\sqrt{5} - 2}{\sqrt{5} - 2}$
 $= \frac{7\sqrt{5} - 14 + 50 - 20\sqrt{5}}{5 - 4}$

M1: multiplies by the conjugate

dM1

$(36 - 13\sqrt{5}) \text{ cm}$

A1

(c) $2(\sqrt{5} + 2 + 36 - 13\sqrt{5}) - 2(3\sqrt{5} - 2 + \sqrt{5} + 4) = (76 - 24\sqrt{5}) - (4 + 8\sqrt{5}) = 72 - 32\sqrt{5}$

B1

STRETCH

E3

(a) $x - 2 \times \sqrt{x} \times \frac{3}{\sqrt{x}} + \frac{9}{x}$

M1: three terms

$x - 6 + \frac{9}{x}$

A1

(b) $\sqrt{x} = \frac{3}{\sqrt{x}}$ gives $x = 3$

B1

(c) $(2 + \sqrt{3})^2 = 4 + 4\sqrt{3} + 3 = 7 + 4\sqrt{3}$, so $\sqrt{x} = 2 + \sqrt{3}$

B1

$\frac{3}{2 + \sqrt{3}} = \frac{3(2 - \sqrt{3})}{4 - 3} = 6 - 3\sqrt{3}$

M1: rationalises

$(2 + \sqrt{3}) - (6 - 3\sqrt{3}) = -4 + 4\sqrt{3}$

A1

(d) $(4\sqrt{3} - 4)^2 = 48 - 32\sqrt{3} + 16$

M1

$64 - 32\sqrt{3}$

A1

