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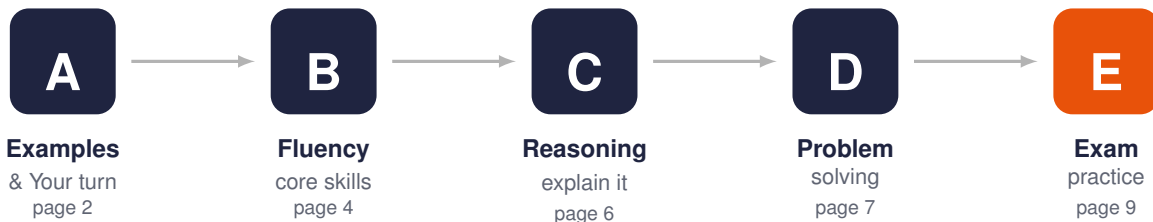
A-LEVEL MATHEMATICS • MECHANICS 1

Vectors in Mechanics

Worksheet

Pearson Edexcel International A Level (WME01) & A Level (9MA0) • Calculator and statistical tables allowed: take $g = 9.8 \text{ m s}^{-2}$ and give answers to 2 or 3 s.f.

Free Vectors in Mechanics worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: find magnitudes, directions and unit vectors • use $\mathbf{r} = \mathbf{r}_0 + \mathbf{vt}$ • find the vector between two moving points • test whether a point lies on a path • *Stretch:* parallel vectors, forces and $\mathbf{F} = m\mathbf{a}$, collisions and closest approach.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Velocities, positions at time t , paths that cross, and how close two ships come.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	19
Stretch marks	16	14	17	12
My score				

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Name

Class

Date



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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1: magnitude and direction

Find the magnitude of $6\mathbf{i} - 8\mathbf{j}$ and the angle it makes with $\mathbf{i}$

$$|a\mathbf{i} + b\mathbf{j}| = \sqrt{a^2 + b^2}; \text{ draw it and use tan}$$

$$\sqrt{36 + 64}; \tan \theta = \frac{8}{6}, \text{ below } \mathbf{i}$$

$$10, 53.1^\circ$$

Your turn A1

Find the magnitude of $-5\mathbf{i} + 12\mathbf{j}$ and the angle it makes with $\mathbf{i}$

Worked example A2: a speed in a direction

A boat moves at 15 km h^{-1} in the direction of $3\mathbf{i} + 4\mathbf{j}$. Find its velocity.

$$\text{Unit vector} \times \text{speed}: |3\mathbf{i} + 4\mathbf{j}| = 5$$

$$\frac{15}{5}(3\mathbf{i} + 4\mathbf{j})$$

$$(9\mathbf{i} + 12\mathbf{j}) \text{ km h}^{-1}$$

Your turn A2

A boat moves at 39 km h^{-1} in the direction of $12\mathbf{i} - 5\mathbf{j}$. Find its velocity.

Worked example A3: position at time t

A particle starts at $2\mathbf{i} + 5\mathbf{j}$ with velocity $3\mathbf{i} - \mathbf{j}$. Find its position when $t = 4$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$$

$$(2 + 12)\mathbf{i} + (5 - 4)\mathbf{j}$$

$$14\mathbf{i} + \mathbf{j}$$

Your turn A3

A particle starts at $-4\mathbf{i} + 7\mathbf{j}$ with velocity $2\mathbf{i} + 3\mathbf{j}$. Find its position when $t = 5$



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STRETCH

Worked example A4: velocity from two positions

A particle moves with constant velocity. At $t = 2$ it is at $3\mathbf{i} + \mathbf{j}$, and at $t = 6$ it is at $11\mathbf{i} - 7\mathbf{j}$. Find its velocity.

Displacement $\div$ time interval (4, not 6)

$$\frac{(8\mathbf{i} - 8\mathbf{j})}{4}$$

$$2\mathbf{i} - 2\mathbf{j}$$

Your turn A4

A particle moves with constant velocity. At $t = 1$ it is at $-2\mathbf{i} + 4\mathbf{j}$, and at $t = 4$ it is at $7\mathbf{i} + 10\mathbf{j}$. Find its velocity.

Worked example A5: the vector between two particles

$\mathbf{r}_A = (1 + 2t)\mathbf{i} + 3t\mathbf{j}$ and $\mathbf{r}_B = (7 - t)\mathbf{i} + (2 + t)\mathbf{j}$. Find AB when $t = 2$

$$\overrightarrow{AB} = \mathbf{r}_B - \mathbf{r}_A$$

$$(5\mathbf{i} + 4\mathbf{j}) - (5\mathbf{i} + 6\mathbf{j}) = -2\mathbf{j}$$

$$2$$

Your turn A5

$\mathbf{r}_A = (1 + 2t)\mathbf{i} + 3t\mathbf{j}$ and $\mathbf{r}_B = (7 - t)\mathbf{i} + (2 + t)\mathbf{j}$. Find AB when $t = 3$

Worked example A6: is a point on the path?

$\mathbf{r} = (1 + 2t)\mathbf{i} + (8 - 3t)\mathbf{j}$. Does the particle pass through $9\mathbf{i} - 4\mathbf{j}$?

Find t from one component, test the other

$$1 + 2t = 9 \text{ gives } t = 4; 8 - 12 = -4 \checkmark$$

$$\text{yes, at } t = 4$$

Your turn A6

$\mathbf{r} = (1 + 2t)\mathbf{i} + (8 - 3t)\mathbf{j}$. Does the particle pass through $7\mathbf{i} - 2\mathbf{j}$?

Worked example A7: a parallel resultant

Forces $(2\mathbf{i} + 3\mathbf{j})\text{ N}$ and $(p\mathbf{i} - \mathbf{j})\text{ N}$ have a resultant parallel to $3\mathbf{i} + \mathbf{j}$. Find p

Parallel means the components are in the same ratio

$$\frac{2+p}{3} = \frac{2}{1}$$

$$p = 4$$

Your turn A7

Forces $(4\mathbf{i} - \mathbf{j})\text{ N}$ and $(p\mathbf{i} + 5\mathbf{j})\text{ N}$ have a resultant parallel to $\mathbf{i} + 2\mathbf{j}$. Find p



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 A particle moves with velocity $(8\mathbf{i} - 15\mathbf{j}) \text{ ms}^{-1}$

- | | |
|---|---|
| (a) Find its speed | (b) Find the bearing of its direction of motion |
| (c) Find a unit vector in its direction | (d) Find its velocity if its speed doubles |

[4 marks]

B2 Forces $(3\mathbf{i} + 4\mathbf{j}) \text{ N}$, $(-7\mathbf{i} + 2\mathbf{j}) \text{ N}$ and $(2\mathbf{i} - 3\mathbf{j}) \text{ N}$ act on a particle of mass 0.5 kg

- | | |
|---------------------------|--|
| (a) Find the resultant | (b) Find its magnitude |
| (c) Find the acceleration | (d) Find the magnitude of the acceleration |

[4 marks]

B3 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

At noon a boat is at $(10\mathbf{i} - 4\mathbf{j}) \text{ km}$, moving with velocity $(6\mathbf{i} + 8\mathbf{j}) \text{ km h}^{-1}$

- | | |
|--------------------------------|--|
| (a) Find its speed | (b) Find $\mathbf{r}$, t hours after noon |
| (c) Find its position at 13:30 | (d) Find when it is due east of O |

[4 marks]

B4 A drone moves with constant velocity. At $t = 3 \text{ s}$ it is at $(4\mathbf{i} + 9\mathbf{j}) \text{ m}$ and at $t = 8 \text{ s}$ it is at $(24\mathbf{i} - 6\mathbf{j}) \text{ m}$

- | | |
|----------------------------------|-----------------------------------|
| (a) Find its velocity | (b) Find its speed |
| (c) Find its position at $t = 0$ | (d) Find $\mathbf{r}$ at time t |

[4 marks]



CORE *continued: relative position*

B5 $\mathbf{r}_A = (2 + 3t)\mathbf{i} + (1 + t)\mathbf{j}$ and $\mathbf{r}_B = (10 - t)\mathbf{i} + (5 - t)\mathbf{j}$

(a) Find $\overrightarrow{AB}$ in terms of t

(b) Show that the particles collide, and state when.

(c) Find where they collide.

[3 marks]

B6 $\mathbf{r} = (-6 + 2t)\mathbf{i} + (4 + 3t)\mathbf{j}$

(a) Find when it is due north of O

(b) Find its position then

(c) Does it pass through $4\mathbf{i} + 19\mathbf{j}$?

(d) Does it pass through $10\mathbf{i} + 26\mathbf{j}$?

[4 marks]

STRETCH**B7** A particle moves with velocity $(p + 2)\mathbf{i} + (2p - 1)\mathbf{j}$, which is parallel to $2\mathbf{i} + 3\mathbf{j}$

(a) Find p

(b) Find the velocity

(c) Find the speed

(d) Find the bearing of its motion

[4 marks]

B8 Forces $\mathbf{F}_1 = (a\mathbf{i} + b\mathbf{j})$ N and $\mathbf{F}_2 = (3\mathbf{i} - 5\mathbf{j})$ N act on a 2 kg particle, which accelerates at $(2\mathbf{i} + \mathbf{j}) \text{ m s}^{-2}$

(a) Find the resultant force

(b) Find a and b

(c) Find $|\mathbf{F}_1|$

(d) Find the angle $\mathbf{F}_1$ makes with $\mathbf{i}$

[4 marks]

B9 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

A ship leaves O with velocity $(4\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$ for 2 hours, then $(-\mathbf{i} + 5\mathbf{j}) \text{ km h}^{-1}$ for 3 hours.

(a) Find its position after 2 hours

(b) Find its position after 5 hours

(c) Find its distance from O

(d) Find its bearing from O

[4 marks]

B10 A particle has velocity $\mathbf{v} = (2\mathbf{i} - \mathbf{j}) + (\mathbf{i} + 2\mathbf{j})t \text{ m s}^{-1}$

(a) Find $\mathbf{v}$ when $t = 3$

(b) Find the speed when $t = 3$

(c) Find when it moves parallel to $\mathbf{i}$

(d) Find the magnitude of its acceleration

[4 marks]



C

Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Ama writes $|3\mathbf{i} - 4\mathbf{j}| = 3 + 4 = 7$
Explain her error, and find $|3\mathbf{i} - 4\mathbf{j}|$ [2 marks]
- C2** A particle is at $\mathbf{i} + 2\mathbf{j}$ when $t = 2$ and at $11\mathbf{i} - 3\mathbf{j}$ when $t = 7$. Ben finds its velocity as $\frac{10\mathbf{i} - 5\mathbf{j}}{7}$
Explain his error, and find the velocity. [2 marks]
- C3** The resultant $(p - 6)\mathbf{i} + (p + 9)\mathbf{j}$ is parallel to $4\mathbf{i} + \mathbf{j}$. Cal writes $p - 6 = 4$ and $p + 9 = 1$
Explain his error, and find p [3 marks]
- C4** A particle has velocity $-3\mathbf{i} - 3\mathbf{j}$. Dee says it moves on a bearing of 045°
Explain her error, and give the correct bearing. [3 marks]

STRETCH

- C5** $\mathbf{r}_A = (1 + 2t)\mathbf{i} + (3 + t)\mathbf{j}$ and $\mathbf{r}_B = (7 - t)\mathbf{i} + (2 + 2t)\mathbf{j}$. Eli finds that the $\mathbf{i}$ -components are equal when $t = 2$ and says the particles collide.
Explain why this is not enough, and decide whether they collide. [3 marks]
- C6** A ship has velocity $(9\mathbf{i} + 12\mathbf{j}) \text{ km h}^{-1}$. Faye says its speed is 15 m s^{-1}
Explain her error, and give the speed in m s^{-1} [3 marks]
- C7** A particle moves at 20 m s^{-1} in the direction of $3\mathbf{i} - 4\mathbf{j}$. Gus writes $\mathbf{v} = 20(3\mathbf{i} - 4\mathbf{j})$
Explain his error, and find $\mathbf{v}$ [3 marks]
- C8** Two particles move so that $\overrightarrow{AB} = (5 - 3t)\mathbf{i} + (4t - 5)\mathbf{j}$
(a) Show that $AB^2 = 25t^2 - 70t + 50$ (2)
(b) By completing the square, find the least distance between the particles and when it occurs. (3)
[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

A particle P moves on a smooth horizontal surface with constant velocity. At time $t = 2$ seconds, P is at the point with position vector $(3\mathbf{i} + 7\mathbf{j})$ m. At time $t = 6$ seconds, P is at the point with position vector $(15\mathbf{i} - \mathbf{j})$ m.

- (a) Find the velocity of P

(2)

- (b) Find the position vector of P at time t seconds.

(1)

[3 marks]

- D2** A particle of mass 4 kg moves on a smooth horizontal plane under the action of three horizontal forces, $\mathbf{F}_1 = (5\mathbf{i} + 2\mathbf{j})$ N, $\mathbf{F}_2 = (-2\mathbf{i} + 6\mathbf{j})$ N and $\mathbf{F}_3 = (p\mathbf{i} + q\mathbf{j})$ N. The acceleration of the particle is $(2\mathbf{i} - \mathbf{j})$ ms⁻²

- (a) Find the value of p and the value of q

(2)

- (b) Find the magnitude of $\mathbf{F}_3$, and the angle it makes with $\mathbf{i}$

(2)

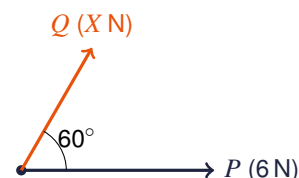
[4 marks]

- D3** Two horizontal forces act on a particle. Force P has magnitude 6 N and force Q has magnitude X newtons. The angle between their directions is 60° , as shown. The resultant has magnitude $\sqrt{76}$ N. (a) Find the value of X

(2)

- (b) Find the angle between the resultant and P

(1)



[3 marks]

- D4** In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

Ship A moves so that, t hours after noon, its position vector is $[(1 + 8t)\mathbf{i} + (6t - 6)\mathbf{j}]$ km. Ship B passes through the point P with position vector $(25\mathbf{i} + 12\mathbf{j})$ km at 14:30.

Show that A also passes through P , and find the time at which it does so.

[3 marks]



STRETCH

D5 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors.

Boat A has velocity $\mathbf{v}_A = [(3\mathbf{i} - 2\mathbf{j}) + (\mathbf{i} + 2\mathbf{j})t] \text{ km h}^{-1}$ at time t hours. Boat B moves with constant acceleration; its velocity is $(5\mathbf{i} + 4\mathbf{j}) \text{ km h}^{-1}$ when $t = 1$ and $(3\mathbf{i} + 8\mathbf{j}) \text{ km h}^{-1}$ when $t = 3$

(a) Find the acceleration of B , and its velocity when $t = 0$

(2)

(b) Find the time at which B is moving due north, and its speed then.

(2)

[4 marks]

D6 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

A ship S is at the point with position vector $(2\mathbf{i} + 10\mathbf{j}) \text{ km}$ at noon, and moves with constant velocity $(6\mathbf{i} - 8\mathbf{j}) \text{ km h}^{-1}$. A buoy is fixed at the point with position vector $(20\mathbf{i} + 2\mathbf{j}) \text{ km}$.

(a) Find the time, t hours after noon, at which S is closest to the buoy.

(2)

(b) Find the least distance between S and the buoy.

(2)

[4 marks]

D7 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

Two particles move so that, at time t seconds, $\mathbf{r}_A = [2t\mathbf{i} + (5 + t)\mathbf{j}] \text{ m}$ and $\mathbf{r}_B = [(8 - t)\mathbf{i} + (1 + 3t)\mathbf{j}] \text{ m}$

(a) Show that the particles never collide.

(2)

(b) Find the distance between them at the instant when B is due east of A

(2)

[4 marks]

D8 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

Two particles move so that, at time t seconds, $\overrightarrow{AB} = [(12 - 4t)\mathbf{i} + (8 - 2t)\mathbf{j}] \text{ m}$

(a) Find the time at which B is due north of A , and the distance AB then.

(2)

(b) Find the time at which B is north-east of A , and the distance AB then.

(3)

[5 marks]



E**Exam practice**

Full-size exam questions in the style of WME01. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question **i** and **j** are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin *O*.

A ship *A* is moving with constant velocity. At noon, the position vector of *A* is $(4\mathbf{i} - 2\mathbf{j})$ km. At 2 pm, the position vector of *A* is $(16\mathbf{i} + 14\mathbf{j})$ km. At time t hours after noon, the position vector of *A* is $\mathbf{r}_A$ km.

- (a) Show that $\mathbf{r}_A = (4 + 6t)\mathbf{i} + (8t - 2)\mathbf{j}$

(3)

- (b) Find the speed of ship *A*, giving your answer in metres per second.

(2)

A second ship *B* moves with constant velocity $(-6\mathbf{i} + 10\mathbf{j})\text{ km h}^{-1}$. At noon, the position vector of *B* is $(31\mathbf{i} + 7\mathbf{j})$ km.

- (c) Show that *A* and *B* both pass through the same point *P*, and find the length of time between *B* passing *P* and *A* passing *P*

(4)

(Total for Question E1 is 9 marks)



E2 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors.

Two boats, A and B , are each moving on a lake with constant acceleration. The boats are modelled as particles. At time t hours after noon, boat A has velocity $\mathbf{v}_A = [(1 + 2t)\mathbf{i} + (4 - t)\mathbf{j}] \text{ km h}^{-1}$

(a) Find the magnitude of the acceleration of A

(2)

When $t = 1$, the velocity of B is $(6\mathbf{i} + 2\mathbf{j}) \text{ km h}^{-1}$, and when $t = 4$, the velocity of B is $(3\mathbf{i} + 8\mathbf{j}) \text{ km h}^{-1}$

(b) Find the acceleration of B , giving your answer in terms of $\mathbf{i}$ and $\mathbf{j}$

(2)

(c) Find the velocity of B at noon.

(2)

(d) Find the time at which both boats are moving with the same speed.

(3)

(e) Find the common speed of the boats at this time.

(1)

(Total for Question E2 is 10 marks)



STRETCH

E3 In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors directed due east and due north respectively, and position vectors are given relative to a fixed origin O .

Two ships, A and B , move with constant velocities. At noon, ship A is at the point with position vector $(2\mathbf{i} + 3\mathbf{j})$ km and ship B is at the point with position vector $(12\mathbf{i} - 7\mathbf{j})$ km. The velocity of A is $(5\mathbf{i} + 2\mathbf{j})$ km h⁻¹ and the velocity of B is $(-\mathbf{i} + 6\mathbf{j})$ km h⁻¹

(a) Show that, t hours after noon, $\vec{AB} = [(10 - 6t)\mathbf{i} + (4t - 10)\mathbf{j}]$ km

(2)

(b) Show that the ships will never collide.

(2)

(c) Find the least distance between the ships, and the time at which it occurs.

(4)

(d) Find the bearing of B from A at that time.

(2)

For safety, the ships must stay at least 3 km apart at all times.

(e) Find, to the nearest minute, the length of time for which they are less than 3 km apart.

(2)

(Total for Question E3 is 12 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Mechanics 1 Chapter 3, Vectors in Mechanics, with a full mark scheme.

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