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INTERNATIONAL A LEVEL MATHEMATICS • PURE 2 (WMA12)

Pure 2 Practice Paper 2

Mark Scheme

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA12/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1, 2, 3, 5, 6, 8 and 9 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A **49** • B **42** • C **35**

Based on recent WMA12 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Binomial coefficients forming an arithmetic sequence; sum of an AP	8	Ch 4 The Binomial Expansion Ch 5 Sequences and Series	
2	Factorising a cubic; a trigonometric equation that hides it	8	Ch 1 Algebraic Methods Ch 6 Trig Identities and Equations	
3	Stationary points; equation of a circle; tangent meeting a cubic	9	Ch 7 Differentiation Ch 2 Coordinate Geometry Ch 1 Algebraic Methods	
4	Logarithms of a geometric sequence; sum of a GP with logs	8	Ch 3 Exponentials and Logarithms Ch 5 Sequences and Series	
5	Equal bases in an exponential equation; trigonometric equations	8	Ch 3 Exponentials and Logarithms Ch 6 Trig Identities and Equations	
6	Increasing functions and the discriminant; areas above and below the axis	9	Ch 7 Differentiation Ch 8 Integration	
7	Trapezium rule for 3^x; area between a curve and a line	8	Ch 8 Integration Ch 3 Exponentials and Logarithms	
8	Binomial expansion with surds; proof for all n	8	Ch 4 The Binomial Expansion Ch 1 Algebraic Methods	
9	Area between a chord and a curve; a maximum point	9	Ch 8 Integration Ch 7 Differentiation	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy or linking the parts together. Paper 3 gives almost no help between the parts and adds a trap to most questions, so check every answer against the conditions in the question.

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Question 1

8 marks

Part	Scheme	Marks
(a)	Coefficients: $\binom{7}{1}k = 7k$, $\binom{7}{2}k^2 = 21k^2$, $\binom{7}{3}k^3 = 35k^3$	B1
	Arithmetic sequence, so the middle term is the mean: $2\left(21k^2\right) = 7k + 35k^3$	M1
	$35k^3 - 42k^2 + 7k = 0$, so $7k\left(5k^2 - 6k + 1\right) = 0$	A1
	$k \neq 0$, so solves $(5k-1)(k-1) = 0$	dM1
	$k = \frac{1}{5}$ or $k = 1$	A1
	Writing $21k^2 - 7k = 35k^3 - 21k^2$ is the same condition. Leaving $k = 0$ in the final answer loses the A1.	(5)
(b)	$k = 1$ gives 7, 21, 35 (increasing); $k = \frac{1}{5}$ gives $\frac{7}{5}$, $\frac{21}{25}$, $\frac{7}{25}$, so $a = \frac{7}{5}$, $d = -\frac{14}{25}$	B1
	$S_{20} = \frac{20}{2}\left[2\left(\frac{7}{5}\right) + 19\left(-\frac{14}{25}\right)\right]$	M1
	$-\frac{392}{5}$ (-78.4)	A1
	The coefficients are $\binom{7}{r}k^r$: forgetting the power of k gives the same three numbers for every k and no equation.	(3)
	Total for Question 1: 8 marks	

Question 2

8 marks

Part	Scheme	Marks
(a)	Finds a factor by the factor theorem, e.g. $f(-1) = -6 - 7 + 7 + 6 = 0$, so $(x+1)$ is a factor	M1
	Divides: $f(x) = (x+1)\left(6x^2 - 13x + 6\right)$	M1
	$f(x) = (x+1)(2x-3)(3x-2)$	A1
		(3)
(b)	Uses $\sin^2\theta = 1 - \cos^2\theta$: $6\cos^3\theta + 7 - 7\cos^2\theta = 7\cos\theta + 1$	M1
	$6\cos^3\theta - 7\cos^2\theta - 7\cos\theta + 6 = 0$, which is $f(\cos\theta) = 0$	A1
	Uses (a): $\cos\theta = -1$, $\frac{2}{3}$, $\frac{3}{2}$	dM1
	$\cos\theta = \frac{2}{3}$: $\theta = 48.2^\circ, 311.8^\circ$	A1
	$\cos\theta = -1$: $\theta = 180^\circ$; and $\cos\theta = \frac{3}{2}$ rejected as $\cos\theta \leqslant 1$. All three:	A1
	$48.2^\circ, 180^\circ, 311.8^\circ$	
	The question gives no hint that the cubic appears, so the key step is spotting $f(\cos\theta) = 0$. Any answer from $\cos\theta = \frac{3}{2}$ loses the final A1.	(5)
	Total for Question 2: 8 marks	

Question 3

9 marks

Part	Scheme	Marks
(a)	$\frac{dy}{dx} = 3x^2 - 12x + 9$	M1
	$3(x-1)(x-3) = 0$, so $x = 1,\ x = 3$	M1
	$\boxed{A(1, 5)}$ and $\boxed{B(3, 1)}$	A1
	The maximum is the left-hand turning point of a positive cubic, so A has $x = 1$. Solutions from a calculator's turning-point function score 0.	(3)
(b)	$r^2 = (3-1)^2 + (1-5)^2 = 20$	M1
	$\boxed{(x-3)^2 + (y-1)^2 = 20}$	A1
		(2)
(c)	Gradient of $BA = \frac{5-1}{1-3} = -2$, so the tangent (perpendicular to the radius) has gradient $\frac{1}{2}$	M1
	Tangent: $y - 5 = \frac{1}{2}(x-1)$, i.e. $y = \frac{1}{2}x + \frac{9}{2}$	A1
	Sets equal to the curve and uses $x = 1$ as a known root: $2x^3 - 12x^2 + 17x - 7 = (x-1)\left(2x^2 - 10x + 7\right) = 0$	M1
	$\boxed{x = \frac{5 \pm \sqrt{11}}{2}}$	A1
	The common error is to use the gradient of the curve at A (which is 0) instead of the radius. The tangent to the circle is not the tangent to the curve.	(4)
		Total for Question 3: 9 marks

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Question 4

8 marks

Part	Scheme	Marks
(a)	$\log_2 u_n = \log_2 a + (n-1)\log_2 r$, so the gradient of the line is $\log_2 r$: $\log_2 r = \dfrac{7-(-2)}{4-1} = 3$	M1
	Common ratio $\boxed{r = 8}$	A1
	First term: $\log_2 u_1 = -2$, so $\boxed{u_1 = \frac{1}{4}}$	B1
		(3)
(b)	$\log_2 u_n = 3n - 5$ forms an arithmetic sequence with first term -2 and $d = 3$: $\dfrac{20}{2}[2(-2) + 19(3)]$	M1
	$\boxed{530}$	A1
		(2)
(c)	$\dfrac{\frac{1}{4}(8^n - 1)}{8-1} > 10^9$, so $8^n > 2.8 \times 10^{10} + 1$	M1
	$n > \dfrac{\log\left(2.8 \times 10^{10} + 1\right)}{\log 8} = 11.57\ldots$	M1
	$\boxed{n = 12}$	A1
	Trial values are fine if they show $S_{11} = 3.07 \times 10^8$ and $S_{12} = 2.45 \times 10^9$.	
		(3)
	Total for Question 4: 8 marks	

Question 5

8 marks

Part	Scheme	Marks
(i)	Writes both sides as powers of 3: $3^{2\sin^2 x} = 3^{3\cos x}$, so $2\sin^2 x = 3\cos x$	M1
	Uses $\sin^2 x = 1 - \cos^2 x$: $2\cos^2 x + 3\cos x - 2 = 0$	M1
	$(2\cos x - 1)(\cos x + 2) = 0$: $\cos x = \frac{1}{2}$, and $\cos x = -2$ is rejected	A1
	$\boxed{x = \dfrac{\pi}{3}, \dfrac{5\pi}{3}}$	A1
		(4)
(ii)	Divides by $\cos(2\theta + 40°)$: $\tan(2\theta + 40°) = \frac{2}{3}$	M1
	$40° \leqslant 2\theta + 40° < 400°$; the principal value $33.7°$ is outside this range, so $2\theta + 40° = 213.7°,\ 393.7°$	M1
	Subtracts $40°$ and halves	dM1
	$\boxed{\theta = 86.8°,\ 176.8°}$	A1
	Using $33.7°$ gives the negative value $-3.2°$, which is out of range. Missing $393.7°$ is the most common error.	
		(4)
	Total for Question 5: 8 marks	

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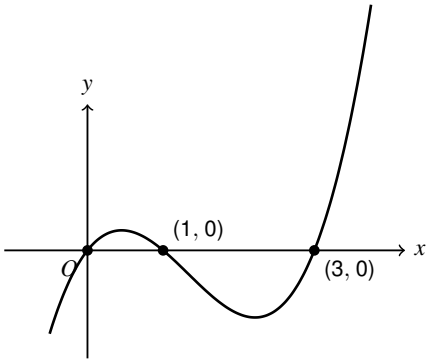
Question 6

9 marks

Part	Scheme	Marks
(a)	$f'(x) = 3x^2 + 2ax + 3$, and f increasing needs $f'(x) \geqslant 0$ for all x	M1
	The quadratic never goes below zero: $b^2 - 4ac \leqslant 0$, i.e. $4a^2 - 36 \leqslant 0$	M1
	$\boxed{-3 \leqslant a \leqslant 3}$	A1
	$-3 < a < 3$ is also accepted. Using $b^2 - 4ac > 0$ or $\geqslant 0$ scores M0.	
		(3)
(b)		B1
	A positive cubic shape, with a maximum between 0 and 1 and a minimum between 1 and 3	
	Meets the x-axis at (0, 0), (1, 0) and (3, 0), from $f(x) = x(x-1)(x-3)$	B1
		(2)
(c)	$\int (x^3 - 4x^2 + 3x)\,dx = \frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}$	M1
	From 0 to 1: $\frac{1}{4} - \frac{4}{3} + \frac{3}{2} = \frac{5}{12}$; from 1 to 3: $\left(\frac{81}{4} - 36 + \frac{27}{2}\right) - \frac{5}{12} = -\frac{8}{3}$	A1
	Treats the two regions separately and adds their sizes: $\frac{5}{12} + \frac{8}{3}$	M1
	$\boxed{\frac{37}{12}}$	A1
	Integrating from 0 to 3 in one go gives $-\frac{9}{4}$: the region below the axis cancels part of the one above. This scores M1 A0 M0 A0.	
		(4)
		Total for Question 6: 9 marks

Question 7

8 marks

Part	Scheme	Marks
(a)	$3^0 = 1 = 2(0)+1$ and $3^1 = 3 = 2(1)+1$, so they meet at $x = 0$ and $x = 1$	**B1** **(1)**
(b)	$h = 0.25$; y values $1,\ 1.31607,\ 1.73205,\ 2.27951,\ 3$ $\frac{1}{2} \times 0.25 \times [1+3+2(1.31607+1.73205+2.27951)]$ 1.832	**B1** **M1** **A1** **(3)**
(c)	Area under the line from 0 to 1: $\int_0^1 (2x+1)\,\mathrm{d}x = \left[x^2+x\right]_0^1 = 2$ (or a trapezium of area 2) Area of $R \approx 2 - 1.832 =$ 0.168	**M1** **A1** **(2)**
(d)	The curve $y = 3^x$ bends upwards (it is convex), so the trapezium rule overestimates the area under the curve R is the line's area minus that, so (c) is an underestimate (needs the first B1) *The common error is to stop at "overestimate". Here the trapezium-rule value is subtracted, so the error is reversed.*	**B1** **B1** **(2)**
		Total for Question 7: 8 marks

Question 8

8 marks

Part	Scheme	Marks
(a)	$(1+\sqrt{2})^5 = 1+5\sqrt{2}+10(\sqrt{2})^2+10(\sqrt{2})^3+5(\sqrt{2})^4+(\sqrt{2})^5$	M1
	$= 1+5\sqrt{2}+20+20\sqrt{2}+20+4\sqrt{2}$	M1
	$\boxed{41+29\sqrt{2}}$	A1
		(3)
(b)	Expands both: $\sum_r \binom{n}{r}(\sqrt{2})^r$ and $\sum_r \binom{n}{r}(-\sqrt{2})^r$	M1
	Terms with r odd cancel; terms with r even are doubled, and $(\sqrt{2})^{2k} = 2^k$	M1
	So the sum is $2\sum_k \binom{n}{2k}2^k$, a sum of integers, so it is an integer for every positive integer n	A1
	Checking a few values of n is not a proof. The argument must work for any n.	(3)
(c)	Changing the sign of $\sqrt{2}$ in (a): $(1-\sqrt{2})^5 = 41-29\sqrt{2}$, and $\lvert 1-\sqrt{2}\rvert < 1$, so this is small: $-0.0122\ldots$	M1
	$\frac{41}{29}-\sqrt{2} = \frac{41-29\sqrt{2}}{29} = -0.00042\ldots$, and $0.00042 < 0.001$	A1
		(2)
		Total for Question 8: 8 marks

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Question 9

9 marks

Part	Scheme	Marks
(a)	$\int \left(6x^{\frac{1}{2}} - x\right) \mathrm{d}x = 4x^{\frac{3}{2}} - \frac{x^2}{2}$	M1
	Area under C from 0 to p: $4p^{\frac{3}{2}} - \frac{p^2}{2}$	A1
	Area of the triangle under OP: $\frac{1}{2} \times p \times \left(6\sqrt{p} - p\right) = 3p^{\frac{3}{2}} - \frac{p^2}{2}$	M1
	Subtracts: $\left(4p^{\frac{3}{2}} - \frac{p^2}{2}\right) - \left(3p^{\frac{3}{2}} - \frac{p^2}{2}\right)$	dM1
	$\boxed{p^{\frac{3}{2}}}$	A1
		(5)
(b)	Area under C from 0 to 36: $4(216) - 648 = 216$, so $p^{\frac{3}{2}} = \frac{216}{8} = 27$	M1
	$\boxed{p = 9}$	A1
		(2)
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^{-\frac{1}{2}} - 1 = 0$ when $\sqrt{x} = 3$, i.e. $x = 9$	M1
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -\frac{3}{2}x^{-\frac{3}{2}} < 0$, so it is a maximum, at $x = p = 9$: $P(9, 9)$ is the maximum point	A1
		(2)
		Total for Question 9: 9 marks

TOTAL FOR PAPER: 75 MARKS

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