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A-LEVEL MATHEMATICS • DECISION 1

Algorithms on Graphs

Worksheet

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Free Algorithms on Graphs worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 11

Tap a section to jump straight to it

By the end of this worksheet you can: use Kruskal's and Prim's algorithms • Prim on a matrix • *Stretch:* Dijkstra with working values, tracing routes, and using final values when the network changes.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Minimum spanning trees by Kruskal and Prim, and shortest routes by Dijkstra, with the follow-ups exams build on them.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: Kruskal

Use Kruskal on AB 5, AC 3, BC 4, BD 6, CD 7, DE 2, CE 8

DE 2 ✓, AC 3 ✓, BC 4 ✓, AB 5 ×, BD 6 ✓

weight 15

Your turn A1

Use Kruskal on PQ 6, PR 2, QR 5, QS 4, RS 7, ST 3, RT 9

Worked example A2: Prim on a network

Use Prim from A on AB 7, AC 4, AD 9, BC 3, BD 5, CD 8

AC 4, CB 3, BD 5

weight 12

Your turn A2

Use Prim from P on PQ 2, PR 6, PS 5, QR 3, QS 7, RS 4

Worked example A3: Prim on a matrix

Use Prim from A on

	A	B	C	D
A	–	8	5	9
B	8	–	6	4
C	5	6	–	7
D	9	4	7	–

AC 5, CB 6, BD 4

weight 15

Your turn A3

Use Prim from D on

	A	B	C	D
A	–	3	7	6
B	3	–	5	9
C	7	5	–	2
D	6	9	2	–

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STRETCH

Worked example A4: Dijkstra

Find the shortest route from S to T: SA 4, SB 7, AB 2, AT 8, BT 3

Labels S 0, A 4, B 6 (7 improved to 6), T 9 (12 improved to 9)

$S–A–B–T$, 9

Your turn A4

Find the shortest route from S to T: SA 3, SB 9, AB 4, AC 8, BC 2, BT 7, CT 6

Worked example A5: tracing back

Final values S 0, A 6, B 10, T 15; arcs SA 6, SB 11, AB 4, AT 10, BT 5. Find the route to T

$15 - 5 = 10$*: BT;* $10 - 4 = 6$*: AB;* $6 - 6 = 0$*: SA*

$S–A–B–T$

Your turn A5

Final values S 0, A 4, B 9, T 14; arcs SA 4, SB 9, AB 6, AT 11, BT 5. Find the route to T

Worked example A6: arcs in a tree

How many arcs are in a minimum spanning tree for 7 vertices?

6

Your turn A6

How many arcs are in a minimum spanning tree for 12 vertices?

Worked example A7: using final values

From S, final values are A 6, B 10. Arc BC is 4 and C can only be reached through B. Find the shortest S to C.

14

Your turn A7

From S, final values are P 9, Q 13. Arc QR is 7 and R can only be reached through Q. Find the shortest S to R.

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 A network has arcs AB 9, AC 4, AD 7, BC 6, BE 10, CD 3, CE 11, DE 12.

(a) List the arcs in Kruskal's order of consideration.

(b) State which arcs are rejected.

(c) Find the weight of the tree.

(d) State the number of arcs in the tree.

[4 marks]

B2 For the network in B1,

(a) use Prim starting at E

(b) state the order the arcs are added

(c) compare the weight with B1

(d) explain why the trees match.

[4 marks]

B3 Use Prim, starting at C, on

	A	B	C	D	E
A	–	14	9	21	17
B	14	–	12	8	19
C	9	12	–	16	11
D	21	8	16	–	13
E	17	19	11	13	–

(a) State the first arc.

(b) List the arcs in order.

(c) Find the weight.

(d) State the vertex added last.

[4 marks]

B4 A network has arcs SA 5, SB 9, AB 3, AC 11, BC 6, BD 12, CD 4, CT 13, DT 5.

(a) Give the order of labelling.

(b) Give the final values.

(c) Give the working values at T

(d) State the shortest route and its length.

[4 marks]

B5 For the network in B4, use the final values

(a) to show that BC is on the shortest route to C

(b) to show that CT is not on the shortest route to T

(c) to state the shortest distance from S to D

[3 marks]

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B6 A connected network has 10 vertices.

(a) State the arcs in its minimum spanning tree.

(b) Explain why Kruskal rejects some arcs.

(c) Explain why Prim never needs to check for cycles.

(d) State what Dijkstra's final values give.

[4 marks]

STRETCH

B7 Use Prim, starting at D, on

	A	B	C	D	E	F
A	–	23	18	27	31	20
B	23	–	15	29	21	26
C	18	15	–	24	19	33
D	27	29	24	–	16	25
E	31	21	19	16	–	28
F	20	26	33	25	28	–

(a) List the arcs in order.

(b) Find the weight.

(c) State the column numbers in order.

(d) Give the arc added last.

[4 marks]

B8 A network has arcs SP 6, SQ 14, SR 4, PQ 7, PU 15, QR 9, QU 5, QT 12, RT 20, UT 3.

(a) Give the order of labelling.

(b) Give every working value at Q, U and T

(c) State the shortest route to T

(d) State its length.

[4 marks]

B9 For the network in B8,

(a) state the shortest distance from S to U

(b) find the shortest route from S to T if UT is closed

(c) state the shortest S to T via R

(d) explain how the final values help.

[4 marks]

B10 The network in B1 has an arc AE of weight x added.

(a) State the path in the tree from A to E

(b) State its largest arc.

(c) Find the values of x that change the tree.

(d) Find the new weight if $x = 5$

[4 marks]

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C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 In B1, Amir gives the weight of the tree as 39. Explain his error. **[2 marks]**

C2 Asked to use Prim on the matrix in B3 starting at C, Bea starts by choosing the smallest entry in the whole table, BD 8. Explain her error. **[2 marks]**

C3 Cal uses Prim on the matrix in A3 (example) by always moving on from the most recently added vertex. Explain why this is not Prim's algorithm, and how the two methods differ. **[3 marks]**

C4 In B4, Dev writes only 23 in the working values at T. Explain why this is not acceptable, and what he should have written. **[3 marks]**

STRETCH

C5 After finding the final values in B8, Eve runs Dijkstra again from S to find the shortest distance to U. Explain why this is unnecessary. **[3 marks]**

C6 To find the route in B4, Fay starts at S and always follows the shortest arc. Explain why this can fail, and the correct method. **[3 marks]**

C7 Gus says a new arc can never change a minimum spanning tree, because the old tree still connects every vertex. Use B10 with $x = 5$ to show that he is wrong. **[3 marks]**

C8 **(a)** Explain why a minimum spanning tree on n vertices has exactly $n - 1$ arcs.

(2)

(b) Explain why Dijkstra's algorithm cannot give a final value that is later improved, given that every arc has a positive weight.

(3)

[5 marks]

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D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 Figure 1 shows a network of cables that could be laid between six buildings. The number on each arc is the cost, in thousands of pounds, of laying the cable.

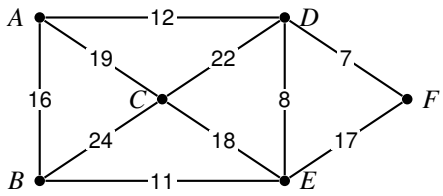


Figure 1

Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them and state whether each one is added to your tree. State the total cost of the cables in your tree.

[3 marks]

D2 The table shows the distances, in km, between six warehouses, P, Q, R, S, T and U.

	P	Q	R	S	T	U
P	–	31	24	40	35	28
Q	31	–	19	27	33	45
R	24	19	–	22	38	30
S	40	27	22	–	17	36
T	35	33	38	17	–	26
U	28	45	30	36	26	–

(a) Use Prim's algorithm, starting at S, to find a minimum spanning tree. You must state the order in which you select the arcs.

(3)

(b) State the weight of your minimum spanning tree.

(1)

[4 marks]

D3 Figure 2 shows a network of paths between six places on a farm, with lengths in metres. Dijkstra's algorithm from A gave the final values A 0, B 9, C 12, D 16, E 27, F 15

(a) Use the final values to find the shortest route from A to E, showing how you checked each arc.

(2)

(b) A new path is built directly from A to E, of length 25 m. State the new shortest distance from A to E, and whether any other final value changes.

(1)

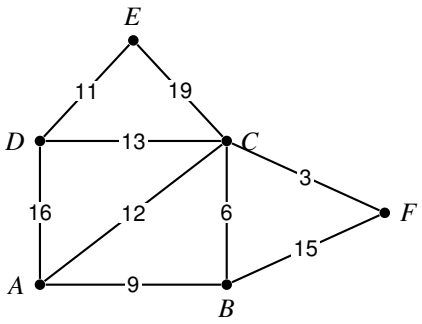


Figure 2

[3 marks]

D4 Figure 3 shows the costs, in hundreds of pounds, of linking seven computers.

(a) Use Prim's algorithm, starting at D, to find a minimum spanning tree, stating the order in which you add the arcs.

(2)

(b) State the weight of the tree, and state what Kruskal's algorithm would give for this weight.

(1)

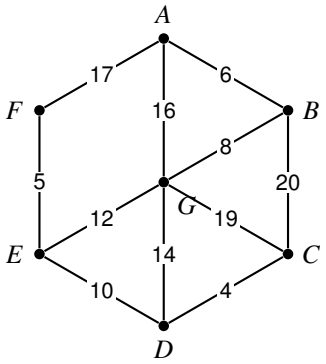


Figure 3

[3 marks]

STRETCH

D5 The table shows the lengths, in metres, of possible pipes between five tanks, J, K, L, M and N. The length of pipe JL is x metres.

	J	K	L	M	N
J	–	12	x	20	15
K	12	–	9	17	22
L	x	9	–	11	18
M	20	17	11	–	14
N	15	22	18	14	–

(a) Taking $x = 15$, use Prim's algorithm starting at J to find a minimum spanning tree and its weight.

(2)

(b) A different value of x gives a minimum spanning tree of weight 41. Determine x, explaining which arcs the tree must contain.

(2)

[4 marks]

D6 Figure 4 shows a grid of streets, with travel times in minutes.

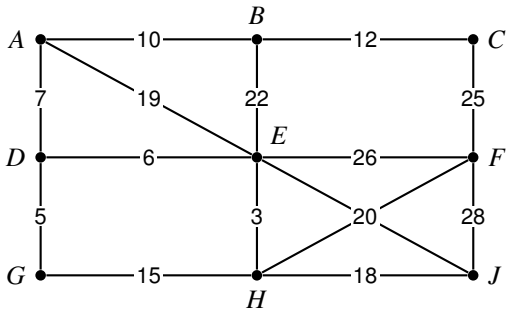


Figure 4

(a) Use Dijkstra's algorithm to find the quickest route from A to J and its time.

(3)

(b) The street EJ is closed for repairs. Find the new quickest route from A to J and its time.

(1)

[4 marks]

D7 Figure 5 shows possible links, with costs, between nine pumping stations.

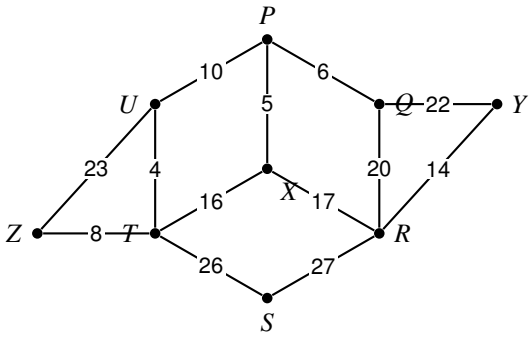


Figure 5

(a) Use Kruskal's algorithm to find a minimum spanning tree, listing the arcs in the order considered and stating whether each is added. State its weight.

(3)

(b) A new link QS of cost 15 becomes possible. Determine the weight of the new minimum spanning tree, explaining your reasoning.

(1)

[4 marks]

D8 Figure 6 shows a network of roads between seven villages, with lengths in km.

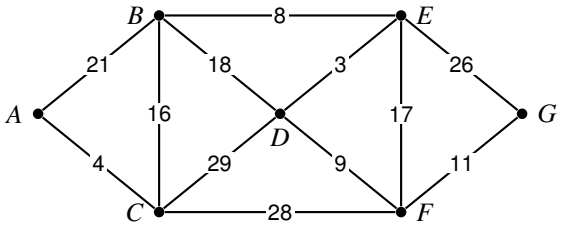


Figure 6

(a) Use Dijkstra's algorithm to find the shortest route from A to G and its length. You must show the working values at each vertex.

(3)

(b) The road EG is to be rebuilt along a new line of length x km. Determine the values of x for which the shortest route from A to G would then use EG.

(2)

[5 marks]

E Exam practice

Exam-style questions in the style of WDM11. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 Figure 7 represents a network of roads between nine towns, S, A, B, C, D, E, F, G and T. The number on each arc is the time, in minutes, to drive along the corresponding road. Mira needs to drive from town S to town T as quickly as possible, and she also wants to know the quickest times from S to the other towns so that she can plan deliveries on the way.

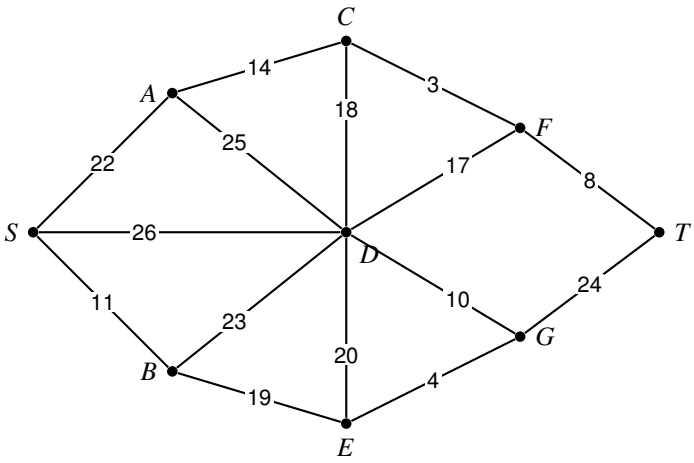


Figure 7

(a) Use Dijkstra's algorithm to find the quickest route from S to T. You should complete the boxes on the answer-book page, showing the order of labelling, the final value and every working value at each vertex.

(5)

(b) State the quickest route and its time, and explain how you used the final values to find the route.

(2)

(c) On one day, Mira must call at town G to collect a parcel on her way from S to T. Use your final values, together with Figure 7, to find the quickest such route and its time, explaining your method clearly. You do not need to apply Dijkstra's algorithm again.

(3)

(Total for Question E1 is 10 marks)

Answer book: Question E1

Work in the space on this page, as you would in the WDM11 answer book.

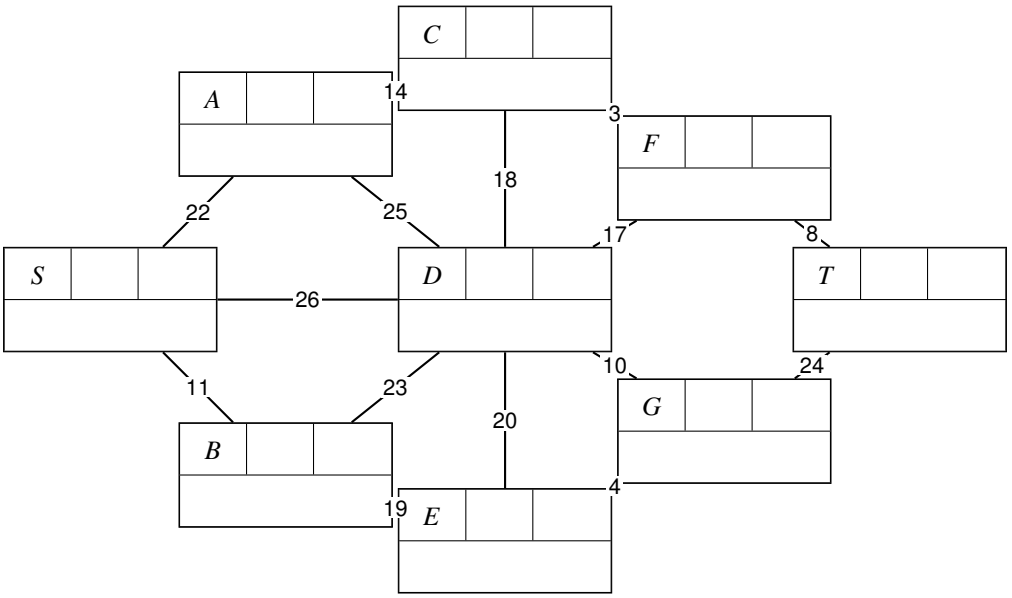


Diagram 1

Key:

Vertex	Order of labelling	Final value
Working values		

Quickest route _____

Time _____

E2 Figure 8 shows the lengths, in km, of the roads between nine villages, A, B, C, D, E, F, G, H and J. A company wants to connect all nine villages to a new fibre network, laying cable along the roads, and wishes to minimise the total length of cable used.

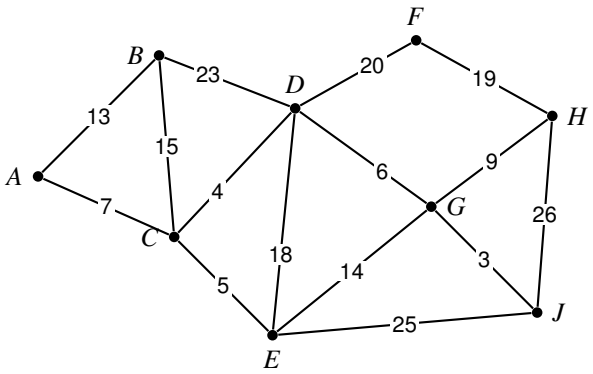


Figure 8

(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them, and state whether each is added to the tree.

(3)

(b) State the minimum total length of cable needed.

(1)

The table of shortest distances between the villages is shown on the answer-book page.

(c) Use Prim's algorithm on the table of shortest distances, starting at A, to find a minimum spanning tree. You must state the order in which you select the arcs.

(3)

(d) State the weight of this tree and explain why it equals your answer to part (b).

(2)

(e) An engineer needs to drive from village B to village F to inspect the work. Use the table to state the shortest distance from B to F, and use Figure 8 to give the corresponding route along the roads, explaining how you found it.

(3)

(Total for Question E2 is 12 marks)

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Answer book: Question E2

Work in the space on this page, as you would in the WDM11 answer book.

	A	B	C	D	E	F	G	H	J
A	–	13	7	11	12	31	17	26	20
B	13	–	15	19	20	39	25	34	28
C	7	15	–	4	5	24	10	19	13
D	11	19	4	–	9	20	6	15	9
E	12	20	5	9	–	29	14	23	17
F	31	39	24	20	29	–	26	19	29
G	17	25	10	6	14	26	–	9	3
H	26	34	19	15	23	19	9	–	12
J	20	28	13	9	17	29	3	12	–

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STRETCH

E3 Figure 9 shows a network of paths in a nature reserve, joining eight viewpoints, A to H. The number on each arc is the length of the path in hundreds of metres. The reserve manager wants the shortest walk from the entrance at A to the viewpoint at H, and also wants to light a set of paths that connects every viewpoint with the least total length. The path between B and D is being rebuilt, and its new length will be x hundred metres.

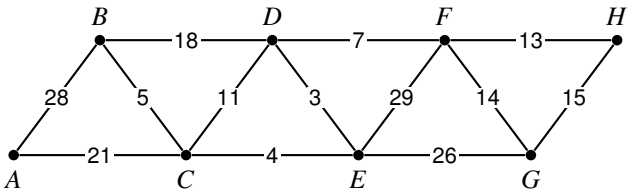


Figure 9

(a) With the current length of BD, use Dijkstra's algorithm to find the shortest walk from A to H and its length. Complete the boxes on the answer-book page.

(5)

(b) Use Kruskal's algorithm, with the current length of BD, to find a minimum spanning tree for the network. List the arcs in the order you consider them, state whether each is added, and state the weight of the tree.

(3)

(c) Determine the values of x for which BD would be included in the minimum spanning tree. You must give a reason for your answer.

(3)

(d) Determine the values of x for which the shortest walk from A to H would use BD, explaining how you use your final values. You may assume that the lengths of all the other paths stay the same and that x is a positive integer.

(3)

(Total for Question E3 is 14 marks)

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Answer book: Question E3

Work in the space on this page, as you would in the WDM11 answer book.

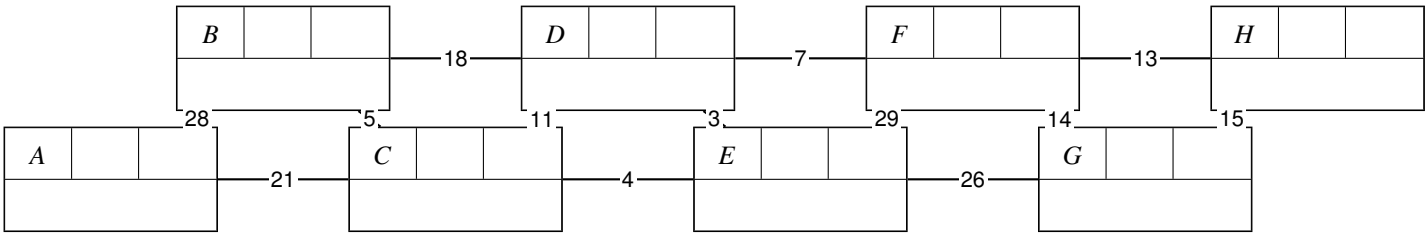


Diagram 2

Key:

Vertex	Order of labelling	Final value
Working values		

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