



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Differentiation

Worksheet

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Free Differentiation worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: standard derivatives and the chain rule • product and quotient rules • *Stretch:* $x = f(y)$, normals and turning points.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH The chain, product and quotient rules, x as a function of y , tangents, normals and turning points.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE*Secure the method***Worked example A1**

Differentiate $y = e^{3x} + \ln 2x$

$$e^{kx} \rightarrow ke^{kx}; \ln 2x = \ln 2 + \ln x \rightarrow \frac{1}{x}$$

$$3e^{3x} + \frac{1}{x}$$

Your turn A1

Differentiate $y = \sin 4x - 2 \ln 5x$

Worked example A2

Differentiate $y = (3x - 1)^5$

Chain rule: power down, reduce by one, then multiply by the inner derivative 3

$$15(3x - 1)^4$$

Your turn A2

Differentiate $y = \sqrt{4x^2 + 1}$

Worked example A3

Differentiate $y = x^2 e^{3x}$, factorising your answer.

Product rule: $u = x^2$, $v = e^{3x}$; $uv' + vu' = 3x^2 e^{3x} + 2x e^{3x}$

$$xe^{3x}(3x + 2)$$

Your turn A3

Differentiate $y = x^3 \ln x$, factorising your answer.



STRETCH

Worked example A4: the quotient rule

Differentiate $y = \frac{2x+1}{x-3}$

$$\frac{vu' - uv'}{v^2} = \frac{2(x-3) - (2x+1)}{(x-3)^2}$$

$$-\frac{7}{(x-3)^2}$$

Your turn A4

Differentiate $y = \frac{x^2}{x+2}$

Worked example A5: trigonometric functions

Differentiate $y = \tan 2x$ and $y = \sec x$

$\tan kx \rightarrow k \sec^2 kx$; $\sec x = (\cos x)^{-1} \rightarrow \sin x (\cos x)^{-2}$

$$2\sec^2 2x; \sec x \tan x$$

Your turn A5

Differentiate $y = \operatorname{cosec} 3x$

Worked example A6: x as a function of y

$x = y^3 + 2y$. Find $\frac{dy}{dx}$ when $y = 1$

$$\frac{dx}{dy} = 3y^2 + 2 = 5, \text{ and } \frac{dy}{dx} = \frac{1}{dx/dy}$$

$$\frac{1}{5}$$

Your turn A6

$x = e^{2y} - y$. Find $\frac{dy}{dx}$ when $y = 0$

Worked example A7: a tangent

Find the tangent to $y = xe^x$ at $x = 0$

$$\frac{dy}{dx} = e^x(1+x) = 1 \text{ at } x = 0, \text{ and } y = 0$$

$$y = x$$

Your turn A7

Find the tangent to $y = x \ln x$ at $x = e$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Differentiate

(a) e^{5x}

(b) $\ln 7x$

(c) $\cos 3x$

(d) $4e^{-\frac{x}{2}}$

[4 marks]

B2 Differentiate

(a) $(2x+5)^6$

(b) $\ln(x^2+1)$

(c) e^{x^2}

(d) $\sin^3 x$

[4 marks]

B3 Differentiate

(a) xe^{2x}

(b) $x^2 \sin x$

(c) $e^x \cos x$

(d) $x \ln 3x$

[4 marks]

B4 Differentiate, simplifying your answers.

(a) $\frac{x}{x+1}$

(b) $\frac{e^x}{x}$

(c) $\frac{\ln x}{x^2}$

(d) $\frac{\sin x}{1+\cos x}$

[4 marks]



CORE *continued: using derivatives*

B5 $y = xe^{-x}$

(a) Find $\frac{dy}{dx}$

(b) Find the exact gradient at $x = \frac{1}{2}$

(c) Find the exact stationary point.

[3 marks]

B6 Differentiate

(a) $\tan 3x$

(b) $\sec 2x$

(c) $\cot x$

(d) $\operatorname{cosec}(x^2)$

[4 marks]

STRETCH

B7 Differentiate, simplifying or factorising your answers.

(a) x^2e^{-x}

(b) $e^{2x} \sin 3x$

(c) $\frac{(x-1)^3}{x+1}$

(d) $\ln(\sin x)$

[4 marks]

B8 Find $\frac{dy}{dx}$ in terms of y , and in (c) and (d) also in terms of x

(a) $x = y^2 + 3y$

(b) $x = \sin 2y$

(c) $x = \ln(y+1)$

(d) $x = \tan y$

[4 marks]

B9 $y = \ln(2x-1)$

(a) Find the gradient of the curve at $x = 1$

(b) Find the equation of the tangent at $(1, 0)$

(c) Find the equation of the normal at $(1, 0)$

(d) Find the x coordinate of the point where the gradient is $\frac{1}{2}$

[4 marks]

B10 $y = xe^{-x^2}$

(a) Show that $\frac{dy}{dx} = e^{-x^2}(1-2x^2)$

(b) Find the exact x coordinates of the stationary points.

(c) Find the exact y coordinate of the stationary point with $x > 0$

(d) Explain why the equation $\frac{dy}{dx} = 0$ has no other solutions.

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

C1 Sam writes $\frac{d}{dx}(\sin 3x) = \cos 3x$

Explain Sam's mistake, and give the correct derivative.

[2 marks]

C2 Lily writes $\frac{d}{dx} \ln(x^2 + 4) = \frac{1}{x^2 + 4}$

Explain Lily's mistake, and give the correct derivative.

[2 marks]

C3 For $y = \frac{x}{x+2}$, Kai writes $\frac{dy}{dx} = \frac{x - (x+2)}{(x+2)^2} = -\frac{2}{(x+2)^2}$

Explain Kai's mistake, and give the correct derivative.

[3 marks]

C4 To find the stationary points of $y = x^2 e^x$, Jo writes $x e^x (x+2) = 0$, divides by $x e^x$, and finds only $x = -2$

Explain what Jo has lost, and why dividing by e^x is safe but dividing by x is not.

[3 marks]



STRETCH

C5 $f(x) = \frac{3x}{x+1}$, $x > -1$. Max writes " $f'(x) = \frac{3}{(x+1)^2}$, so f is increasing."

Explain what Max must add for a complete answer.

[3 marks]

C6 A curve has equation $x = y^2 + 1$. Ali writes $\frac{dy}{dx} = 2y$

Explain Ali's mistake, and find the gradient of the curve at the point (5, 2)

[3 marks]

C7 The tangent to a curve at P has gradient 3. Kai writes the normal at P with gradient -3

Explain Kai's mistake, and give the gradient of the normal.

[3 marks]

C8 (a) Using the quotient rule, prove that $\frac{d}{dx}(\tan x) = \sec^2 x$

(2)

(b) Prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$

(2)

(c) Hence show that $\frac{d}{dx} \ln(\sec x) = \tan x$

(1)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

A curve has equation $y = \frac{x+4}{2x-1}$, $x \neq \frac{1}{2}$. The curve has two separate branches, one on each side of its vertical asymptote.

- (a) Use the quotient rule to show that $\frac{dy}{dx} = \frac{k}{(2x-1)^2}$, where k is a constant to be found.

(2)

- (b) Hence explain why the curve has no stationary points, and why y decreases as x increases on each branch of the curve.

(1)

[3 marks]

- D2** Figure 1 shows the curve with equation $x = y^2 - 4y + 1$, in which x is given as a function of y . At the point P the tangent to the curve is parallel to the y -axis.

- (a) Find $\frac{dx}{dy}$

(1)

- (b) Find the coordinates of P

(2)

- (c) Find the gradient of the curve at the point $(1, 4)$

(1)

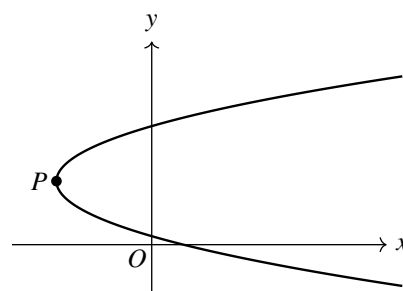


Figure 1

[4 marks]



- D3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 2 shows part of the curve with equation $y = 2x \ln 3x$, $x > 0$. The curve meets the x -axis at P and has a minimum point Q .

Use calculus to find the exact coordinates of Q , showing each step of your working and giving each coordinate in terms of e .

(3)

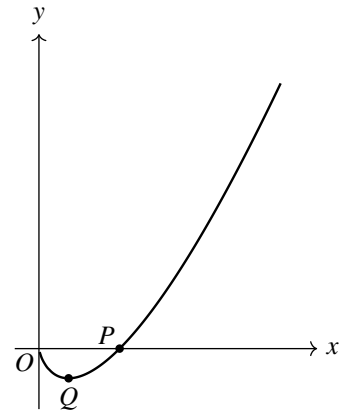


Figure 2

[3 marks]

- D4** A curve has equation $y = e^{2x} - 3x$. The tangent to the curve at the point where $x = 0$ meets the x -axis at the point R .

Find the equation of the tangent in the form $y = mx + c$, and hence the coordinates of R , showing your method clearly and stating the gradient of the curve at the point where $x = 0$.

[3 marks]

STRETCH

- D5** Figure 3 shows the curve C with equation

$$y = \frac{x^2}{\sqrt{2x+1}}, \quad x > -\frac{1}{2}$$

(a) Show that $\frac{dy}{dx} = \frac{x(3x+2)}{(2x+1)^{\frac{3}{2}}}$

(3)

(b) Using your answer to part (a), explain why the origin O is the only stationary point on C .

(1)

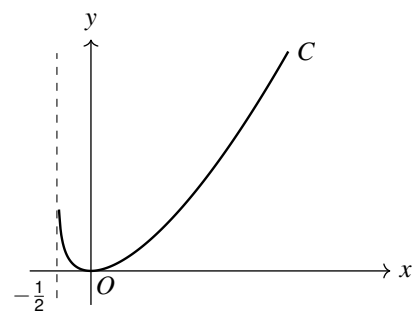


Figure 3

[4 marks]



- D6** Figure 4 shows the curve with equation $x = e^{2y} + e^y$.
The curve meets the x -axis at $P(2, 0)$

(a) Find $\frac{dx}{dy}$, and hence the gradient of the curve at P

(2)

(b) The normal to the curve at P meets the y -axis at the point N . Find the coordinates of N , showing clearly how you find the gradient of the normal.

(2)

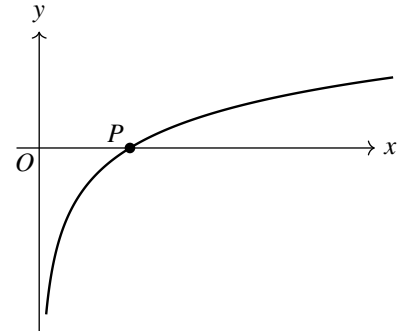


Figure 4

[4 marks]

- D7** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

A curve has equation $y = \frac{6}{(3x - k)^2}$, where k is a constant and $x \neq \frac{k}{3}$

(a) Use the chain rule to find $\frac{dy}{dx}$ in terms of x and k , without expanding the bracket.

(2)

(b) Given that the gradient of the curve at $x = 1$ is $-\frac{9}{2}$, find the value of k

(2)

[4 marks]

- D8** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 5 shows the curve with equation $y = e^{-x} \sin x$, $0 \leq x \leq \pi$. The point M is the maximum point of the curve.

(a) Show that $\frac{dy}{dx} = e^{-x}(\cos x - \sin x)$

(2)

(b) Find the exact coordinates of M

(2)

(c) Find the exact gradient of the curve at the point where $x = \pi$

(1)

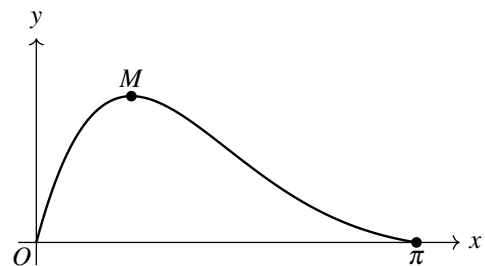


Figure 5

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA13. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{x^2 + 5}{x - 2} \quad x \neq 2$$

- (a) Use the quotient rule to show that $\frac{dy}{dx} = \frac{(x-a)(x+b)}{(x-2)^2}$, where a and b are positive integers to be found.

.....
(3)

- (b) Hence find the set of values of x for which y is increasing, explaining the role of the denominator in your answer.

.....
(2)

(Total for Question E1 is 5 marks)



E2 Figure 6 shows part of the curve C with equation

$$y = 3 \ln(2x - 3) \quad x > \frac{3}{2}$$

The curve meets the x -axis at the point $P(2, 0)$. The line l is the normal to C at P

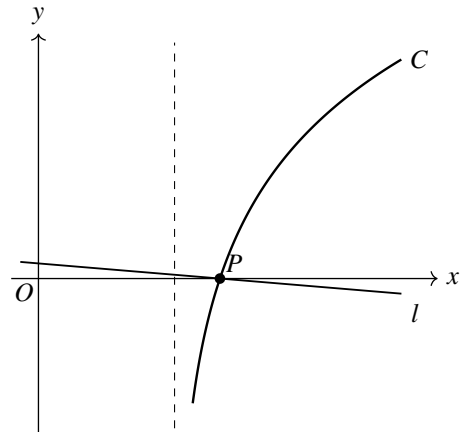


Figure 6

(a) Find $\frac{dy}{dx}$, and hence the gradient of C at P

.....
(2)

(b) Find an equation for l , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

.....
(3)

(c) Find the exact coordinates of the point where l meets the y -axis.

.....
(1)

(Total for Question E2 is 6 marks)



STRETCH

E3 Figure 7 shows a sketch of the curve C with equation

$$y = (x^2 - 3)e^x \quad x \in \mathbb{R}$$

The curve has a maximum point A and a minimum point B

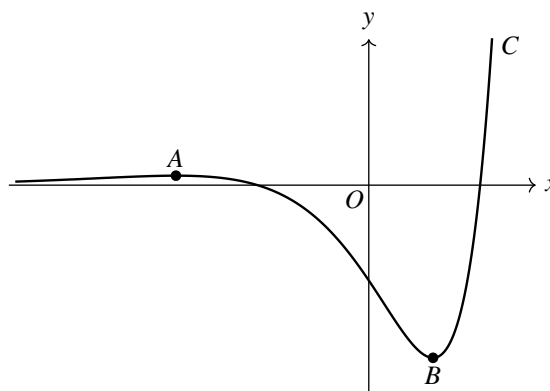


Figure 7

(a) Show that $\frac{dy}{dx} = e^x(x+p)(x+q)$, where p and q are integers to be found.

.....
(2)

(b) Find the exact coordinates of A and of B

.....
(3)

(c) Using your answer to (b) and Figure 7, state the range of the function $f(x) = (x^2 - 3)e^x$, $x \in \mathbb{R}$

.....
(1)

(d) The tangent to C at the point where $x = 0$ meets the x -axis at T . Find the coordinates of T

.....
(2)

(Total for Question E3 is 8 marks)

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