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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 2

# Further Pure 2

## Chapter 7: Maclaurin and Taylor Series

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM02/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM02 paper, with a full mark scheme


| Chapters (tap a chapter to jump to it) | | Qs | Marks | Page | My score |
|---|---|---|---|---|---|
| 1 | **Inequalities**<br>Rational and modulus inequalities, critical values, graphs | 10 | 80 | separate file | |
| 2 | **Series**<br>Method of differences, partial fractions, telescoping sums | 10 | 80 | separate file | |
| 3 | **Complex Numbers**<br>Exponential form, de Moivre's theorem, trigonometric identities, $n$th roots | 10 | 80 | separate file | |
| 4 | **Further Argand Diagrams**<br>Loci, regions and transformations of the complex plane | 10 | 80 | separate file | |
| 5 | **First-Order Differential Equations**<br>Integrating factors, substitutions, general and particular solutions | 10 | 80 | separate file | |
| 6 | **Second-Order Differential Equations**<br>Auxiliary equations, complementary functions, particular integrals, substitutions | 10 | 80 | separate file | |
| 7 | **Maclaurin and Taylor Series**<br>Higher derivatives, standard and compound series, Taylor series, series solutions | 10 | 80 | 2 | |
| 8 | **Polar Coordinates**<br>Polar curves, areas, tangents parallel and perpendicular to the initial line | 10 | 80 | separate file | |



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CHAPTER 7 • KEY SKILLS

# 7 Maclaurin and Taylor Series

Specification FP2: Maclaurin and Taylor series • 10 questions • 80 marks • about 96 minutes

### 1 Higher derivatives

Use the product and chain rules, and simplify each derivative before differentiating again.

### 2 Maclaurin series

$f(x) =$

$$f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

Find the values at 0 first, then substitute.

### 3 Standard series

Use $e^x$, $\sin x$, $\cos x$, $\ln(1+x)$ and $(1+x)^n$ from the formula book, with their ranges of validity.

### 4 Compound functions

Substitute (e.g. $2x$ for $x$), use $\ln\frac{a}{b} = \ln a - \ln b$, or multiply series, discarding powers beyond the one asked for.

### 5 Taylor series

About $x = a$:

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

### 6 Differential equations

Differentiate the equation itself to get higher derivatives, then substitute the known values at the point.

### 7 Using a relation

If $y^{(4)} = -4y$ then $y^{(5)} = -4y'$ and $y^{(6)} = -4y''$: no need to differentiate again.

### 8 Approximations

Substitute a small value; percentage error $= \dfrac{|\text{approx} - \text{true}|}{\text{true}} \times 100$

### 9 Limits

Replace each function by its series, cancel the lowest power of $x$, then let $x \to 0$

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|---|---|---|---|---|---|---|---|---|---|---|
| *Marks* | 6 | 6 | 6 | 6 | 6 | 10 | 10 | 10 | 10 | 10 |
| 1 Higher derivatives | • | | • | | • | • | • | | | • |
| 2 Maclaurin series | • | | | | | • | | | | • |
| 3 Standard series | | • | | • | | | | • | • | |
| 4 Compound functions | | • | | • | | | | • | • | |
| 5 Taylor series | | | • | | • | | • | | | |
| 6 Differential equations | | | | | • | | • | | | |
| 7 Using a relation | | | | | | • | | | | • |
| 8 Approximations | | | • | | | | • | • | | |
| 9 Limits | | | | • | | | | | • | |
| **Interleaved:** differentiation rules (P3) | • | | • | | • | • | • | | | • |
| **Interleaved:** logarithms and binomial expansion (P2, P4) | • | • | | | | | | • | • | • |
| **Interleaved:** differential equations (Chapters 5, 6) | | | | | • | | • | | | |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** List the values $f(a)$, $f'(a)$, $f''(a)$, … before writing the series, show the factorials, and finish with "$y = \ldots$". When told "hence", use the result just found.

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**1.**

$$f(x) = \ln(\cos x), \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

**(a)** Find $f'(x)$, $f''(x)$, $f'''(x)$ and $f''''(x)$.

**(4)**

**(b)** Deduce that, for small values of $x$, $\ln(\cos x) \approx -\frac{1}{2}x^2 - \frac{1}{12}x^4$

**(2)**

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**Question 1 continued**

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**(Total for Question 1 is 6 marks)**

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**2.** **(a)** Using standard series, find the series expansion of $\ln\left(\frac{1+2x}{1-x}\right)$ in ascending powers of $x$, up to and including the term in $x^3$

**(4)**

**(b)** State the range of values of $x$ for which the expansion is valid.

**(2)**

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**Question 2 continued**

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**(Total for Question 2 is 6 marks)**

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**3.** **(a)** Find the Taylor series of $\tan x$ about $x = \frac{\pi}{4}$, up to and including the term in $\left(x - \frac{\pi}{4}\right)^2$

**(4)**

**(b)** Use your series to estimate $\tan 0.8$, giving your answer to 5 decimal places.

**(2)**

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**Question 3 continued**

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**4.** **(a)** Using standard series, show that $e^x \sin x = x + x^2 + \frac{1}{3}x^3 + \cdots$, and show that the coefficient of $x^4$ is zero.

**(4)**

**(b)** Hence find $\lim_{x \to 0} \frac{e^x \sin x - x}{x^2}$

**(2)**

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**Question 4 continued**

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**5.** The function $y = f(x)$ satisfies the differential equation $\frac{dy}{dx} = x^2 + y^2$, with $y = 1$ when $x = 0$

**(a)** Find the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 0$

**(4)**

**(b)** Hence find a series solution for $y$ in ascending powers of $x$, up to and including the term in $x^3$

**(2)**

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**Question 5 continued**

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**6.**

$$y = e^x \cos x$$

**(a)** Show that $\dfrac{d^4y}{dx^4} = -4y$

**(4)**

**(b)** Hence find the values of $\dfrac{d^5y}{dx^5}$ and $\dfrac{d^6y}{dx^6}$ when $x = 0$

**(2)**

**(c)** Write down the expansion of $e^x \cos x$ in ascending powers of $x$ as far as the term in $x^5$, showing the value of each derivative used.

**(4)**

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**Question 6 continued**

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**7.**

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

At $x = 1$, $y = 1$ and $\frac{dy}{dx} = 0$

**(a)** Find the value of $\frac{d^2y}{dx^2}$ at $x = 1$

**(1)**

**(b)** Show that $\frac{d^3y}{dx^3} = -2\frac{dy}{dx} - x\frac{d^2y}{dx^2}$

**(2)**

**(c)** Find the value of $\frac{d^4y}{dx^4}$ at $x = 1$

**(2)**

**(d)** Hence find a series solution for $y$ in ascending powers of $(x-1)$, up to and including the term in $(x-1)^4$

**(3)**

**(e)** Use your series to estimate the value of $y$ at $x = 1.2$, giving your answer to 4 decimal places.

**(2)**

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**Question 7 continued**

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**8.**

$$f(x) = \frac{e^{2x}}{1-x}$$

**(a)** By multiplying two standard series, show that $f(x) = 1 + 3x + 5x^2 + \frac{19}{3}x^3 + \cdots$

**(4)**

**(b)** State the range of values of $x$ for which the expansion is valid.

**(1)**

**(c)** Use the series with $x = 0.1$ to estimate $f(0.1)$, and find the percentage error in this estimate.

**(3)**

**(d)** Use the series to estimate $\int_0^{0.1} f(x)\,dx$

**(2)**

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**Question 8 continued**

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**9.** Use series expansions to find the values of the following limits, showing your working.

**(a)** Show that $1 - \cos 2x = 2x^2 - \frac{2}{3}x^4 + \cdots$

**(2)**

**(b)** $\lim_{x \to 0} \frac{x \sin x}{1 - \cos 2x}$

**(3)**

**(c)** $\lim_{x \to 0} \frac{e^x - 1 - x}{x \sin x}$

**(3)**

**(d)** $\lim_{x \to 0} \frac{\ln(1 + x) - x}{x^2}$

**(2)**

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**Question 9 continued**

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**10.**

$$y = \ln\left(1 + e^{x}\right)$$

**(a)** Show that $\frac{dy}{dx} = \frac{e^{x}}{1 + e^{x}}$ and that $\frac{d^2y}{dx^2} = \frac{dy}{dx}\left(1 - \frac{dy}{dx}\right)$

**(3)**

**(b)** Hence find the Maclaurin series of $y$, up to and including the term in $x^4$

**(5)**

**(c)** By showing that $\ln(1 + e^{x}) - \frac{x}{2} = \ln\left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right)$, explain why your series has no terms in $x^3$ or $x^5$

**(2)**

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**TOTAL FOR CHAPTER 7 IS 80 MARKS**

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