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INTERNATIONAL A LEVEL MATHEMATICS • PURE 1 (WMA11)

Pure 1 Practice Paper 1

Mark Scheme

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA11/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 1, 2, 3, 4, 6, 7 and 10 say this. There, a correct final answer with no algebraic working scores **no marks**.

Rough grade guide (out of 75): A **54** • B **47** • C **40**

Based on recent WMA11 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Question paper | Exam questions by topic | More practice papers

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	Index laws to form simultaneous equations; rationalising a denominator	7	Ch 1 Algebraic Expressions	
2	Completing the square, sketching, and roots read from the graph	7	Ch 2 Quadratics; Ch 4 Graphs and Transformations	
3	Intersection of a line and a curve; a region bounded by three graphs	7	Ch 3 Equations and Inequalities	
4	Perpendicular bisector; area of a triangle from coordinates	7	Ch 5 Straight-line Graphs	
5	Cosine rule, the ambiguous case of the sine rule, area	8	Ch 6 Trigonometric Ratios	
6	Differentiating fractional and negative powers; tangent and normal	8	Ch 8 Differentiation; Ch 1 Algebraic Expressions	
7	Integrating to find f(x) from a point; a normal	8	Ch 9 Integration; Ch 8 Differentiation	
8	Arc length, sector and segment area in radians	7	Ch 7 Radians; Ch 6 Trigonometric Ratios	
9	Translations and stretches; roots of f(x) + k = 0	7	Ch 4 Graphs and Transformations	
10	Factorising and sketching a cubic; discriminant with a parameter	9	Ch 4 Graphs and Transformations; Ch 2 Quadratics; Ch 3 Equations and Inequalities	
	Total	**75**		

Where did the marks go? If you lost the first marks of a question, the method needs work: go back to that chapter's key skills page. If you lost the final marks, the method is fine and the gap is accuracy or linking the parts together. Papers 2 and 3 give less help between the parts, so that is the skill to build next.

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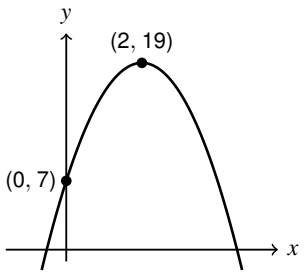
Question 1

7 marks

Part	Scheme	Marks
(a)	Writes the first equation with base 2: $2^{2x} \times 2^{y} = 2^{4} \times 2^{\frac{1}{2}}$, so $2x + y = \frac{9}{2}$	M1
	Writes the second equation with base 3: $\frac{3^{2x}}{3^{3y}} = 3^{\frac{1}{2}}$, so $2x - 3y = \frac{1}{2}$	M1
	Solves the two linear equations, e.g. subtracts: $4y = 4$	dM1
	$\boxed{x = \frac{7}{4},\ y = 1}$	A1
	Both equations must come from correct index work. $4^{x} = 2^{2x}$ and $27^{y} = 3^{3y}$ are the steps most often missed.	(4)
(b)	$(3 - \sqrt{5})^{2} = 9 - 6\sqrt{5} + 5 = 14 - 6\sqrt{5}$	M1
	Multiplies numerator and denominator by $\sqrt{5} + 1$: $\frac{(14 - 6\sqrt{5})(\sqrt{5} + 1)}{5 - 1} = \frac{14\sqrt{5} + 14 - 30 - 6\sqrt{5}}{4}$	M1
	$= \frac{-16 + 8\sqrt{5}}{4} = \boxed{-4 + 2\sqrt{5}}$	A1
	Expanding the bracket as $9 - 5$ or $9 + 5$ with no $\sqrt{5}$ term is the common error and loses the first mark.	(3)
		Total for Question 1: 7 marks

Question 2

7 marks

Part	Scheme	Marks
(a)	$f(x) = -3\left(x^2 - 4x\right) + 7 = -3\left[(x-2)^2 - 4\right] + 7$ $b = -3$ $\boxed{f(x) = 19 - 3(x-2)^2}$, so $a = 19$, $b = -3$, $c = -2$	M1 B1 A1 (3)
(b)	 A $\cap$ shape with its maximum at (2, 19) Crosses the y-axis at (0, 7) *The maximum must be in the first quadrant. The x-intercepts are not needed.*	B1 B1 (2)
(c)	Uses the graph: $k < 19$ for two roots, and $k > 7$ so that both roots are to the right of the y-axis $\boxed{7 < k < 19}$ *The common wrong answer is $k < 19$. At $k = 7$ the roots are 0 and 4, and 0 is not positive, so $k = 7$ is excluded. Using the product of the roots, $\frac{k-7}{3} > 0$, is also fine.*	M1 A1 (2)
		Total for Question 2: 7 marks

Question 3

7 marks

Part	Scheme	Marks
(a)	$2x^2 - 5x - 3 = 2x + 1$ $2x^2 - 7x - 4 = 0$ *	M1 A1* (2)
(b)	$(2x+1)(x-4) = 0 \Rightarrow x = -\frac{1}{2},\ x = 4$ $\boxed{A\left(-\frac{1}{2}, 0\right)}$ and $\boxed{B(4, 9)}$ *Solving with the quadratic formula on a calculator with no working scores 0 here.*	M1 A1 (2)
(c)	$y \leqslant 2x + 1$ $y \geqslant 2x^2 - 5x - 3$ $y \geqslant 0$ *Strict inequalities are also accepted. All three are needed: without $y \geqslant 0$ the region would include the part below the x-axis between A and $(3, 0)$.*	B1 B1 B1 (3)
		Total for Question 3: 7 marks

Question 4

7 marks

Part	Scheme	Marks
(a)	Gradient of $AB = \frac{8-4}{5-(-3)} = \frac{1}{2}$, so the perpendicular gradient is -2 Midpoint of AB is $(1, 6)$: $y - 6 = -2(x - 1)$ $2x + y - 8 = 0$ *	M1 M1 A1* (3)
(b)	Solves $2x + y - 8 = 0$ and $y = x + 2$: $3x - 6 = 0$ $C(2, 4)$ A complete method for the area, e.g. AC is horizontal, so area $= \frac{1}{2} \times AC \times$ height of B above $AC = \frac{1}{2} \times 5 \times 4$ $\boxed{\text{Area} = 10}$ *Alternative: C is on the perpendicular bisector, so the height is $MC = \sqrt{5}$ and area $= \frac{1}{2} \times AB \times MC = \frac{1}{2} \times 4\sqrt{5} \times \sqrt{5} = 10$. The shoelace formula is also fine.*	M1 A1 M1 A1 (4)
		Total for Question 4: 7 marks

Question 5

8 marks

Part	Scheme	Marks
(a)	$AC^2 = 120^2 + 85^2 - 2 \times 120 \times 85\cos 108°$	M1
	$\boxed{AC = 167.1\text{ m}}$	A1
		(2)
(b)	$\dfrac{\sin ADC}{167.1} = \dfrac{\sin 32°}{110}$	M1
	$\sin ADC = 0.805\ldots$, giving $53.6°$	A1
	Angle ADC is obtuse, so $ADC = 180° - 53.6° = \boxed{126.4°}$	A1
	This is the ambiguous case. Giving $53.6°$ as the final answer scores M1 A1 A0.	
		(3)
(c)	Area $ABC = \frac{1}{2} \times 120 \times 85\sin 108° = 4850\ldots$	M1
	Angle $ACD = 180° - 32° - 126.4° = 21.6°$; area $ACD = \frac{1}{2} \times 167.1 \times 110\sin 21.6° = 3386\ldots$	M1
	$\boxed{\text{Area of the field} = 8240\text{ m}^2}$	A1
	Using $53.6°$ from part (b) gives the wrong triangle ACD and the wrong total. Other correct methods for area ACD, e.g. finding AD first, are fine.	
		(3)
	Total for Question 5: 8 marks	

Question 6

8 marks

Part	Scheme	Marks
(a)	Writes $y = x^{\frac{3}{2}} - 16x^{-1} - 4x^{\frac{1}{2}} + 10$ and differentiates: at least one power reduced by 1	M1
	Two of the three terms correct	A1
	$\boxed{\dfrac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 16x^{-2} - 2x^{-\frac{1}{2}}}$	A1
	Any equivalent simplest form, e.g. $\frac{3}{2}\sqrt{x} + \frac{16}{x^2} - \frac{2}{\sqrt{x}}$. The constant 10 must disappear.	
		(3)
(b)	At $x = 4$: $\frac{3}{2}(2) + \frac{16}{16} - \frac{2}{2} = 3 + 1 - 1 = 3$	M1
	$y - 6 = 3(x - 4) \Rightarrow y = 3x - 6$ *	A1*
		(2)
(c)	Normal gradient $-\frac{1}{3}$: $y - 6 = -\frac{1}{3}(x - 4)$	M1
	Normal meets the x-axis at $Q(22, 0)$; tangent meets it at $R(2, 0)$	A1
	Area $= \frac{1}{2} \times (22 - 2) \times 6 = \boxed{60}$	A1
	The height of the triangle is the y coordinate of P, which is 6.	
		(3)
	Total for Question 6: 8 marks	

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Question 7

8 marks

Part	Scheme	Marks
(a)	Splits the fraction: $f'(x) = \frac{5}{2}x^{\frac{3}{2}} - \frac{9}{2}x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$	M1
	Integrates: at least one power increased by 1	M1
	$x^{\frac{5}{2}} - 3x^{\frac{3}{2}} - 4x^{\frac{1}{2}}$ $(+c)$, all three terms correct	A1
	Substitutes $(1, -3)$: $1 - 3 - 4 + c = -3$	M1
	$\boxed{f(x) = x^{\frac{5}{2}} - 3x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 3}$	A1
	Integrating the numerator and the denominator separately is not a valid method and scores M0. The final A1 needs $c = 3$ and the whole of $f(x)$ written out.	(5)
(b)	$f(4) = 32 - 24 - 8 + 3 = 3$ and $f'(4) = \frac{80 - 36 - 4}{4} = 10$	M1
	Normal gradient $-\frac{1}{10}$: $y - 3 = -\frac{1}{10}(x - 4)$	M1
	$\boxed{x + 10y - 34 = 0}$	A1
	Any non-zero integer multiple is accepted. Using the gradient 10 for the normal scores M1 M0 A0.	(3)
	Total for Question 7: 8 marks	

Question 8

7 marks

Part	Scheme	Marks
(a)	$\cos AOB = \frac{6^2 + 6^2 - 9^2}{2 \times 6 \times 6} = -\frac{1}{8}$	M1
	$AOB = 1.6961\ldots = 1.696$ (3 d.p.) *	A1*
	Alternative: $\sin\left(\frac{1}{2}AOB\right) = \frac{4.5}{6}$. *A value of at least 4 decimal places must be seen before rounding.*	(2)
(b)	Major arc $= 6(2\pi - 1.696) = 27.52\ldots$, then adds the chord: $27.52 + 9$	M1
	$\boxed{\text{Perimeter} = 36.5\,\text{cm}}$	A1
	Using the minor arc, 6×1.696, is the common error and scores M0.	(2)
(c)	Minor segment $= \frac{1}{2} \times 6^2 \times (1.696 - \sin 1.696) = 12.67\ldots$, or the major sector $= \frac{1}{2} \times 6^2 \times (2\pi - 1.696) = 82.56\ldots$	M1
	A complete method: $\pi \times 6^2 - 12.67$, or major sector + triangle AOB $= 82.56 + \frac{1}{2} \times 6^2 \sin 1.696$	M1
	$\boxed{\text{Area} = 100\,\text{cm}^2}$ $(100.4\ldots)$	A1
	The calculator must be in radians. Using degrees in $\sin 1.696$ *gives a different, wrong area.*	(3)
	Total for Question 8: 7 marks	

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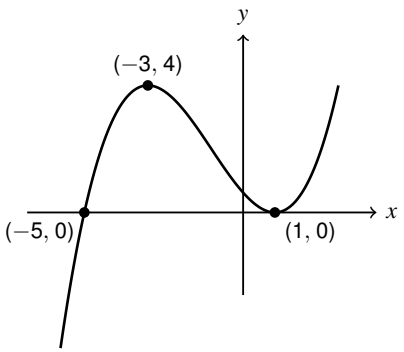
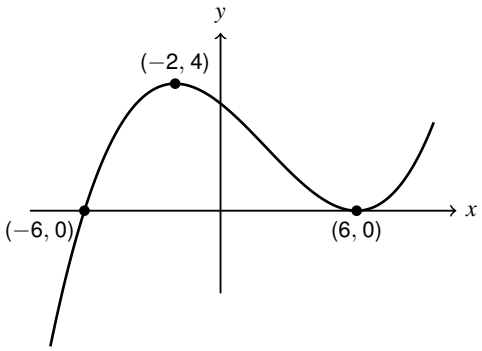
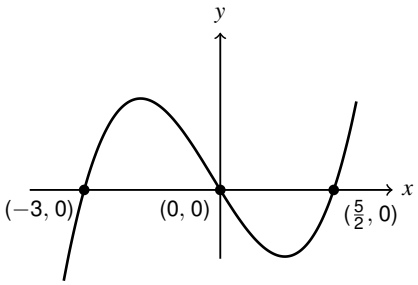
Question 9

7 marks

Part	Scheme	Marks
(a)	 The same shape translated 2 units to the left Meets the x-axis at $(-5, 0)$ and touches it at $(1, 0)$ Maximum point at $(-3, 4)$ *A shift to the right, giving $(-1, 0)$, $(1, 4)$ and $(5, 0)$, is the common error and scores B1 B0 B0 at most.*	B1 B1 B1 (3)
(b)	 A stretch parallel to the x-axis: meets the x-axis at $(-6, 0)$ and touches it at $(6, 0)$ Maximum point at $(-2, 4)$, with the correct shape *The y coordinates do not change. Halving the x coordinates instead of doubling them scores B0 B0.*	B1 B1 (2)
(c)	One correct region, $k > 0$ or $k < -4$, from moving the curve up or down $\boxed{k < -4 \text{ or } k > 0}$ *For $k > 0$ the minimum lifts off the axis; for $k < -4$ the maximum drops below it. $k = 0$ and $k = -4$ both give two points, so they are excluded.*	M1 A1 (2)
		Total for Question 9: 7 marks

Question 10

9 marks

Part	Scheme	Marks
(a)	$f(x) = x\left(2x^2 + x - 15\right)$, then factorises the quadratic $f(x) = x(2x-5)(x+3)$	**M1** **A1** **(2)**
(b)	 A positive cubic shape: up on the right, down on the left, with two turning points Meets the axes at $(-3, 0)$, $(0, 0)$ and $\left(\frac{5}{2}, 0\right)$ only	**B1** **B1** **(2)**
(c)	$2x^3 + x^2 - 15x = kx$, and divides by x as $x \neq 0$ $2x^2 + x - 15 - k = 0$, i.e. $2x^2 + x - (15 + k) = 0$ *	**M1** **A1*** **(2)**
(d)	Two distinct roots: $b^2 - 4ac > 0$, i.e. $1 + 8(15 + k) > 0$ $k > -\frac{121}{8}$ $k \neq -15$, since then $x = 0$ is a root of the quadratic and the line meets the curve at only two points. Final answer $k > -\frac{121}{8},\ k \neq -15$ *Most candidates stop at $k > -\frac{121}{8}$. The B1 is for spotting that one of the two roots must not be $x = 0$.*	**M1** **A1** **B1** **(3)**
		Total for Question 10: 9 marks

TOTAL FOR PAPER: 75 MARKS

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