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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Chapter 1: Complex Numbers

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	2	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	separate file	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	separate file	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	separate file	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	separate file	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	separate file	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	separate file	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	separate file	

Full mark scheme | Free courses | More worksheets

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CHAPTER 1 • KEY SKILLS

1 Complex Numbers

Specification FP1: Complex numbers • 10 questions • 80 marks • about 96 minutes

1 Arithmetic

$i^2 = -1$. Add and subtract real and imaginary parts; multiply out brackets and replace i^2 by -1

2 Conjugate and division

$z = a + bi$ has conjugate $z^* = a - bi$. To divide, multiply top and bottom by the conjugate of the denominator.

3 Equating parts

Two complex numbers are equal when their real parts are equal and their imaginary parts are equal: two equations.

4 Argand diagram

Plot $a + bi$ at (a, b): real numbers on the real axis, conjugate pairs reflected in it.

5 Modulus

$|a + bi| = \sqrt{a^2 + b^2}$, always positive. Give it exactly (surd form) unless told otherwise.

6 Argument

Sketch first, then use $\tan^{-1}\frac{|b|}{|a|}$ and adjust for the quadrant:

$-\pi < \arg z \leqslant \pi$, in radians.

7 Modulus–argument form

$z = r(\cos\theta + i\sin\theta)$. $|zw| = |z||w|$ and $\arg(zw) = \arg z + \arg w$

8 Quadratic equations

Use the formula or complete the square; a negative discriminant gives the conjugate pair $p \pm qi$

9 Cubics and quartics

Real coefficients: complex roots come in conjugate pairs. Multiply $(z - \alpha)(z - \alpha^*)$ to get a real quadratic factor.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Arithmetic	•			•	•		•	•	•	
2 Conjugate and division	•			•				•		
3 Equating parts				•	•		•	•		
4 Argand diagram			•			•	•			•
5 Modulus		•	•	•				•	•	
6 Argument		•						•	•	•
7 Modulus–argument form		•							•	
8 Quadratic equations			•				•		•	•
9 Cubics and quartics						•	•		•	
Interleaved: simultaneous equations and quadratics (P1)			•	•	•		•	•		•
Interleaved: factor theorem and algebraic division (P2)						•	•		•	
Interleaved: trigonometry, exact values and areas (P1)		•				•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Show the algebra: a calculator answer alone scores nothing when working is required. Sketch an Argand diagram before finding any argument, and give final answers as complex numbers in the form asked for.

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1. The complex numbers z_1 and z_2 are given by $z_1 = 3 - 2i$ and $z_2 = 1 + 4i$. Without using a calculator, and showing your working,

(a) find $z_1 z_2$ in the form $a + bi$

(2)

(b) find $\frac{z_1}{z_2}$ in the form $a + bi$, where a and b are rational numbers,

(3)

(c) find $z_1 + z_2^*$

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The complex number $z = -3 + 3\sqrt{3}\,\mathrm{i}$

(a) Find $|z|$.

(1)

(b) Find $\arg z$, giving your answer as an exact multiple of π.

(2)

(c) Write z in the form $r(\cos\theta + \mathrm{i}\sin\theta)$

(1)

The complex number $w = 1 - \mathrm{i}$

(d) Find the exact values of $|zw|$ and $\arg(zw)$.

(2)

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Question 2 continued

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3. **(a)** Solve the equation $z^2 + 6z + 13 = 0$, giving your answers in the form $p + qi$

(3)

(b) Show the roots on an Argand diagram.

(1)

(c) Find the exact modulus of each root.

(1)

(d) Explain why the roots of any quadratic equation with real coefficients and negative discriminant have equal moduli.

(1)

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Question 3 continued

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4. **(a)** Find the complex number z which satisfies $2z + iz^* = 5 + 4i$

(4)

(b) Find the exact value of $|z - 3|$.

(2)

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Question 4 continued

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5. **(a)** Given that $(a+b\text{i})^2 = 5 - 12\text{i}$, where a and b are real, find the possible values of a and b.

(5)

(b) Hence write down the solutions of $z^2 = 5 - 12\text{i}$

(1)

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Question 5 continued

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6.

$$f(z) = z^3 - 5z^2 + 17z - 13$$

Given that $2 + 3i$ is a root of the equation $f(z) = 0$,

(a) write down another complex root of the equation,

(1)

(b) find the quadratic factor of $f(z)$ with these two roots,

(2)

(c) solve completely the equation $f(z) = 0$,

(2)

(d) show all the roots of $f(z) = 0$ on a single Argand diagram.

(2)

The points representing the roots are the vertices of a triangle.

(e) Find the area of this triangle.

(3)

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Question 6 continued

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Question 6 continued

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7.

$$f(z) = z^4 + pz^2 + qz + 50$$

where p and q are real constants. Given that $1 + 2i$ is a root of $f(z) = 0$,

(a) find the value of p and the value of q,

(4)

(b) find the other three roots of $f(z) = 0$,

(4)

(c) show all four roots on a single Argand diagram.

(2)

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Question 7 continued

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Question 7 continued

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8.

$$w = \frac{k + 6\text{i}}{2 - \text{i}}$$

where k is a real constant.

(a) Express w in the form $a + b\text{i}$, where a and b are in terms of k.

(3)

Given that $\arg w = \frac{\pi}{4}$

(b) find the value of k,

(3)

(c) find the exact value of $|w|$.

(1)

(d) Find the possible values of the real constant c for which $|w + c\text{i}| = 10$

(3)

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Question 8 continued

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9. The complex number $z = 1 + \sqrt{3}\mathrm{i}$

(a) Find $|z|$ and $\arg z$.

(2)

(b) Find z^2 in the form $a + b\mathrm{i}$, and show that $|z^2| = |z|^2$ and $\arg(z^2) = 2\arg z$

(3)

(c) Show that $z^3 = -8$

(2)

(d) Hence, or otherwise, find all the roots of the equation $z^3 + 8 = 0$

(3)

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Question 9 continued

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Question 9 continued

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10. The equation $z^2 - 2kz + k^2 + 16 = 0$, where k is a real constant, has roots z_1 and z_2, where $\text{Im}(z_1) > 0$

(a) Show that the roots are not real for any value of k.

(2)

(b) Find z_1 and z_2 in terms of k.

(2)

(c) Given that $\arg z_1 = \frac{\pi}{4}$, find the value of k.

(2)

The points P and Q represent z_1 and z_2 on an Argand diagram, and O is the origin. Given instead that $k > 0$ and triangle OPQ is equilateral,

(d) find the exact value of k,

(3)

(e) find the exact area of triangle OPQ.

(1)

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Question 10 continued

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TOTAL FOR CHAPTER 1 IS 80 MARKS

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