



eClassroom

A-LEVEL MATHEMATICS • STATISTICS 1

The Normal Distribution

Worked Solutions

Pearson Edexcel International A Level (WST01) & A Level (9MA0) • Calculator and statistical tables allowed: show every step

Full worked solutions for the free The Normal Distribution worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Statistics 1 The Normal Distribution worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dm** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $z = \frac{108 - 120}{10} = -1.2$: $1 - \Phi(1.2) = 1 - 0.8849$: **0.1151**

A2 $z = -0.6$: $P(Z > -0.6) = \Phi(0.6)$: **0.7257**

A3 $\Phi(1.5) - \Phi(-1) = 0.9332 - 0.1587$: **0.7745**

STRETCH

A4 10% below: $z = -1.2816$: $400 - 1.2816 \times 25$: **$b = 368.0$**

A5 $\frac{30 - \mu}{4} = -1.96$: **$\mu = 37.84$**

A6 $\mu - 0.5244\sigma = 12$, $\mu + 2.3263\sigma = 30$; $2.8507\sigma = 18$: **$\sigma = 6.31$, $\mu = 15.3$**

A7 $\frac{\Phi(-1.6)}{0.5} = \frac{0.0548}{0.5}$: **0.1096**

B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\Phi(1)$: **0.8413** (b) $\Phi(0.75)$: **0.7734** (c) $\Phi(1.5) - \Phi(-0.5)$: **0.6247** (d) **0**

A continuous variable has zero probability at a single value.

B2 (a) **0.8907** (b) $\Phi(0.85)$: **0.8023** (c) $0.6554 - 0.1151$: **0.5403** (d) $2(1 - 0.9772)$: **0.0456**

Use $\Phi(-z) = 1 - \Phi(z)$ for negative values.



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- B3** (a) $70 + 1.2816 \times 12$: **85.38** (b) $70 - 1.6449 \times 12$: **50.26**
 (c) $70 \pm 0.6745 \times 12$: **61.91 and 78.09** (d) **16.19**

Below the mean, the z value is negative.

- B4** (a) $\frac{35 - 40}{\sigma} = -0.5244$: **9.53** (b) $\frac{50 - \mu}{3} = 1.96$: **44.12** (c) 1.6449×6 : **9.87** (d) $1 - \Phi(1)$: **0.1587**

Each unknown comes from one standardised equation.

CORE

- B5** (a) **0.5** (b) $2\Phi(1) - 1$: **0.6826** (c) $1 - \Phi(2)$: **0.0228**

These hold for every normal distribution.

- B6** (a) $z = 1.33$: $1 - 0.9082$: **0.0918** (b) 400×0.0918 : **37** (c) by symmetry: **0.0918** (d) $400(1 - 0.1836)$: **327**

Expected number = $n \times$ probability, rounded to a whole number of bolts.

STRETCH

- B7** (a) $\mu - 0.8416\sigma = 28$, $\mu + 1.2816\sigma = 52$ (b) $2.1232\sigma = 24$: **11.30** (c) **37.51**
 (d) $z = \frac{45 - 37.51}{11.30} \approx 0.66$: $1 - 0.7454$: **0.2546**

Subtract the equations to remove μ

- B8** (a) $1 - \Phi(1.25)$: **0.1056** (b) $1 - \Phi(0.5)$: **0.3085** (c) $\frac{0.1056}{0.3085}$: **0.3423**
 (d) $\frac{\Phi(0.5) - \Phi(-0.5)}{\Phi(0.5)} = \frac{0.3830}{0.6915}$: **0.5539**

The numerator is the overlap of the two events.

- B9** (a) $z = -1.67$: $1 - 0.9525$: **0.0475** (b) 0.9525^2 : **0.9073**
 (c) $3 \times 0.0475 \times 0.9525^2$: **0.1293** (d) $1 - 0.9525^3$: **0.1358**

"Exactly one of three" has three arrangements.

- B10** (a) $50 + 0.6745 \times 6$: **54.05** (b) **45.95** (c) $54.05 + 1.5 \times 8.09$: **66.19**
 (d) $z = 2.70$: $2(1 - 0.9965)$: **0.0070**

The outlier limits are symmetrical about the mean.



C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 16 is the variance; divide by $\sigma = 4$: $\Phi(1) = 0.8413$

C2 44 is below the mean, so $z = -1.5$: $1 - 0.9332 = 0.0668$

C3 A value with 10% below it is below the mean, so $z = -1.2816$: $70 - 1.2816 \times 12 = 54.62$

C4 Table values must be quoted in full (1.6449): $400 + 1.6449 \times 20 = 432.90$

STRETCH

C5 42 has a probability of 0.2 below it, so it is below the mean: $\frac{42 - \mu}{5} = -0.8416$

$$\mu = 46.21$$

C6 There are $\binom{4}{2} = 6$ ways to choose the 2 faulty items: $6 \times 0.1^2 \times 0.9^2 = 0.0486$

C7 The events are not independent; " $X > 70$ " lies inside " $X > 64$ ": $\frac{0.1056}{0.3085} = 0.3423$

C8 (a) Standardise: $P(-k < Z < k) = \Phi(k) - \Phi(-k) = \Phi(k) - (1 - \Phi(k)) = 2\Phi(k) - 1$

(b) $2\Phi(2) - 1 = 2(0.9772) - 1 = 0.9544$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $P(B \geq 38) = P(Z \geq \frac{38 - 32}{4}) = 1 - \Phi(1.5) = 1 - 0.9332 = 0.0668$

(b) $50 \times 0.0668 = 3.34$: about 3 phones



- D2** (a) $\Phi(1) - \Phi(-0.5) = 0.8413 - 0.3085$: **0.5328**
 (b) $P(T < d) = 0.15$, so $z = -1.0364$: $42 - 1.0364 \times 6$: **$d = 35.78$**

- D3** (a) $\frac{825.63 - 800}{\sigma} = 1.2816$, so $\sigma = \frac{25.63}{1.2816} \approx 19.998$: **$\sigma = 20$**
 (b) $\Phi(-1) = 1 - 0.8413$: **0.1587**

- D4** (a) The fastest 30% are below the cut-off: $z = -0.5244$: $55 - 0.5244 \times 5$: **52.4 minutes**
 (b) $\Phi(-1)$: **0.1587**

STRETCH

- D5** (a) $\Phi(2) = 0.9772$ and $\Phi(-0.5) = 0.3085$: **$\mu + 2\sigma = 92$, $\mu - 0.5\sigma = 56$**
 (b) $2.5\sigma = 36$: **$\sigma = 14.4$, $\mu = 63.2$**

- D6** $p = P(X > 210) = 1 - \Phi(1.25) = 0.1056$
 $\binom{5}{2} p^2 (1-p)^3 = 10 \times 0.1056^2 \times 0.8944^3$: **0.0798**

- D7** (a) $2\Phi\left(\frac{6}{\sigma}\right) - 1 = 0.8$, so $\frac{6}{\sigma} = 1.2816$: **$\sigma = 4.68$**
 (b) $z = \frac{3}{4.68} \approx 0.64$: $\frac{1 - \Phi(0.64)}{\Phi(0.64)} = \frac{0.2611}{0.7389}$: **0.3534**

- D8** (a) The two tails are equal, so the z values are opposite: $\frac{k - 40}{5} = -\frac{2k - 90}{10}$
 $2k - 80 = 90 - 2k$: **$k = 42.5$**
 (b) $z = \frac{42.5 - 40}{5} = 0.5$: $1 - \Phi(0.5)$: **$p = 0.3085$**

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.



CORE

Exam questions for everyone

- E1** (a) $P(W < 820) = P(Z < \frac{820 - 805}{10}) = \Phi(1.5)$ M1
- 0.9332 A1*
- (b) $\Phi(1.5) - \Phi(-1.5) = 0.9332 - 0.0668$ M1
- 0.8664 A1
- (c) $\frac{k}{10} = 1.6449$ M1
- $k = 16.45$ A1
- (d) $z = -0.8416$: $805 - 0.8416 \times 10$ M1
- 796.6 grams A1
- (e) $1 - 0.8^3$ M1
- 0.488 A1
-
- E2** (a) $z = \frac{27 - 30}{2.4} = -1.25$: $1 - 0.8944$ M1
- 0.1056 A1
- (b) $30 \pm 0.6745 \times 2.4$: $Q_3 = 31.62, Q_1 = 28.38$ M1 A1
- (c) IQR = 3.238; $31.62 + 4.857$ and $28.38 - 4.857$ M1
- 36.48 and 23.52 A1
- (d) $z = \frac{36.48 - 30}{2.4} = 2.70$: $2\Phi(2.70) - 1 = 2(0.9965) - 1$ M1
- 0.993 A1*
- (e) $P(33 < L < 36.48) = \Phi(2.70) - \Phi(1.25) = 0.9965 - 0.8944 = 0.1021$ M1 M1
- $\frac{0.1021}{0.993}$: 0.103 A1
-



STRETCH

- E3 (a) $\frac{r-300}{k} = -0.8416$ M1 A1
- $\frac{2r-220-300}{k} = 1.96$ A1
- (b) $r = 300 - 0.8416k$, so $2(300 - 0.8416k) - 520 = 1.96k$ M1
- $80 = 3.6432k$: $k = 21.96, r = 281.5$ A1 A1
- (c) $z = \frac{20}{21.96} \approx 0.91$: $1 - 0.8186$ M1
- 0.1814 A1
- (d) $4 \times 0.1814 \times 0.8186^3$ M1
- 0.398 A1
- (e) $\frac{0.1814}{0.5}$ M1
- 0.3628 A1

