



eClassroom

A-LEVEL MATHEMATICS • PURE 2

# Trigonometric Identities and Equations

Worked Solutions

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Trigonometric Identities and Equations worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

**Full worked solutions** for every question on the Pure 2 Trigonometric Identities and Equations worksheet, in the same order and with the same question numbers.

**How to mark your work.** Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

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**A****Your turn answers**

Compare each line of your working with the solution.

**CORE**

**A1**  $-\sin 45^\circ, -\cos 60^\circ, \tan 30^\circ$ :  $-\frac{\sqrt{2}}{2}, -\frac{1}{2}, \frac{\sqrt{3}}{3}$

**A2**  $\sin^2 \theta = 1 - \frac{144}{169} = \frac{25}{169}$ ; third quadrant, so  $\sin \theta < 0$ :  $\sin \theta = -\frac{5}{13}, \tan \theta = \frac{5}{12}$

**A3**  $\cos x = -0.75$ : principal value  $138.6^\circ$ , and  $360^\circ - 138.6^\circ$ :  $138.6^\circ, 221.4^\circ$

**STRETCH**

**A4**  $\tan x = -4$ :  $x = -76.0^\circ$  is outside, so add  $180^\circ$  twice:  $104.0^\circ, 284.0^\circ$

**A5**  $-40^\circ \leq x - 40^\circ < 320^\circ$ :  $x - 40^\circ = 60^\circ$  or  $300^\circ$ :  $x = 100^\circ, x = 340^\circ$

**A6**  $2(1 - \cos^2 x) = 1 + \cos x$ , so  $2\cos^2 x + \cos x - 1 = 0$ ,  $(2\cos x - 1)(\cos x + 1) = 0$ :  $60^\circ, 180^\circ, 300^\circ$

**A7**  $-\frac{\pi}{3} \leq x - \frac{\pi}{3} < \frac{5\pi}{3}$ :  $x - \frac{\pi}{3} = -\frac{\pi}{3}$  or  $\frac{4\pi}{3}$ :  $x = 0, x = \frac{5\pi}{3}$



## B

## Fluency solutions

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

## CORE

B1 (a)  $-\frac{1}{2}$  (b)  $-\frac{\sqrt{2}}{2}$  (c)  $-\sqrt{3}$  (d)  $-\frac{1}{2}$

*Sine is positive in quadrants 1 and 2, cosine in 1 and 4, tangent in 1 and 3.*

B2 (a) 0.8 (b) 0.75 (c) -0.8 (d) -0.75

*The identity gives the size; the quadrant gives the sign.*

B3 (a) 3 (b)  $\frac{\sin^2 x}{\sin x} = \sin x$  (c)  $\sin x \times \frac{\cos x}{\sin x} = \cos x$  (d)  $\sin x$

*Replace  $\tan x$  by  $\frac{\sin x}{\cos x}$  and  $1 - \cos^2 x$  by  $\sin^2 x$*

B4 (a)  $23.6^\circ, 156.4^\circ$  (b)  $107.5^\circ, 252.5^\circ$  (c)  $116.6^\circ, 296.6^\circ$  (d)  $270^\circ$

*Second values:  $180^\circ - \alpha$  for sine,  $360^\circ - \alpha$  for cosine,  $\alpha + 180^\circ$  for tangent.*

B5 (a)  $\sin^2 x + 2 \sin x \cos x + \cos^2 x = 1 + 2 \sin x \cos x$  as required

(b)  $(1 - \sin^2 x) - \sin^2 x = 1 - 2 \sin^2 x$  as required

(c)  $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$  as required

*Start from one side and change it, step by step, into the other.*

B6 (a) 2 (b) 4 (c) 6 (d) 2

*$\sin kx$  has  $k$  cycles in  $360^\circ$ , each giving two solutions;  $\sin \frac{x}{2}$  has only half a cycle.*

## STRETCH

B7 (a)  $30^\circ, 150^\circ$  (b)  $22.5^\circ, 112.5^\circ$  (c)  $60^\circ$  (d)  $2x - 30^\circ = 90^\circ, 270^\circ: 60^\circ, 150^\circ$

*Change the interval for the whole argument before solving.*

B8 (a)  $\sin x(2 \sin x - 1) = 0: 0^\circ, 30^\circ, 150^\circ, 180^\circ$  (b)  $\cos x = \pm \frac{\sqrt{3}}{2}: 30^\circ, 150^\circ, 210^\circ, 330^\circ$

(c)  $\cos x = 1$  or  $-\frac{1}{2}: 0^\circ, 120^\circ, 240^\circ$  (d)  $\tan x = \pm \sqrt{3}: 60^\circ, 120^\circ, 240^\circ, 300^\circ$

*A square root gives both signs; factorise rather than divide.*



B9 (a)  $\frac{\pi}{4}, \frac{3\pi}{4}$  (b)  $\frac{2\pi}{3}, \frac{4\pi}{3}$  (c)  $\frac{\pi}{3}, \frac{4\pi}{3}$  (d)  $2x = 0, \pi, 2\pi, 3\pi$ :  $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

In radians,  $\pi$  replaces  $180^\circ$  in every rule.

B10 (a)  $24^\circ\text{C}$  (b)  $12^\circ\text{C}$  (c)  $15t = 180, t = 12$ : **12:00 (noon)** (d)  $\cos(15t)^\circ = -\frac{1}{2}, 15t = 120$ : **08:00**

The maximum is where  $\cos = -1$ ; a value of  $t$  is a time of day.

## C

## Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

## CORE

C1 Dividing by  $\sin x$  loses the solutions of  $\sin x = 0$ . Factorise:  $\sin x(\cos x - 1) = 0$

$x = 0^\circ$  and  $x = 180^\circ$

C2 An obtuse angle is in the second quadrant, where cosine is negative:  $\cos \theta = -0.8$

C3 Every term must be multiplied by  $\cos x$ :  $\sin x + 2\sin x \cos x = 0$ , so  $\sin x(1 + 2\cos x) = 0$

$\sin x = 0$  or  $\cos x = -\frac{1}{2}$ :  $0^\circ, 120^\circ, 180^\circ, 240^\circ$

C4  $2x$  runs from  $0^\circ$  to  $720^\circ$ , so there are four values:  $2x = 60^\circ, 120^\circ, 420^\circ, 480^\circ$

$30^\circ, 60^\circ, 210^\circ, 240^\circ$

## STRETCH

C5 The interval is in radians, so the answers must be in radians:  $\frac{\pi}{3}, \frac{5\pi}{3}$

C6  $x = 60^\circ$ :  $\sin 90^\circ = 1$ , but  $\sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}+1}{2} \neq 1$ , so it is false.

$30^\circ \leq x + 30^\circ < 390^\circ$ :  $x + 30^\circ = 30^\circ$  or  $150^\circ$ :  $x = 0^\circ$ ,  $x = 120^\circ$



**C7**  $(2\cos x - 1)(\cos x + 3) = 0$ ;  $\cos x = -3$  is impossible because  $-1 \leq \cos x \leq 1$

$$\cos x = \frac{1}{2}: \boxed{60^\circ, 300^\circ}$$

**C8** (a)  $\sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = \sin^2 x - \cos^2 x$  as required

(b)  $\sin^2 x - (1 - \sin^2 x) = \frac{1}{2}$ , so  $\sin^2 x = \frac{3}{4}$ ,  $\sin x = \pm \frac{\sqrt{3}}{2}$ :  $60^\circ, 120^\circ, 240^\circ, 300^\circ$

## D

### Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

#### CORE

**D1** A right-angled triangle with opposite  $\sqrt{5}$  and adjacent 2 has hypotenuse  $\sqrt{5+4} = 3$

Second quadrant: sine positive, cosine negative:  $\sin \theta = \frac{\sqrt{5}}{3}$ ,  $\cos \theta = -\frac{2}{3}$

**D2** (a)  $4(1 - \cos^2 x) + 4\cos x = 5$ , so  $4\cos^2 x - 4\cos x + 1 = 0$ , so  $(2\cos x - 1)^2 = 0$  as required

(b) A repeated root  $\cos x = \frac{1}{2}$ , which has two solutions in the interval:  $60^\circ, 300^\circ$

**D3**  $10^\circ \leq 2x + 10^\circ < 370^\circ$ ;  $\sin(2x + 10^\circ) = 0.6$ :  $2x + 10^\circ = 36.87^\circ$  or  $143.13^\circ$

$x = 13.4^\circ$ ,  $x = 66.6^\circ$

**D4** If  $\cos x = 0$ , then  $\sin x = \pm 1$  and the equation would say  $\pm\sqrt{3} = 0$ , which is false; so  $\cos x \neq 0$

$\tan x = \frac{1}{\sqrt{3}}$ :  $30^\circ, 210^\circ$

#### STRETCH

**D5**  $-\frac{\pi}{6} \leq 2x - \frac{\pi}{6} < \frac{11\pi}{6}$ , and  $\cos\left(2x - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$2x - \frac{\pi}{6} = \frac{5\pi}{6}$  or  $\frac{7\pi}{6}$ , so  $2x = \pi$  or  $\frac{4\pi}{3}$ :  $x = \frac{\pi}{2}$ ,  $x = \frac{2\pi}{3}$



**D6**  $\frac{\sin x}{\cos x} = 4 \sin x$ , so  $\sin x = 4 \sin x \cos x$ ,  $\sin x(1 - 4 \cos x) = 0$  (factorise, do not divide by  $\sin x$ )

$\sin x = 0$ :  $0^\circ, 180^\circ$ ;  $\cos x = \frac{1}{4}$ :  $75.5^\circ, 284.5^\circ$

**D7** (a)  $\sin(30t)^\circ = 1$  gives  $8\text{ m}$ , first when  $30t = 90$ :  $03:00$

(b)  $\sin(30t)^\circ = 0.5$ :  $30t = 30, 150, 390, 510$ , so  $t = 1, 5, 13, 17$ :  $01:00, 05:00, 13:00, 17:00$

**D8** (a)  $(\sin x + \cos x)(1 - \sin x \cos x) = \sin x + \cos x - \sin^2 x \cos x - \sin x \cos^2 x$

$= \sin x(1 - \cos^2 x) + \cos x(1 - \sin^2 x) = \sin^3 x + \cos^3 x$  **as required**

(b)  $\sin x \cos x \leq \frac{1}{2}$  for all  $x$  (it is never 1), so  $1 - \sin x \cos x \neq 0$ . So  $\sin x + \cos x = 0$ ,  $\tan x = -1$ :  $135^\circ, 315^\circ$

**E****Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

**CORE**

Exam questions for everyone

**E1** (a)  $3(1 - \cos^2 x) + 7 \cos x = 5$

M1: uses the identity

$3 \cos^2 x - 7 \cos x + 2 = 0$  **as required**

A1\*

(b)  $(3 \cos x - 1)(\cos x - 2) = 0$ ;  $\cos x = 2$  is impossible

M1

$\cos x = \frac{1}{3}$ :  $70.5^\circ$

A1

$289.5^\circ$

A1

**E2** (a)  $\frac{4 \sin \theta}{\cos \theta} = 3 \cos \theta$ , so  $4 \sin \theta = 3 \cos^2 \theta = 3 - 3 \sin^2 \theta$

M1

$3 \sin^2 \theta + 4 \sin \theta - 3 = 0$  **as required**

A1\*

(b)  $\sin \theta = \frac{-4 + \sqrt{52}}{6} = \frac{-2 + \sqrt{13}}{3}$  (the other root is less than  $-1$ )

M1

$x + 20^\circ = 32.36^\circ$  or  $147.64^\circ$  (with  $20^\circ \leq x + 20^\circ < 380^\circ$ )

dM1 A1

$x = 12.4^\circ$ ,  $x = 127.6^\circ$

A1



## STRETCH

- E3** (a)  $\boxed{5}$  and  $\boxed{-3}$  B1
- (b)  $2\theta - \frac{\pi}{6} = \frac{\pi}{2}$ , so  $\theta = \frac{\pi}{3}$ :  $\boxed{P\left(\frac{\pi}{3}, 5\right)}$  M1 A1
- (c)  $4\sin\left(-\frac{\pi}{6}\right) + 1 = -1$ :  $\boxed{(0, -1)}$  B1
- (d)  $\sin\left(2\theta - \frac{\pi}{6}\right) = \frac{1}{2}$ , with  $-\frac{\pi}{6} \leq 2\theta - \frac{\pi}{6} \leq \frac{11\pi}{6}$  M1
- $2\theta - \frac{\pi}{6} = \frac{\pi}{6}$  or  $\frac{5\pi}{6}$  dM1
- $\boxed{\theta = \frac{\pi}{6}}$ ,  $\boxed{\theta = \frac{\pi}{2}}$  A1
- (e) A line through  $P$  or through  $Q$  touches the curve once:  $\boxed{k = 5 \text{ or } k = -3}$  B1

