



eClassroom

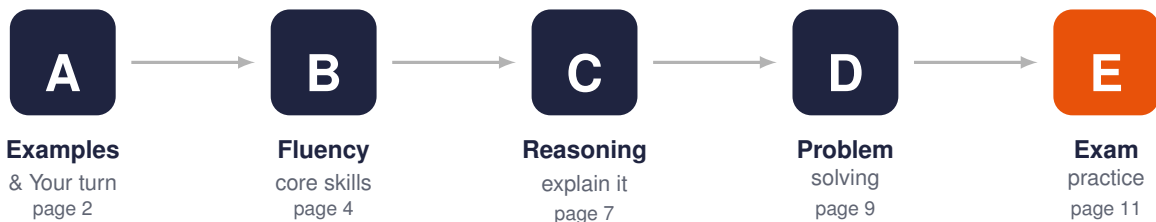
GCSE MATHEMATICS • ALGEBRA

Functions

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Free Functions worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: use function machines forwards and backwards • evaluate $f(x)$ and solve $f(x) = k$ • recognise linear, quadratic, cubic and reciprocal graphs • *Higher:* composite functions $fg(x)$, inverse functions $f^{-1}(x)$, and transformations of graphs.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Higher only: composite and inverse functions, graph transformations and exponential graphs. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	23	10	13	15
Higher marks	16	14	17	15
My score				

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A

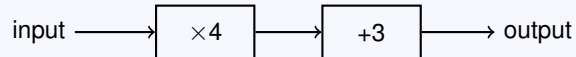
Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION

Everyone starts here

Worked example A1

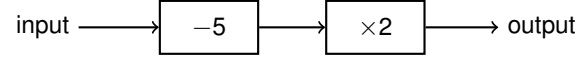


The input is 6. Find the output. Then find the input when the output is 35

Forwards: $6 \times 4 + 3$. Backwards: -3 then $\div 4$

27; 8

Your turn A1



The input is 11. Find the output. Then find the input when the output is 30

Worked example A2

$f(x) = x^2 + 2x$. Find $f(3)$ and $f(-4)$

Replace x with the number in brackets.

$9 + 6$ $16 - 8$

$f(3) = 15$, $f(-4) = 8$

Your turn A2

$g(x) = 10 - 3x$. Find $g(2)$ and $g(-2)$

Worked example A3

Complete the table for $y = x^3$ for $x = -2$ to 2

Cube each value: a negative cubed is negative.

-8, -1, 0, 1, 8

Your turn A3

Complete the table for $y = \frac{12}{x}$ for $x = 1, 2, 3, 4$ and 6



HIGHER*Higher only: composite and inverse functions, and transformations***Worked example A4: a composite function** $f(x) = 3x - 1$ and $g(x) = x + 4$. Find $fg(x)$ $fg(x)$ means do g first, then f : $f(x + 4)$

$$3(x + 4) - 1$$

$$fg(x) = 3x + 11$$

Your turn A4 $f(x) = 2x$ and $g(x) = x^2 - 1$. Find $gf(x)$ **Worked example A5: an inverse function** $f(x) = 4x + 5$. Find $f^{-1}(x)$ Write $y = 4x + 5$ and make x the subject: $x = \frac{y - 5}{4}$

$$f^{-1}(x) = \frac{x - 5}{4}$$

Your turn A5 $f(x) = \frac{x + 3}{2}$. Find $f^{-1}(x)$ **Worked example A6: transforming a point** $P(2, 5)$ is on $y = f(x)$. Find the image of P on $y = f(x) - 3$ and on $y = f(x + 1)$ $f(x) - 3$ moves the graph down 3. $f(x + 1)$ moves it left 1

$$(2, 2) \text{ and } (1, 5)$$

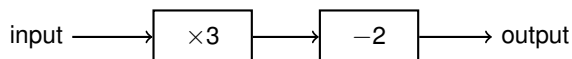
Your turn A6 $Q(-1, 4)$ is on $y = f(x)$. Find the image of Q on $y = -f(x)$ and on $y = f(-x)$ **Worked example A7: solving with a composite** $f(x) = 2x + 3$ and $g(x) = x^2$. Solve $gf(x) = 25$ $gf(x) = (2x + 3)^2 = 25$, so $2x + 3 = 5$ or $2x + 3 = -5$

$$x = 1 \text{ or } x = -4$$

Your turn A7 $f(x) = x - 1$ and $g(x) = x^2$. Solve $fg(x) = 8$ 

B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION*Everyone starts here***B1**

(a) Find the output when the input is 4 (b) Find the output when the input is -1

(c) Find the input when the output is 13 (d) Find the input when the output is -8

[4 marks]**B2**

$$f(x) = 5 - 2x$$

(a) Find $f(1)$

(b) Find $f(-3)$

(c) Find $f(0.5)$

(d) Solve $f(x) = -9$

[4 marks]**B3**

$$f(x) = x^2 - 4$$

(a) Find $f(3)$

(b) Find $f(-3)$

(c) Find $f(0)$

(d) Solve $f(x) = 12$

[4 marks]**B4**

Work out the value of y .

(a) $y = x^3 - x$ when $x = 2$

(b) $y = x^3 - x$ when $x = -2$

(c) $y = \frac{6}{x}$ when $x = 3$

(d) $y = \frac{6}{x}$ when $x = -2$

[4 marks]

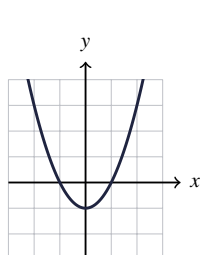
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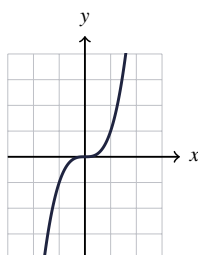
FOUNDATION

continued: graphs and machine expressions

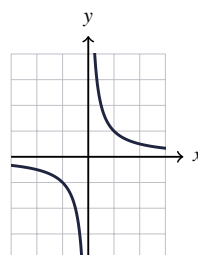
B5 Match each graph to one of the equations $y = x^3$, $y = \frac{1}{x}$, $y = x^2 - 1$



Graph P



Graph Q



Graph R

[3 marks]

- B6** (a) A machine adds 3, then multiplies by 2. Write an expression for the output when the input is x .
 (b) Another machine multiplies by 2, then adds 3. Write an expression for its output.
 (c) Can the two machines ever give the same output for the same input? Explain.
 (d) Write an expression for the input of the second machine when its output is x

[4 marks]

HIGHER

Higher tier only

B7 $f(x) = x + 3$ and $g(x) = 2x$

(a) Find $fg(1)$ (b) Find $gf(1)$ (c) Find $fg(x)$ (d) Find $gf(x)$

[4 marks]

B8 Find $f^{-1}(x)$

(a) $f(x) = x + 7$ (b) $f(x) = 5x$ (c) $f(x) = 3x - 4$ (d) $f(x) = 2(x - 1)$

[4 marks]



HIGHER*continued: transformations and exponential graphs*

B9 The point $(3, -2)$ is on the graph of $y = f(x)$. Find its image on

(a) $y = f(x) + 4$

(b) $y = f(x - 2)$

(c) $y = -f(x)$

(d) $y = f(-x)$

[4 marks]

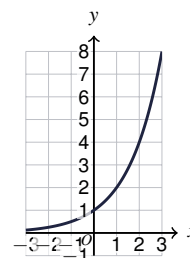
B10 $y = 2^x$

(a) Find y when $x = 3$

(b) Find y when $x = 0$

(c) Find y when $x = -1$

(d) Solve $2^x = 32$



[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

- C1** $f(x) = 3x^2$. Tom writes $f(-2) = -12$
Explain Tom's mistake, and find the correct value. [2 marks]
- C2** A machine multiplies by 4, then adds 1. Sam says the reverse is "divide by 4, then subtract 1"
Explain why Sam is wrong, and write the correct reverse. [2 marks]
- C3** Explain why the graph of $y = \frac{1}{x}$ has no point where $x = 0$, and why it never crosses the x -axis. [3 marks]
- C4** Machine A adds 5, then multiplies by 2. Machine B multiplies by 2, then adds 10
Show that the two machines always give the same output. [3 marks]



HIGHER*Higher tier only*

- C5** $f(x) = x + 1$ and $g(x) = x^2$. Jo says, " $fg(x)$ and $gf(x)$ are the same."

Show that Jo is wrong.

[3 marks]

- C6** $f(x) = 2x + 6$. Kai says, " $f^{-1}(x) = \frac{1}{2x+6}$ "

Explain Kai's mistake, and find $f^{-1}(x)$

[3 marks]

- C7** $f(x) = \frac{x+4}{3}$

Find $f^{-1}(x)$ and show that $ff^{-1}(x) = x$

[3 marks]

- C8** $f(x) = x^2 - 6x + 10$ for $x \geq 3$

(a) Write $f(x)$ in the form $(x - a)^2 + b$

(b) Hence find $f^{-1}(x)$, and explain why the condition $x \geq 3$ is needed.

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** A taxi fare is worked out by a machine: £2.50, plus £1.20 for each kilometre.

A fare is £10.90. How far was the journey?

[3 marks]

D2

Machine A: $x \longrightarrow \boxed{\times 3} \longrightarrow \boxed{-5} \longrightarrow \text{output}$

Machine B: $x \longrightarrow \boxed{+1} \longrightarrow \boxed{\times 2} \longrightarrow \text{output}$

The same input gives the same output from both machines. Find the input and the output.

[4 marks]

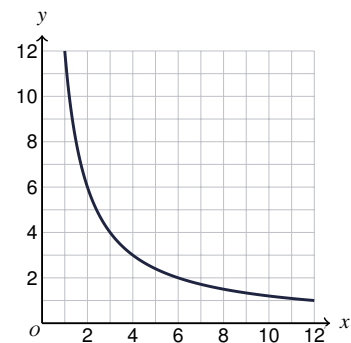
- D3** $f(x) = x^2 + 1$ and $g(x) = 5x - 3$. Find the values of x for which $f(x) = g(x)$

[3 marks]

- D4** The graph shows $y = \frac{12}{x}$ for $x > 0$

(a) $(a, 3)$ and $(4, b)$ are on the graph. Find a and b .

(b) How many points with positive whole-number coordinates are on the graph?



[3 marks]



HIGHER*Higher tier only*

D5 $f(x) = 2x + 1$. Find the value of a for which $f(a) = f^{-1}(a)$

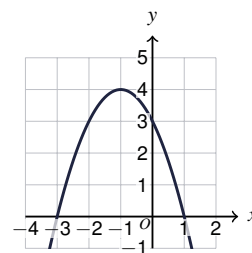
[4 marks]

D6 The graph of $y = f(x)$ has a maximum at $(-1, 4)$ and passes through $(0, 3)$.

(a) Write down the maximum point of $y = f(x + 2)$

(b) Write down the minimum point of $y = -f(x)$

(c) Where does $y = f(x) - 3$ cross the y -axis?

**[4 marks]**

D7 $f(x) = 3x - 2$

(a) Find $ff(x)$ **(b)** Solve $ff(x) = x$

[4 marks]

D8 $g(x) = 2^x$

Find the value of x for which $g(x) \times g(x + 1) = 128$

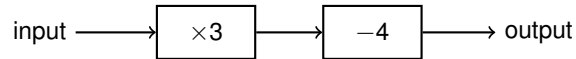
[5 marks]

E**Exam practice**

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION*Foundation and Higher crossover*

E1 Here is a function machine.



(a) Work out the output when the input is 5

.....
(1)

(b) Work out the input when the output is 20

.....
(1)

(Total for Question E1 is 2 marks)

E2 $f(x) = 2x^2 - 3$

(a) Find $f(4)$

.....
(1)

(b) Find $f(-1)$

.....
(1)

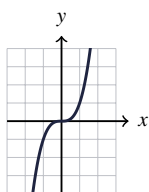
(c) Solve $f(x) = 15$

.....
(1)

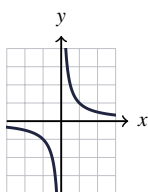
(Total for Question E2 is 3 marks)



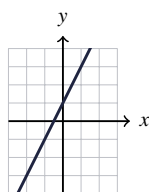
- E3** Match each graph to its equation. $A: y = 2x + 1$ $B: y = x^2$ $C: y = x^3$ $D: y = \frac{1}{x}$



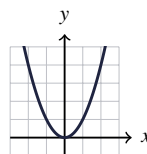
Graph 1



Graph 2



Graph 3



Graph 4

(Total for Question E3 is 2 marks)

- E4** A machine adds 2 to the input, then multiplies by 5

(a) Write an expression for the output when the input is x

.....
(1)

(b) Find the input that gives an output equal to the input.

.....
(2)

(Total for Question E4 is 3 marks)

- E5** Complete the table of values for $y = x^3 - 3x$

x	-2	-1	0	1	2
y					

(Total for Question E5 is 3 marks)

- E6** $f(x) = 3x + 7$

Find the value of x for which $f(x) = f(2) + 12$

.....
(Total for Question E6 is 2 marks)



HIGHER*Higher tier only*

E7 $f(x) = 2x + 1$ and $g(x) = x^2$

(a) Find $fg(3)$

.....
(1)

(b) Find $gf(x)$. Give your answer in its simplest form.

.....
(2)

(Total for Question E7 is 3 marks)

E8 $f(x) = \frac{3x-2}{5}$

Find $f^{-1}(x)$

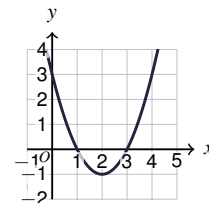
.....
(Total for Question E8 is 3 marks)

E9 The graph of $y = f(x)$ has a turning point at $(2, -1)$.

Write down the coordinates of the turning point of

(a) $y = f(x) + 3$ **(b)** $y = f(x - 4)$

(c) $y = -f(x)$ **(d)** $y = f(-x)$



(a) **(b)**

(c) **(d)**

(Total for Question E9 is 4 marks)



E10 $f(x) = 3x - 1$ and $g(x) = \frac{2}{x}$

(a) Find $f^{-1}(x)$

.....

(2)

(b) Solve $f^{-1}(x) = g(x)$

.....

(3)

(Total for Question E10 is 5 marks)

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