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A-LEVEL MATHEMATICS • FURTHER PURE 2

Inequalities

Worked Solutions

Pearson Edexcel International A Level (WFM02) • Calculator allowed: show every step

Full worked solutions for the free Inequalities worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 4

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 2 Inequalities worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $\dfrac{2x-1-(x+2)}{x+2} = \dfrac{x-3}{x+2} \leqslant 0$; -2 excluded: $\boxed{-2 < x \leqslant 3}$

A2 $\times(x+1)^2$: $(x+1)\big(4-(x-2)(x+1)\big) \geqslant 0$, so $(x+1)(x-3)(x+2) \leqslant 0$: $\boxed{x \leqslant -2 \text{ or } -1 < x \leqslant 3}$

A3 $\dfrac{2x(x+1)-3(x-4)}{(x-4)(x+1)} = \dfrac{2x^2-x+12}{(x-4)(x+1)} < 0$; the numerator is always positive: $\boxed{-1 < x < 4}$

STRETCH

A4 $x \geqslant -2$: $x+2 \geqslant 3x-2$, $x \leqslant 2$; $x < -2$: the right side is negative, always true: $\boxed{x \leqslant 2}$

A5 $x^2-2x > 3$: $x < -1$ or $x > 3$; $x^2-2x < -3$ has no solutions: $\boxed{x < -1 \text{ or } x > 3}$

A6 $|x+2| \leqslant 2$, $x \neq -2$: $\boxed{-4 \leqslant x \leqslant 0,\ x \neq -2}$

A7 $4 = (x+2)(x-1)$ at $x = -3, 2$; asymptote $x = 1$: $\boxed{-3 \leqslant x < 1 \text{ or } x \geqslant 2}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{x < -4 \text{ or } x > 3}$ **(b)** $\boxed{x \leqslant -2 \text{ or } 1 \leqslant x < 5}$ **(c)** $\boxed{x > -3}$ **(d)** $\boxed{x < 2}$

A squared factor never changes sign; a denominator value is never included.

B2 **(a)** $\dfrac{2(1-x)}{x+1} < 0$: $\boxed{x < -1 \text{ or } x > 1}$ **(b)** $\boxed{x < -1 \text{ or } x \geqslant 1}$ **(c)** $\boxed{-1 < x \leqslant 1}$ **(d)** $\boxed{x = 1}$

Only the endpoint $x = 1$ changes between strict and non-strict.

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B3 **(a)** $(x+3)(x+4)(x-1)<0$: $\boxed{x<-4 \text{ or } -3<x<1}$ **(b)** $\boxed{-4\leqslant x<-3 \text{ or } x\geqslant 1}$

(c) $x(x-1)(x^2-2x+2)>0$: $\boxed{x<0 \text{ or } x>1}$ **(d)** $(x-2)(x-3)(x+3)>0$: $\boxed{-3<x<2 \text{ or } x>3}$

Multiply by the square of the denominator, then factorise.

B4 **(a)** $\boxed{1<x<7}$ **(b)** $\boxed{x>\frac{4}{3}}$ **(c)** $\boxed{x\leqslant\frac{4}{3}}$ **(d)** square both sides: $\boxed{x>1}$

Both cases of the modulus, unless both sides are moduli.

B5 **(a)** $x^2=16$ or 2: $\boxed{x=\pm4,\pm\sqrt{2}}$ **(b)** $2<x^2<16$: $\boxed{-4<x<-\sqrt{2} \text{ or } \sqrt{2}<x<4}$

(c) $\boxed{x<-4,\ -\sqrt{2}<x<\sqrt{2} \text{ or } x>4}$

The equation gives every critical value.

B6 **(a)** $\boxed{\{x:-2<x<3\}}$ **(b)** $\boxed{\{x:x\leqslant-1\}\cup\{x:x>4\}}$ **(c)** $\boxed{\{x:x<0\}\cup\{x:x>1\}}$

(d) $\boxed{\{x:0<x<2\}}$

Two separate intervals are joined with $\cup$

STRETCH

B7 **(a)** $\frac{3}{(x-1)(x+2)}>0$: $\boxed{x<-2 \text{ or } x>1}$ **(b)** $\frac{3x}{(x-1)(x+2)}\geqslant0$: $\boxed{-2<x\leqslant0 \text{ or } x>1}$

(c) $\frac{x-5}{(x+1)(x-1)}\leqslant0$: $\boxed{x<-1 \text{ or } 1<x\leqslant5}$ **(d)** $\frac{7x-5}{(x-2)(x+1)}>0$: $\boxed{-1<x<\frac{5}{7} \text{ or } x>2}$

One fraction, then every critical value from top and bottom.

B8 **(a)** $\boxed{0<x<2}$ **(b)** $\boxed{x<0,\ 1<x<2 \text{ or } x>\frac{3+\sqrt{17}}{2}}$ **(c)** $\boxed{0\leqslant x\leqslant2}$ **(d)** $\boxed{-2<x<-\frac{2}{3} \text{ or } x>2}$

Split at the zero of the modulus and at every asymptote.

B9 **(a)** $x^2+x-6=0$: $\boxed{x=2,-3}$ **(b)** $\boxed{x<-3 \text{ or } -1<x<2}$ **(c)** $|x|<2$: $\boxed{-2<x<2}$

(d) $\boxed{-3\leqslant x<-1 \text{ or } x\geqslant2}$

The graph shows which side of the asymptote each interval is.

B10 **(a)** $\frac{6-x}{x-3}>0$: $\boxed{3<x<6}$ **(b)** $x>2x-6$ gives $x<6$, which wrongly includes $x<3$ **(c)** $\boxed{3<x\leqslant6}$

(d) $\boxed{x<3 \text{ or } x\geqslant6}$

Multiplying by a negative reverses the sign.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $x-2$ may be negative, which reverses the inequality: $\dfrac{5-x}{x-2} < 0$: $\boxed{x < 2 \text{ or } x > 5}$

C2 $x = -3$ makes the denominator zero, so it cannot be included: $\boxed{-3 < x \leqslant 1}$

C3 The modulus needs both cases: $-(2x-1) < x-2 < 2x-1$ gives $x > 1$ and $x > -1$: $\boxed{x > 1}$

C4 Dividing by x loses the critical value 0 and ignores its sign: $\dfrac{x(x-1)}{x+1} \geqslant 0$: $\boxed{-1 < x \leqslant 0 \text{ or } x \geqslant 1}$

STRETCH

C5 Squaring is valid only when both sides are non-negative; for $x < 3$ the right side is negative and the inequality always holds: $\boxed{\text{all real } x}$

C6 No number is in both sets; the solution is $x < -1$ *or* $x > 2$: $\boxed{\{x : x < -1\} \cup \{x : x > 2\}}$

C7 For $x < 0$, $\dfrac{1}{x}$ is negative, so the inequality fails; $\dfrac{2-x}{2x} > 0$: $\boxed{0 < x < 2}$

C8 **(a)** $x^2+1 > 0$, so $f(x) \leqslant \dfrac{3}{2} \iff 2x^2+2x+2 \leqslant 3x^2+3 \iff (x-1)^2 \geqslant 0$, true; equality at $\boxed{x = 1}$

(b) $2x^2+2x+2 \geqslant x^2+1 \iff (x+1)^2 \geqslant 0$, true: $\boxed{\text{range } \dfrac{1}{2} \leqslant f(x) \leqslant \dfrac{3}{2}}$

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D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\dfrac{x^2-2x-2}{x+1} < 0$; critical values -1, $1\pm\sqrt{3}$: $\boxed{\{x : x < -1\} \cup \{x : 1-\sqrt{3} < x < 1+\sqrt{3}\}}$

D2 $\dfrac{(x+2)(x+1)-3(x-1)}{(x-1)(x+1)} = \dfrac{x^2+5}{(x-1)(x+1)} \leqslant 0$; $x^2+5>0$: $\boxed{-1 < x < 1}$

D3 **(a)** $x(x+4) = 3(x+2)$, $x^2+x-6=0$: $\boxed{x=2,\ x=-3}$

(b) $\dfrac{(x+3)(x-2)}{(x+2)(x+4)} < 0$; critical values $-4, -3, -2, 2$: $\boxed{-4 < x < -3 \text{ or } -2 < x < 2}$

D4 **(a)** $\dfrac{x^2+x-6}{x} > 0$, $\dfrac{(x+3)(x-2)}{x} > 0$: $\boxed{-3 < x < 0 \text{ or } x > 2}$

(b) $|x| > 0$, so only $|x| > 2$: $\boxed{x < -2 \text{ or } x > 2}$

STRETCH

D5 **(a)** $\times(x-2)^2$: $(x-2) \leqslant kx(x-2)^2$, so $(x-2)(kx^2-2kx-1) \geqslant 0$: $\boxed{p=k,\ q=-2k,\ r=-1}$

(b) $3x^2-6x-1=0$: $x = 1\pm\dfrac{2\sqrt{3}}{3}$; $x \neq 2$: $\boxed{1-\dfrac{2\sqrt{3}}{3} \leqslant x < 2 \text{ or } x \geqslant 1+\dfrac{2\sqrt{3}}{3}}$

D6 **(a)** $x^2-9 = x+3$: $x = 4, -3$; $9-x^2 = x+3$: $x = 2, -3$: $\boxed{x=-3,\ 2,\ 4}$

(b) From the figure, the line is above the curve between 2 and 4 (at $x=-3$ they only meet): $\boxed{2 < x < 4}$

D7 **(a)** $\dfrac{x^2+2x-8}{x+2} > 0$, $\dfrac{(x+4)(x-2)}{x+2} > 0$: $\boxed{-4 < x < -2 \text{ or } x > 2}$

(b) $|x| \geqslant 0$, so only $|x| > 2$: $\boxed{x < -2 \text{ or } x > 2}$

D8 **(a)** $\dfrac{1}{4} - \dfrac{n(n+3)}{4(n+1)(n+2)} = \dfrac{(n+1)(n+2)-n(n+3)}{4(n+1)(n+2)} = \dfrac{2}{4(n+1)(n+2)}$, as required

(b) $\dfrac{1}{2(n+1)(n+2)} < 0.001$: $n^2+3n-498>0$, $n > \dfrac{-3+\sqrt{2001}}{2} = 20.86\ldots$: $\boxed{n=21}$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1

$\dfrac{x^2-1-(2-x)(x+4)}{x+4} \geqslant 0$ *M1*

$\dfrac{2x^2+2x-9}{x+4} \geqslant 0$ *A1*

Critical values -4 and $\dfrac{-1\pm\sqrt{19}}{2}$ *M1 A1*

$\left\{x : -4 < x \leqslant \dfrac{-1-\sqrt{19}}{2}\right\} \cup \left\{x : x \geqslant \dfrac{-1+\sqrt{19}}{2}\right\}$ *M1 A1*

E2

(a) $2x(x-2) = x+1$ *M1*

$2x^2-5x-1=0$ *A1*

$x = \dfrac{5\pm\sqrt{33}}{4}$ *A1*

(b) Critical values -1, 2 and $\dfrac{5\pm\sqrt{33}}{4}$ *M1*

$-1 < x < \dfrac{5-\sqrt{33}}{4}$ **or** $2 < x < \dfrac{5+\sqrt{33}}{4}$ *A1*

(c) With $t = |x| \geqslant 0$, only $2 < t < \dfrac{5+\sqrt{33}}{4}$ is possible *M1*

$-\dfrac{5+\sqrt{33}}{4} < x < -2$ **or** $2 < x < \dfrac{5+\sqrt{33}}{4}$ *A1*

STRETCH

E3 **(a)** $\dfrac{(x-3)(x+2)-4x}{x+2} > 0$ *M1*

$\dfrac{x^2-5x-6}{x+2} = \dfrac{(x-6)(x+1)}{x+2} > 0$ *A1*

Critical values -2, -1, 6 *M1 A1*

$\boxed{-2 < x < -1 \text{ or } x > 6}$ *A1*

(b) For $x \geqslant 3$ the answer is as part (a): $x > 6$ *B1*

For $x < 3$: $3 - x > \dfrac{4x}{x+2}$, $\dfrac{x^2+3x-6}{x+2} < 0$, critical values -2 and $\dfrac{-3 \pm \sqrt{33}}{2}$ *M1 A1*

$\boxed{x < \dfrac{-3-\sqrt{33}}{2}, -2 < x < \dfrac{-3+\sqrt{33}}{2} \text{ or } x > 6}$ *A1*

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