



eClassroom

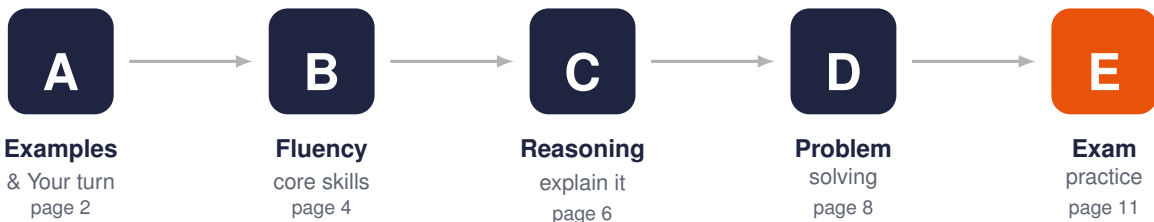
A-LEVEL MATHEMATICS • PURE 2

Coordinate Geometry

Worksheet

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Free Coordinate Geometry worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: write the equation of a circle from its centre and radius • complete the square to find the centre and the radius • find where a line meets a circle • use tangent and chord properties: the tangent is perpendicular to the radius, and the perpendicular from the centre bisects a chord • *Stretch:* use the discriminant for tangents, the angle in a semicircle, perpendicular bisectors and the distance between centres.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Tangency from the discriminant, circles touching the axes or each other, three points on a circle, and tangents from a point.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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Name

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Find the centre and the radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$

Complete the square in x and in y , then move the constants.

$$(x-3)^2 - 9 + (y+2)^2 - 4 - 12 = 0, \text{ so } (x-3)^2 + (y+2)^2 = 25$$

$$(3, -2), r = 5$$

Your turn A1

Find the centre and the radius of the circle $x^2 + y^2 + 8x - 2y + 1 = 0$

Worked example A2

A circle has centre $(1, 4)$ and passes through $(4, 8)$. Find its equation.

The radius is the distance from the centre to the point.

$$r^2 = 3^2 + 4^2 = 25$$

$$(x-1)^2 + (y-4)^2 = 25$$

Your turn A2

A circle has centre $(-2, 3)$ and passes through $(3, -9)$. Find its equation.

Worked example A3

Find the tangent to $(x-1)^2 + (y-2)^2 = 25$ at $P(4, 6)$, in the form $ax + by + c = 0$

Radius gradient $\frac{6-2}{4-1} = \frac{4}{3}$; the tangent is

perpendicular: $-\frac{3}{4}$

$$y-6 = -\frac{3}{4}(x-4)$$

$$3x + 4y - 36 = 0$$

Your turn A3

Find the tangent to $(x+2)^2 + (y-1)^2 = 20$ at $P(2, 3)$, in the form $ax + by + c = 0$



STRETCH

Worked example A4: a diameter

$A(-1, 4)$ and $B(5, 2)$ are the ends of a diameter.
Find the equation of the circle.

Centre = midpoint $(2, 3)$; radius = half of AB , or the distance from the centre to A

$$r^2 = (2+1)^2 + (3-4)^2 = 10$$

$$(x-2)^2 + (y-3)^2 = 10$$

Your turn A4

$A(-4, 1)$ and $B(2, -3)$ are the ends of a diameter.
Find the equation of the circle.

Worked example A5: a line meets a circle

Find where the line $y = 2x - 5$ meets the circle $x^2 + y^2 = 25$

Substitute the line into the circle.

$$x^2 + (2x - 5)^2 = 25, \text{ so } 5x^2 - 20x = 0, x = 0 \text{ or } 4$$

$$(0, -5), (4, 3)$$

Your turn A5

Find where the line $y = x + 2$ meets the circle $x^2 + y^2 = 10$

Worked example A6: a tangent from the discriminant

The line $y = x + k$ is a tangent to $x^2 + y^2 = 8$. Find k
 $x^2 + (x + k)^2 = 8$, so $2x^2 + 2kx + k^2 - 8 = 0$

$$\text{One point of contact: } b^2 - 4ac = 0: 4k^2 - 8(k^2 - 8) = 0$$

$$k = \pm 4$$

Your turn A6

The line $y = 2x + k$ is a tangent to $x^2 + y^2 = 5$. Find k

Worked example A7: a chord

$(x - 1)^2 + (y - 4)^2 = 25$ has a horizontal chord of length 8 above the centre. Find its equation.

*The perpendicular from the centre bisects the chord:
half-chord 4, radius 5, so the distance is 3*

$$y = 4 + 3 = 7$$

Your turn A7

$(x - 2)^2 + (y + 1)^2 = 13$ has a horizontal chord of length 6 below the centre. Find its equation.



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 Find the centre and the radius of each circle.

(a) $(x - 3)^2 + (y + 1)^2 = 16$

(b) $(x + 2)^2 + y^2 = 7$

(c) $x^2 + y^2 - 10y = 0$

(d) $x^2 + y^2 + 4x - 6y = 3$

[4 marks]

B2 Find the equation of the circle with

(a) centre (0, 0) and radius 6

(b) centre (2, -3) and radius 4

(c) centre (2, -3), passing through (5, 1)

(d) a diameter from (-1, -3) to (5, 5)

[4 marks]

B3 The circle C has equation $(x - 2)^2 + (y - 1)^2 = 25$. State whether each point is inside C , on C or outside C , giving a reason.

(a) (5, 5)

(b) (4, 3)

(c) (-3, 1)

(d) (7, -3)

[4 marks]

B4 Find the equation of the tangent to $x^2 + y^2 = 25$ at

(a) (3, 4)

(b) (-3, 4)

(c) (5, 0)

(d) (0, -5)

[4 marks]



CORE*continued: lines meeting circles***B5** Find where each line meets the circle $x^2 + y^2 = 20$

(a) $y = 2$

(b) $y = x + 2$

(c) $y = 2x + 10$

[3 marks]**B6** State the number of points where each line meets the circle $x^2 + y^2 = 9$, showing your method.

(a) $y = x$

(b) $y = x + 6$

(c) $y = 3$

(d) $y = x + 3\sqrt{2}$

[4 marks]**STRETCH****B7** $A(0, 0)$, $B(8, 0)$ and $C(0, 6)$ lie on a circle.**(a)** Find the perpendicular bisector of AB **(b)** Find the perpendicular bisector of AC **(c)** Find the centre of the circle.**(d)** Find its radius, and explain why BC is a diameter.**[4 marks]****B8** Find the values of k for which each line is a tangent to the given circle.

(a) $y = k$ and $(x - 1)^2 + (y - 2)^2 = 9$

(b) $x = k$ and $(x - 1)^2 + (y - 2)^2 = 9$

(c) $y = x + k$ and $x^2 + y^2 = 18$

(d) $y = 3x + k$ and $x^2 + y^2 = 10$

[4 marks]**B9** The circle C has equation $(x - 3)^2 + (y - 4)^2 = 25$ **(a)** Show that C passes through O , and find the length of the chord of C on the x -axis.**(b)** Find the length of the chord of C on the y -axis.**(c)** Find the distance from the centre of C to the chord on the x -axis.**(d)** The chords meet C again at A and B . Show that AB is a diameter.**[4 marks]****B10****(a)** $A(-5, 0)$, $B(5, 0)$ and $C(3, 4)$ lie on $x^2 + y^2 = 25$. Show that angle $ACB = 90^\circ$ **(b)** Find the area of triangle ABC **(c)** The triangle PQR has $P(1, 1)$, $Q(7, 1)$, $R(1, 9)$ and a right angle at P . Find the centre of the circle through P , Q and R **(d)** Find the equation of this circle.**[4 marks]**

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

- C1** Sam says the circle $x^2 + y^2 + 6x - 8y = 0$ has centre $(3, -4)$
Explain Sam's mistake, and give the correct centre and radius. [2 marks]
- C2** Lily says the circle $(x - 2)^2 + (y + 1)^2 = 20$ has radius 20
Explain Lily's mistake, and give the radius in its simplest form. [2 marks]
- C3** Kai completes the square for $x^2 + y^2 - 6x + 2y = 15$ and writes $(x - 3)^2 + (y + 1)^2 = 15$, so the radius is $\sqrt{15}$
Explain Kai's mistake, and find the correct radius. [3 marks]
- C4** Jo finds the tangent to $(x - 1)^2 + (y + 2)^2 = 25$ at $P(4, 2)$ using the gradient $\frac{4}{3}$ of the radius.
Explain Jo's mistake, and find the correct tangent in the form $ax + by + c = 0$ [3 marks]



STRETCH

- C5** Show that the circles $x^2 + y^2 = 4$ and $(x - 6)^2 + (y - 8)^2 = 64$ touch, explaining your reasoning with the values you use. [3 marks]
- C6** $A(2, 7)$ lies on the circle $(x + 1)^2 + (y - 3)^2 = 25$. Show this, and find the other end of the diameter through A [3 marks]
- C7** Max says the line $y = 7 - x$ is a tangent to $x^2 + y^2 = 25$, because it touches the circle at $(3, 4)$. Show that Max is wrong. [3 marks]
- C8** (a) Show that the circle $x^2 + y^2 - 2ax - 2by = 0$ passes through the origin for all values of the constants a and b (1)
- (b) Show that a circle with centre (a, b) , where $b > 0$, that touches the x -axis has equation $x^2 + y^2 - 2ax - 2by + a^2 = 0$ (2)
- (c) Find the equation of the circle that touches the x -axis and passes through $(0, 1)$ and $(2, 1)$ (2)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

D1 The circle C has equation

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

(a) By completing the square, find the coordinates of the centre of C and the radius of C .

(2)

(b) Show that the point $(7, 1)$ lies outside C , giving a reason based on the distance from the centre.

(1)

[3 marks]

D2 In this question you must show all stages of your working.

The circle C has equation $x^2 + y^2 - 6x + 4y - 3 = 0$

(a) Find the centre and the radius of C .

(2)

(b) Find the exact x coordinates of the points where C meets the x -axis, giving your answers in simplest surd form.

(2)

[4 marks]

D3 The points $(-3, 4)$ and $(5, 10)$ are the ends of a diameter of the circle C .

(a) Find an equation for C .

(2)

(b) Hence find the coordinates of the point on C that is nearest to the x -axis, explaining how you know.

(1)

[3 marks]

D4 The circle C has centre $(1, -2)$ and radius 5. The chord MN of C is parallel to the x -axis, has length 8 and lies below the centre.

(a) By considering the perpendicular from the centre to the chord, find an equation of the line through M and N .

(2)

(b) Find the coordinates of M and N .

(1)

[3 marks]



STRETCH

D5 A circle with radius r lies in the first quadrant and touches both coordinate axes. It passes through the point $(2, 1)$

(a) Explain why its centre is (r, r) , and show that $r^2 - 6r + 5 = 0$

(2)

(b) Hence find the equations of the two circles that satisfy these conditions, giving each in the form $(x - a)^2 + (y - b)^2 = r^2$

(2)

[4 marks]

D6 Figure 1 shows the points $A(1, 7)$, $B(7, 5)$ and $C(-1, 1)$, which lie on a circle. The centre of the circle lies on the perpendicular bisector of every chord.

(a) Find the perpendicular bisector of AB , giving your answer in the form $y = mx + c$

(2)

(b) Given that the perpendicular bisector of AC is $y = 4 - \frac{1}{3}x$, find the coordinates of the centre of the circle and an equation of the circle.

(2)

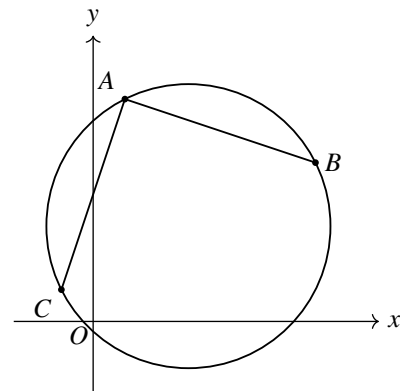


Diagram NOT accurately drawn

Figure 1

[4 marks]



D7 Figure 2 shows the circle with centre $P(2, 4)$ and radius 5, and the points $A(-2, 1)$, $B(6, 7)$ and $C(5, 0)$ on the circle.

(a) Show that AB is a diameter, and use gradients to show that angle $ACB = 90^\circ$

(2)

(b) Find the exact area of triangle ABC , showing your method clearly.

(2)

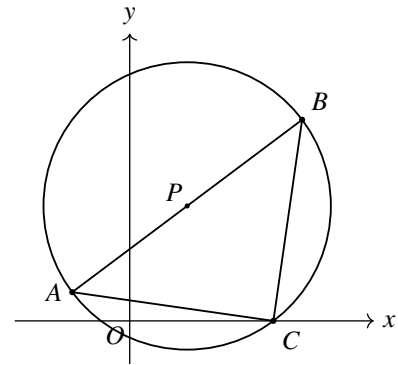


Diagram NOT accurately drawn

Figure 2

[4 marks]

D8 The circle C_1 has equation $x^2 + y^2 - 2x - 4y - 20 = 0$

The circle C_2 has centre $(9, 8)$ and radius r

(a) Find the centre and the radius of C_1

(2)

(b) Find the two values of r for which C_2 touches C_1 , explaining your reasoning. You may find it helpful to draw a sketch.

(3)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA12. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 A circle C has centre $(3, -1)$

Given that the point $P(7, 2)$ lies on C

(a) find an equation for C

.....

(2)

(b) Find the equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

.....

(3)

(Total for Question E1 is 5 marks)



E2 The circle C has equation

$$x^2 + y^2 + 6x - 4y - 12 = 0$$

(a) Find (i) the coordinates of the centre of C (ii) the radius of C

(2)

The line l with equation $y = x$ meets C at the points A and B , as shown in Figure 3.

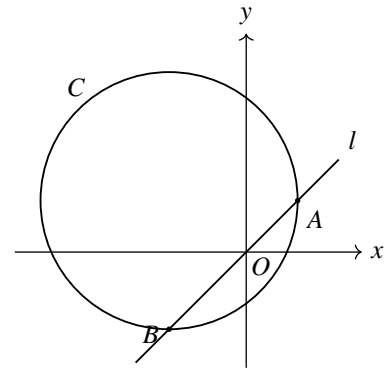


Diagram NOT accurately drawn

Figure 3

(b) Use algebra to find the coordinates of A and B .

.....

(3)

(c) Find the exact length of the chord AB , giving your answer as a simplified surd.

.....

(1)

(Total for Question E2 is 6 marks)



STRETCH

E3 The circle C has equation $x^2 + y^2 - 4x - 2y - 5 = 0$

The point T has coordinates $(6, 3)$

(a) Find the centre and the radius of C , and show that T lies outside C

(2)

A line through T with gradient m is a tangent to C

(b) Show that $3m^2 - 8m - 3 = 0$

(3)

(c) Hence show that the two tangents from T to C are perpendicular.

(2)

(d) Explain why T , the centre of C and the two points of contact form a square, and find its area.

(1)

(Total for Question E3 is 8 marks)

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