



eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA13 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	own PDF	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	own PDF	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	own PDF	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	own PDF	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	own PDF	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	2	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	own PDF	
8 Numerical Methods Locating roots, iteration	10	77	own PDF	

[Full mark scheme](#)

[Free courses](#)

[More worksheets](#)

Name

Class

Date



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

6

CHAPTER 6 • KEY SKILLS

Differentiation

Specification P3: Differentiation • 10 questions • 76 marks • about 90 minutes

1 Standard derivatives

$e^{kx} \rightarrow ke^{kx}$, $\ln x \rightarrow \frac{1}{x}$,
 $\sin kx \rightarrow k \cos kx$, $\cos kx \rightarrow -k \sin kx$,
 $\tan kx \rightarrow k \sec^2 kx$

2 The chain rule

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, e.g. $\ln(2x+1) \rightarrow \frac{2}{2x+1}$

3 The product rule

$y = uv$: $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$. Factorise the answer (e.g. take out e^{kx}).

4 The quotient rule

$y = \frac{u}{v}$: $\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ (the order matters)

5 x as a function of y

$\frac{dy}{dx} = \frac{1}{dx/dy}$. Use an identity to write the answer in terms of x .

6 Tangents and normals

Gradient of the normal = $-\frac{1}{m}$. Keep exact values, e.g. $y = 4e^{-2}(2-x)$

7 Stationary points

Solve $\frac{dy}{dx} = 0$; $e^{kx} \neq 0$, so only the other factors give solutions. Give exact coordinates.

8 Increasing and decreasing

Find the sign of $\frac{dy}{dx}$ and finish with a statement: "so g is increasing".

9 Using the derivative

Rearrange $f'(x) = 0$ into a given form, or use the maximum to compare values.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	10	10	10	10	11
1 Standard derivatives	•		•	•	•	•	•	•	•	•
2 The chain rule	•		•	•	•	•	•	•		•
3 The product rule	•			•		•	•			•
4 The quotient rule	•	•			•			•	•	
5 x as a function of y			•							•
6 Tangents and normals		•				•	•	•		•
7 Stationary points				•		•		•	•	•
8 Increasing and decreasing					•				•	
9 Using the derivative						•	•		•	
Interleaved: transformations and number of solutions (Chapter 2)						•				
Interleaved: trigonometric identities (Chapters 3, 4)			•					•		
Interleaved: e and $\ln$, exact values (Chapter 5)				•	•	•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. "Use calculus" means differentiate and set equal to zero. In "show that" questions, write the unsimplified product or quotient rule line first, then simplify.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

1. Differentiate with respect to x

(a) $y = e^{3x} \ln(2x)$, $x > 0$

(2)

(b) $y = \frac{2x+1}{\cos x}$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, simplifying your answer.

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 1 is 5 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = \frac{3x-1}{x+2}$, $x \neq -2$

(a) Show that $\frac{dy}{dx} = \frac{a}{(x+2)^2}$, where a is an integer to be found.

(2)

(b) Find an equation of the tangent to C at the point where $x = -1$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 2 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 2 is 5 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

3. The curve C has equation $x = 2\tan 3y$, $-\frac{\pi}{6} < y < \frac{\pi}{6}$

(a) Find $\frac{dy}{dx}$ in terms of y .

(2)

(b) Hence show that $\frac{dy}{dx} = \frac{2}{3(x^2 + 4)}$

(3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 3 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 3 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = xe^{-2x}$

(a) Use calculus to find the exact coordinates of the stationary point of C .

(4)

(b) Hence write down the maximum value of xe^{-2x}

(1)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Question 4 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 4 is 5 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

5.

$$g(x) = \ln(2x + 1) + \frac{4}{2x + 1} \quad x > -\frac{1}{2}$$

Find the set of values of x for which g is increasing.

(5)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Question 5 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 5 is 5 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

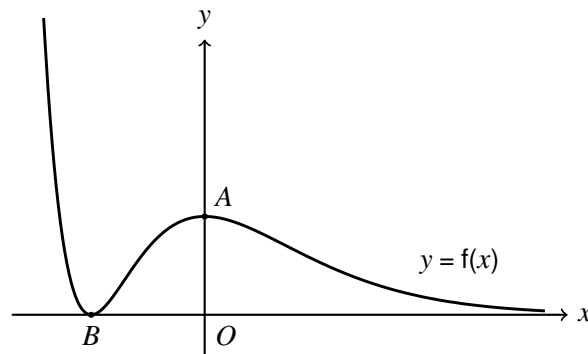


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x+1)^2 e^{-2x} \quad x \in \mathbb{R}$$

The curve has a maximum turning point at A and a minimum turning point at B .

- (a) Show that $f'(x) = -2x(x+1)e^{-2x}$ (3)
- (b) Use calculus to find the coordinates of A and B . (2)
- (c) State the y coordinate of the minimum point on the curve with equation $y = 3f(x) - 2$ (1)
- (d) Find the set of values of k for which the equation $f(x) = k$ has exactly three solutions. (2)
- (e) Find, in terms of e , an equation of the tangent to the curve at the point where $x = 1$ (2)



Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 6 is 10 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

$$f(x) = 2x \cos\left(\frac{x}{2}\right) \quad 0 < x < 2\pi$$

- (a)** Find $f'(x)$. **(2)**
- (b)** Show that the equation $f'(x) = 0$ can be written as $x = 2 \cot\left(\frac{x}{2}\right)$. **(2)**
- The normal to the curve $y = f(x)$ at the point P where $x = \pi$ meets the y -axis at the point Q .
- (c)** Find an equation of the normal at P . **(3)**
- (d)** Find the exact area of the triangle OPQ , where O is the origin. **(3)**

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 7 is 10 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Solutions relying on calculator technology are not acceptable.

$$y = \frac{1 + \sin 2x}{\cos 2x} \quad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

- (a) Show that $\frac{dy}{dx} = \frac{2}{1 - \sin 2x}$ (5)
- (b) Find an equation of the tangent to C at the point where $x = \frac{\pi}{12}$, giving your answer in the form $y = mx + c$, where m and c are exact. (3)
- (c) Explain why C has no stationary points. (2)

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{\ln x}{x} \quad x > 0$$

- (a) Find $f'(x)$. (2)
- (b) Find the exact coordinates of the stationary point of $y = f(x)$ and show that it is a maximum. (3)
- (c) Hence explain why $\frac{\ln \pi}{\pi} < \frac{1}{e}$ (2)
- (d) Hence show that $\pi^e < e^\pi$ (3)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 9 is 10 marks)



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

Solutions relying on calculator technology are not acceptable.

(a) Find $\frac{dy}{dx}$ in terms of y .

(3)

(b) Find the exact x coordinate of P and an equation of the tangent to C at P .

(4)

(c) The tangent at P crosses the x -axis at the point Q . Find the exact x coordinate of Q .

(2)

(d) Find the exact coordinates of the point on C at which the tangent is parallel to the y -axis.

(2)

Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 10 is 11 marks)**TOTAL FOR CHAPTER 6 IS 76 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk