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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 3 (WFM03)

# Further Pure 3 Practice Paper 2

Mark Scheme

**AIMING FOR A**

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM03/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

**Mark every line, not just the answer.** Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

| Code | Meaning |
|---|---|
| **M1** | Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark. |
| **dM1** | Dependent method mark: can only be given if the method mark before it has been given. |
| **A1** | Accuracy mark: for a correct answer or line of working. It depends on the method mark before it. |
| **B1** | Independent mark: for a correct result or statement, whatever method was used. |
| * | The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso). |
| **cao / cso** | Correct answer only / correct solution only, with no errors anywhere in the working. |
| **ft / oe** | Follow through from your own earlier answer / or any correct equivalent form. |

**"Solutions relying on calculator technology are not acceptable."** Questions 1 and 5 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

**Rough grade guide (out of 75):** A* **63** • A **57** • B **50** • C **43**

Based on recent WFM03 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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# What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

| Q | What it tested | Marks | Practise it | Score |
|---|---|---|---|---|
| 1 | An identity from the exponential definitions; a quadratic in $\sinh x$ | 7 | Ch 1 Hyperbolic Functions | |
| 2 | A substitution you choose yourself; limits changed to $u$ | 8 | Ch 4 Integration | |
| 3 | A determinant as a volume scale factor; the image of a line | 9 | Ch 6 Further Matrix Algebra · Ch 5 Vectors | |
| 4 | The line where two planes meet; the distance to a skew line | 9 | Ch 5 Vectors | |
| 5 | Arc length and surface area for a parametric curve | 10 | Ch 4 Integration | |
| 6 | A normal to a hyperbola; the locus of a midpoint | 10 | Ch 2 Further Coordinate Systems | |
| 7 | An unknown entry from an eigenvector; diagonalising | 11 | Ch 6 Further Matrix Algebra | |
| 8 | A reduction formula for $\cosh^n x$ with exact limits | 11 | Ch 4 Integration · Ch 3 Differentiation | |
| | **Total** | **75** | | |

**Where did the marks go?** At grade A in FP3 marks go where the paper stops telling you what to do: a substitution not given, limits not changed when the variable changes, a direction vector transformed without its point, a locus given without the part of the curve it covers, or $\mathbf{P}$ and $\mathbf{D}$ written in different orders. Find the line where your working first differs from the scheme, then practise that chapter.

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## Question 1

7 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $\cosh 2x = \frac{1}{2}\left(e^{2x}+e^{-2x}\right)$ | M1 |
| | $2\sinh^2 x = 2\times\frac{1}{4}\left(e^{2x}-2+e^{-2x}\right) = \frac{1}{2}\left(e^{2x}+e^{-2x}\right)-1$ | M1 |
| | $\boxed{\cosh 2x = 1+2\sinh^2 x}$ | A1 |
| | | (3) |
| (b) | $1+2\sinh^2 x-3\sinh x = 3,\ 2\sinh^2 x-3\sinh x-2=0$ | M1 |
| | $(2\sinh x+1)(\sinh x-2)=0,\ \sinh x = 2$ or $\sinh x = -\frac{1}{2}$ | A1 |
| | $x = \operatorname{arsinh} s = \ln\left(s+\sqrt{s^2+1}\right)$ | M1 |
| | $\boxed{x = \ln\left(2+\sqrt{5}\right),\ x = \ln\dfrac{\sqrt{5}-1}{2}}$ | A1 |
| | *Both values of* $\sinh x$ *give a solution:* sinh *takes every real value, so neither is rejected.* | |
| | | (4) |
| | | **Total for Question 1: 7 marks** |

## Question 2

8 marks

| Part | Scheme | Marks |
|---|---|---|
| | $x = 3\sinh u,\ \dfrac{dx}{du} = 3\cosh u$ | M1 |
| | $\sqrt{9\sinh^2 u+9} = 3\cosh u,$ integrand $9\cosh^2 u$ | A1 |
| | $\cosh^2 u = \frac{1}{2}(1+\cosh 2u)$ | M1 |
| | $\frac{9}{2}\left[u+\frac{1}{2}\sinh 2u\right] = \frac{9}{2}[u+\sinh u\cosh u]$ | A1 |
| | Limits $u=0$ and $u = \operatorname{arsinh}\frac{4}{3} = \ln 3$ | M1 |
| | At $u=\ln 3$: $\sinh u = \frac{4}{3}$, $\cosh u = \frac{5}{3}$ | A1 |
| | $\frac{9}{2}\left(\ln 3+\frac{20}{9}\right)$ | M1 |
| | $\boxed{10+\frac{9}{2}\ln 3}$ | A1 |
| | | (8) |
| | | **Total for Question 2: 8 marks** |

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## Question 3

9 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $k(2k-0)-1(0-1)+0$ | M1 |
| | $\boxed{\det\mathbf{M}=2k^2+1}$ | A1 |
| | $2k^2+1\geqslant 1$ for every real $k$, so $\det\mathbf{M}\neq 0$ | B1 |
| | | (3) |
| (b) | $2\left(2k^2+1\right)=38,\ k^2=9$ | M1 |
| | $\boxed{k=3}$ | A1 |
| | | (2) |
| (c) | $l:\ \mathbf{r}=(1,0,-2)+t(2,-1,1)$ | M1 |
| | $\mathbf{M}\begin{pmatrix}1\\0\\-2\end{pmatrix}=\begin{pmatrix}3\\-2\\-5\end{pmatrix}$ | M1 |
| | $\mathbf{M}\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}5\\-1\\5\end{pmatrix}$ | M1 |
| | $\boxed{\frac{x-3}{5}=\frac{y+2}{-1}=\frac{z+5}{5}}$ | A1 |
| | | (4) |
| | | **Total for Question 3: 9 marks** |

## Question 4

9 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $(1,1,-1)\times(2,-1,1)$ | M1 |
| | $=(0,-3,-3)$, direction $(0,1,1)$ | A1 |
| | A common point: adding the equations, $3x=6,\ x=2$; then $y-z=-1$ | M1 |
| | $(2,0,1)$ (or any point on $L$) | A1 |
| | $\boxed{\mathbf{r}=2\mathbf{i}+\mathbf{k}+\lambda(\mathbf{j}+\mathbf{k})}$ | A1 |
| | | (5) |
| (b) | Common perpendicular $(0,1,1)\times(1,0,2)=(2,1,-1)$ | M1 |
| | Vector between the lines: $(1,2,0)-(2,0,1)=(-1,2,-1)$ | M1 |
| | $\frac{\lvert(-1,2,-1)\cdot(2,1,-1)\rvert}{\sqrt{6}}=\frac{1}{\sqrt{6}}$ | M1 |
| | $\boxed{\frac{\sqrt{6}}{6}}$ | A1 |
| | | (4) |
| | | **Total for Question 4: 9 marks** |

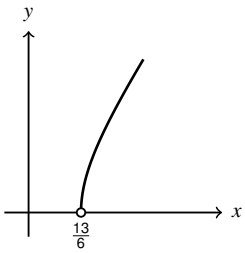
## Question 5

10 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $\frac{dx}{dt} = 6t^2,\ \frac{dy}{dt} = 6t$ | M1 |
| | $\sqrt{36t^4 + 36t^2} = 6t\sqrt{1+t^2}$ | M1 |
| | $\int_0^1 6t\sqrt{1+t^2}\,dt = \left[2\left(1+t^2\right)^{3/2}\right]_0^1$ | A1 |
| | $4\sqrt{2}-2$ | A1 |
| | | (4) |
| (b) | $S = 2\pi\int y\,ds = 2\pi\int_0^1 3t^2 \times 6t\sqrt{1+t^2}\,dt$ | M1 |
| | $= 36\pi\int_0^1 t^3\sqrt{1+t^2}\,dt$ | A1 |
| | $u = 1+t^2,\ du = 2t\,dt,\ t^2 = u-1,$ limits 1 and 2 | M1 |
| | $18\pi\int_1^2 (u-1)u^{1/2}\,du$ | A1 |
| | $18\pi\left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^2 = 18\pi\left(\frac{8\sqrt{2}}{5} - \frac{4\sqrt{2}}{3} + \frac{4}{15}\right)$ | M1 |
| | $\frac{24\pi}{5}\left(\sqrt{2}+1\right)$ | A1 |
| | *The factor $2\pi$ and the square root are the marks most often lost. No end discs are added: the surface is the curved part only.* | |
| | | (6) |
| | **Total for Question 5: 10 marks** | |

## Question 6

10 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $\frac{dy}{dx} = \frac{2\cosh t}{3\sinh t}$ | M1 A1 |
| | Normal gradient $-\frac{3\sinh t}{2\cosh t}$ | M1 |
| | $y - 2\sinh t = -\frac{3\sinh t}{2\cosh t}(x - 3\cosh t)$ | M1 |
| | $3x\sinh t + 2y\cosh t = 13\sinh t\cosh t$ | A1 |
| | | (5) |
| (b) | $A\left(\frac{13}{3}\cosh t, 0\right)$ | B1 |
| | $B\left(0, \frac{13}{2}\sinh t\right)$ | B1 |
| | | (2) |
| (c) | Midpoint $\left(\frac{13}{6}\cosh t, \frac{13}{4}\sinh t\right)$ | M1 |
| | $\cosh^2 t - \sinh^2 t = 1$: $\left(\frac{6x}{13}\right)^2 - \left(\frac{4y}{13}\right)^2 = 1$ | M1 |
| | $36x^2 - 16y^2 = 169$, with $x > \frac{13}{6}$, $y > 0$ | |
| |   | A1 |
| | *$t > 0$ gives $\cosh t > 1$ and $\sinh t > 0$: the locus is only part of one branch.* | |
| | | (3) |
| | | **Total for Question 6: 10 marks** |

## Question 7

11 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $\mathbf{A}\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}4+a\\2+2a\\4+a\end{pmatrix}$ | M1 |
| | $4+a = 2+2a$ | M1 |
| | $a = 2$, eigenvalue 6 | A1 |
| | | (3) |
| (b) | $\det(\mathbf{A}-\lambda\mathbf{I})$ expanded | M1 |
| | $-\lambda^3+8\lambda^2-12\lambda = 0$ | A1 |
| | $-\lambda(\lambda-6)(\lambda-2) = 0$, using $\lambda = 6$ | M1 |
| | $\lambda = 2,\ \lambda = 0$ | A1 |
| | | (4) |
| (c) | $\lambda = 2$: $(1, 0, -1)$; $\lambda = 0$: $(1, -2, 1)$ | M1 A1 |
| | Normalise: $\frac{1}{\sqrt{3}}(1,1,1)$, $\frac{1}{\sqrt{2}}(1,0,-1)$, $\frac{1}{\sqrt{6}}(1,-2,1)$ | M1 |
| | $\mathbf{P} = \begin{pmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}}\end{pmatrix}$, $\mathbf{D} = \begin{pmatrix}6&0&0\\0&2&0\\0&0&0\end{pmatrix}$ | A1 |
| | *The columns of $\mathbf{P}$ must be in the same order as the eigenvalues in $\mathbf{D}$.* | |
| | | (4) |
| | | **Total for Question 7: 11 marks** |

## Question 8

11 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $I_n = \int \cosh^{n-1} x \cosh x \, \mathrm{d}x$, by parts | M1 |
| | $= \left[\sinh x \cosh^{n-1} x\right] - (n-1)\int \sinh^2 x \cosh^{n-2} x \, \mathrm{d}x$ | M1 A1 |
| | $\sinh^2 x = \cosh^2 x - 1$: $I_n = [\,] - (n-1)(I_n - I_{n-2})$ | M1 |
| | $x = \ln 2$: $\sinh x = \frac{3}{4}$, $\cosh x = \frac{5}{4}$; $x = 0$: $\sinh x = 0$ | M1 |
| | $nI_n = \frac{3}{4}\left(\frac{5}{4}\right)^{n-1} + (n-1)I_{n-2}$ | A1* |
| | | (6) |
| (b) | $I_0 = \ln 2$ | B1 |
| | $2I_2 = \frac{3}{4} \times \frac{5}{4} + I_0$ | M1 |
| | $I_2 = \frac{15}{32} + \frac{1}{2}\ln 2$ | A1 |
| | $4I_4 = \frac{3}{4}\left(\frac{5}{4}\right)^3 + 3I_2 = \frac{375}{256} + \frac{45}{32} + \frac{3}{2}\ln 2$ | M1 |
| | $\boxed{I_4 = \frac{735}{1024} + \frac{3}{8}\ln 2}$ | A1 |
| | | (5) |
| | | Total for Question 8: 11 marks |

**TOTAL FOR PAPER: 75 MARKS**

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