GCSE Mathematics

Circles & Circle Theorems

Worked Solutions

Pearson Edexcel GCSE & iGCSE Mathematics

Section A — Foundation — Worked Solutions

[Fluency] Question 1

$\frac{45}{360} \times 2\pi \times 8 = \frac{1}{8} \times 16\pi = 2\pi$

$\therefore$ $\approx$ **6.28 cm**

[Fluency] Question 2

$\frac{120}{360} \times \pi \times 100 = \frac{100\pi}{3}$

$\therefore$ $\approx$ **104.7 cm²**

[Fluency] Question 3

The tangent is perpendicular to the radius at the point of contact.

$\therefore$ **90°**

[Fluency] Question 4

Angle at circumference = angle at centre ÷ 2 = 140 ÷ 2

$\therefore$ **70°**

[Fluency] Question 5

Angle in a semicircle theorem: angle subtended by diameter = 90°

$\therefore$ **APB = 90°**

[Reasoning] Question 6

(a) Opposite angles: A + C = 180 $\rightarrow$ C = 93°

(b) B + D = 180 $\rightarrow$ D = 70°

$\therefore$ **(a) 93° (b) 70°**

[Reasoning] Question 7

Angle OAP = 90° (tangent $\perp$ radius)

$$OP = \sqrt{OA^2 + PA^2} = \sqrt{36 + 64} = \sqrt{100}$$

$\therefore$ **OP = 10 cm**

[Reasoning] Question 8

Angles in same segment equal: x = 2x – 30

30 = x

∴ **x = 30°**

[Problem Solving] Question 9

Half-chord = 5 cm

$$d = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144}$$

∴ **12 cm**

[Problem Solving] Question 10

Angle OPT = 90°

$$OT = \sqrt{6^2 + 8^2} = \sqrt{100}$$

∴ **OT = 10 cm**

Section B — Higher — Worked Solutions

[Fluency] Question 1

3x + (2x+20) = 180 → 5x = 160 → x = 32

∴ **x = 32; angles 96° and 84°**

[Fluency] Question 2

(a) $d = \sqrt{13^2 - 5^2} = 12$ cm

(b) $\sin(\theta/2) = 5/13 \Rightarrow \theta/2 = 22.6° \Rightarrow \theta = 45.2°$

∴ **(a) 12 cm (b) ≈ 45.2°**

[Fluency] Question 3

Alternate segment theorem

∴ **48°**

[Reasoning] Question 4

OAP = OBP = 90°. Angles in quad: 90+90+50+AOB=360 → AOB=130°

(b) 90+90+50+130=360° → OAPB is cyclic ✓

∴ **(a) AOB = 130° (b) Shown ✓**

[Reasoning] Question 5

OA=OB=OP (radii) → triangles OAP and OBP isosceles.

Let ∠OAP=∠OPA=α and ∠OBP=∠OPB=β.

∠AOB = 2α+2β = 2(α+β) = 2∠APB ✓

∴ **Angle at centre = 2 × angle at circumference ✓**

[Reasoning] Question 6

(a) x = 70÷2 = 35°

(b) Reflex = 360 – 70 = 290°

(c) Major arc angle = reflex/2 = 145°

∴ **(a) 35° (b) 290° (c) 145°**

[Problem Solving] Question 7

Arc AB subtends 2∠D at centre (D on minor arc) and 2∠B (B on major arc).

2∠D + 2∠B = 360° → ∠B + ∠D = 180° ✓

∴ **Opposite angles sum to 180° ✓**

[Problem Solving] Question 8

OA=OB (radii), triangle OAB isosceles → ∠OBA = ∠OAB = 25°

∠AOB = 180–25–25 = 130°

∠ACB = 130 ÷ 2 = 65°

∴ **∠ACB = 65°**

[Problem Solving] Question 9

(a) Angle at circumference = 80 ÷ 2 = 40°

(b) By alternate segment theorem, this equals the angle between tangent at A and chord AB.

∴ **(a) 40° (b) Alternate segment theorem ✓**

[Problem Solving] Question 10

OA ⊥ PA and OB ⊥ PB (tangent ⊥ radius).

In triangles OAP and OBP: OA=OB (radii), OP common, ∠OAP=∠OBP=90°

By RHS: OAP ≅ OBP → PA = PB ✓

∴ **PA = PB ✓**